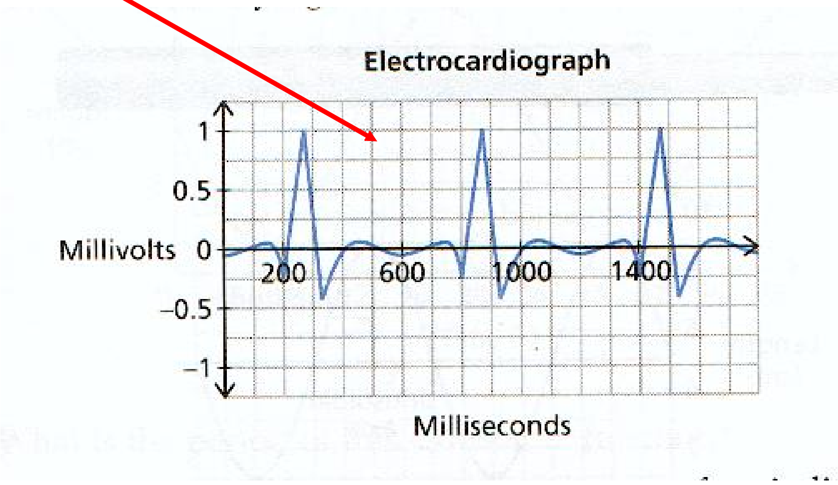
Sinusoidal Relations

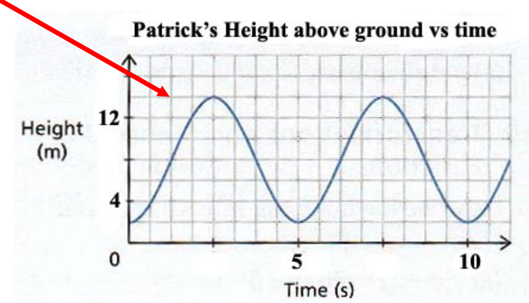
Periodic Function: A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

Example of periodic behavior

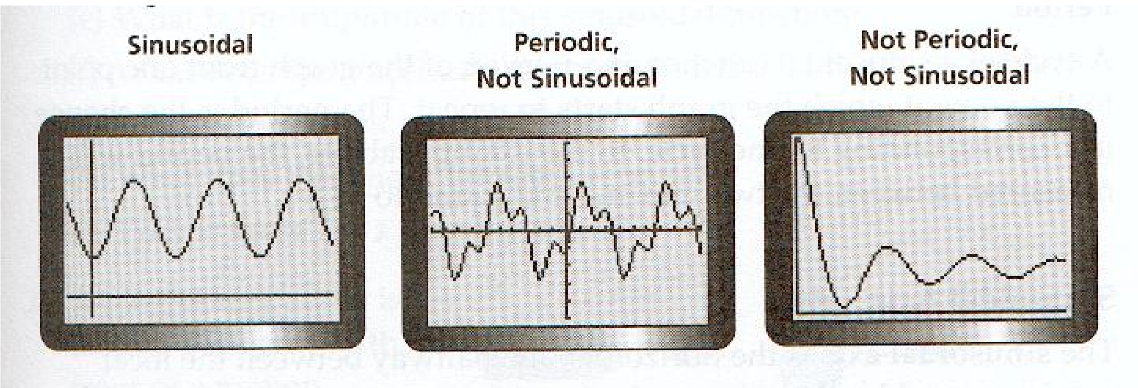


Sinusoidal Function: A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

Example of sinusoidal behavior

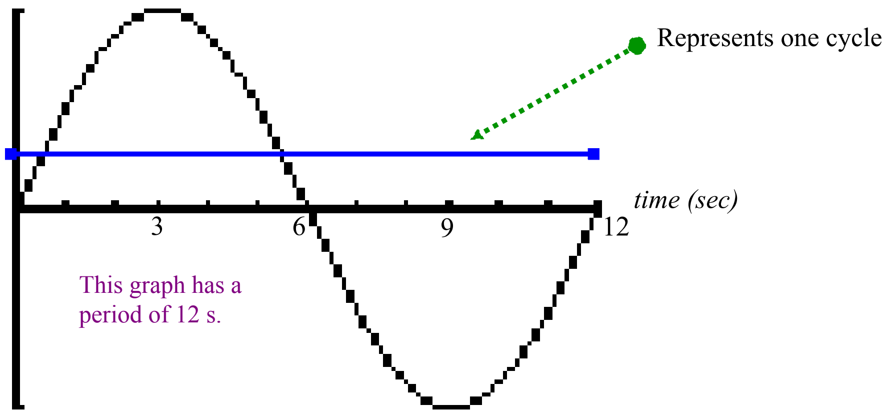


These illustrations should summarize periodic and sinusoidal...

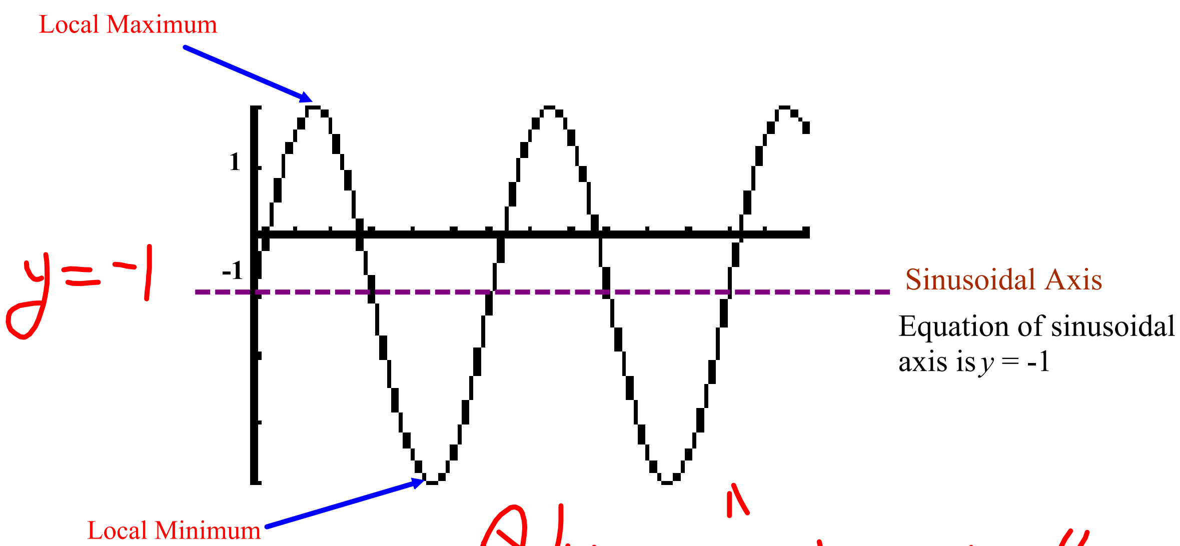


Vocabulary of Sinusoidal Functions

I. Period: The change in x corresponding to one cycle.

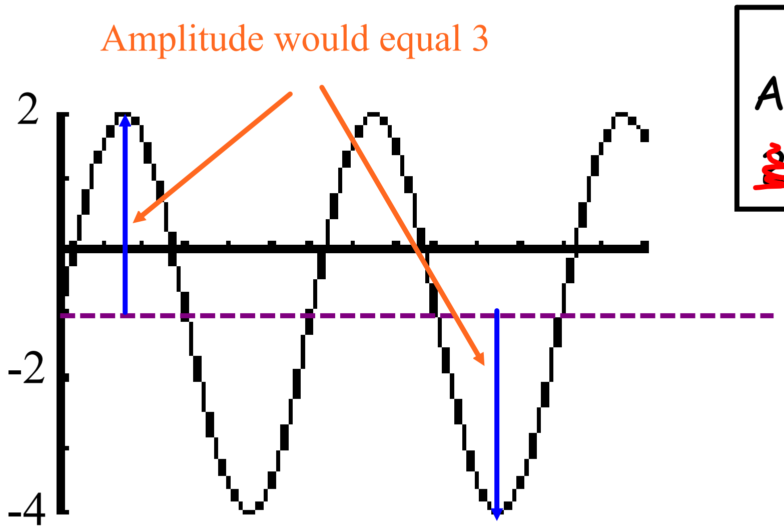


II. Sinusoidal Axis: The horizontal line halfway between the local maximum and local minimum.



Always "$\underline{y = \#}$"

III. Amplitude:
The vertical distance from the sinusoidal axis to a local maximum or local minimum.

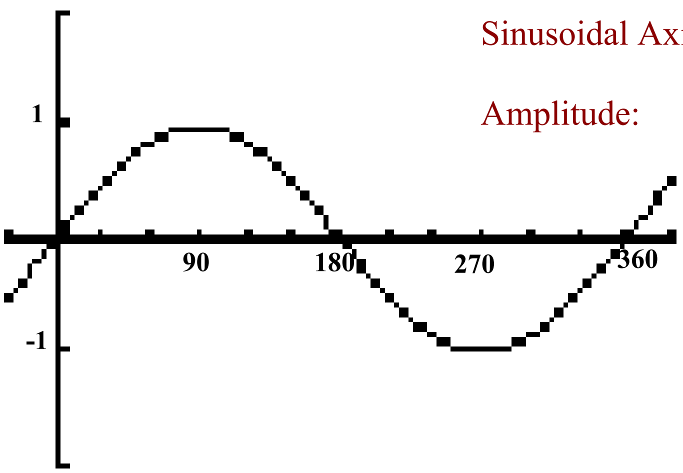


$$\boxed{\text{Amp} = \frac{\text{Max} - \text{Min}}{2}}$$

$$\frac{2-(-4)}{2} = \frac{6}{2} = 3$$

Summarize...

Here is the graph of $y = \sin\theta$



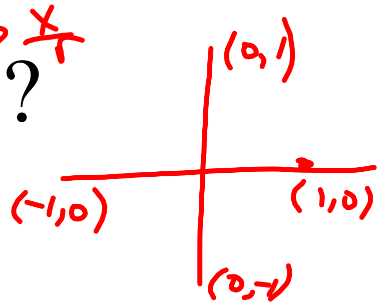
Period : $360°$

Sinusoidal Axis: $y = 0$

Amplitude: 1

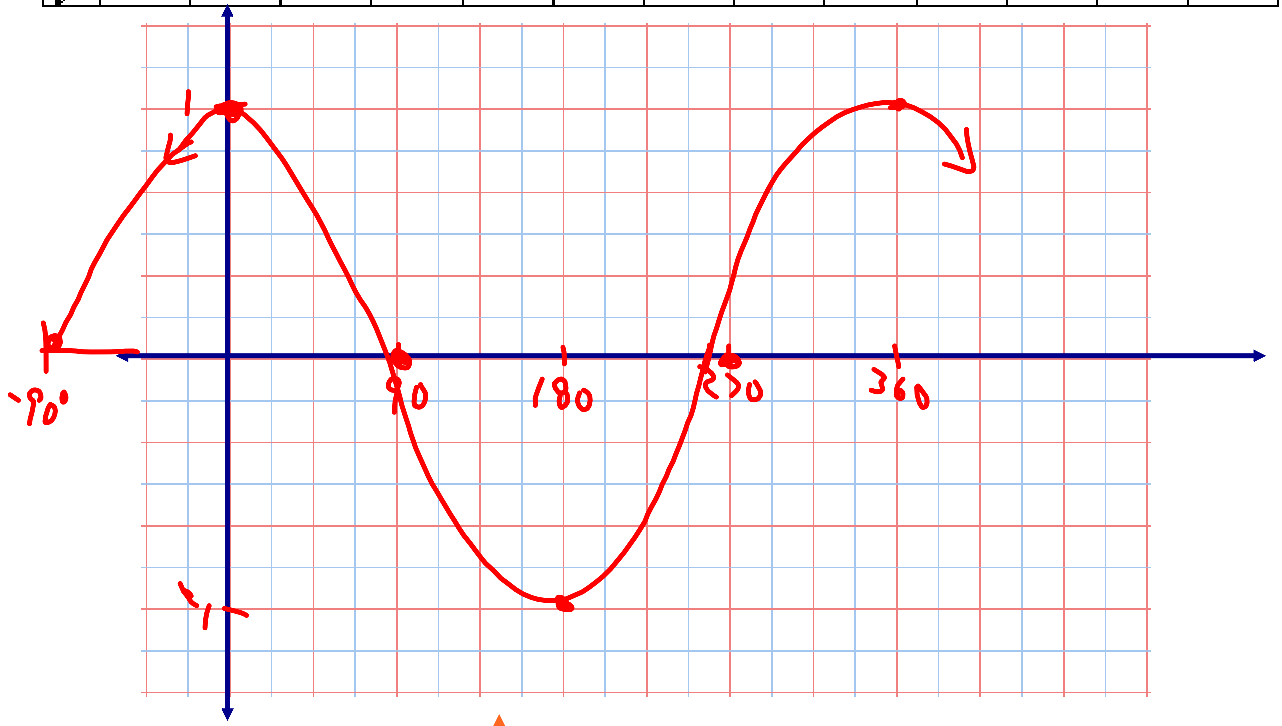
What about $y = \cos\theta$?

$\frac{x}{r}$



Complete the table of values and sketch below

θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	1			0			-1			0			1



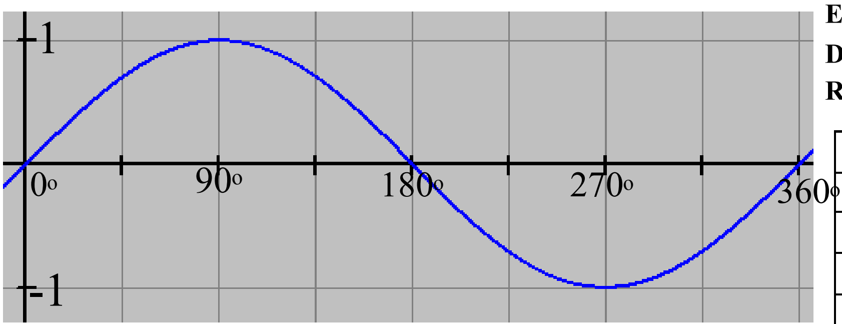
Is this a sinusoidal function?

What about the period, sinusoidal axis, and amplitude?

Per = 360°
S.Axis = y = 0
Amp = 1

Basic Trig Graphs

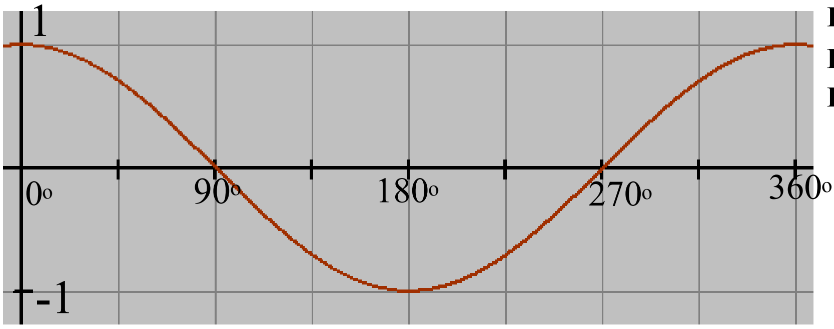
y = sin θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R }
Range: { -1 ≤ y ≤ 1 } [-1, 1]

θ	y
0°	0
90°	1
180°	0
270°	−1
360°	0

y = cos θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R }
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	1
90°	0
180°	−1
270°	0
360°	1

Transformations of the Sinusoidal Function

Recall...

$$y = -2(x-3)^2 + 4$$

reflection in the x-axis

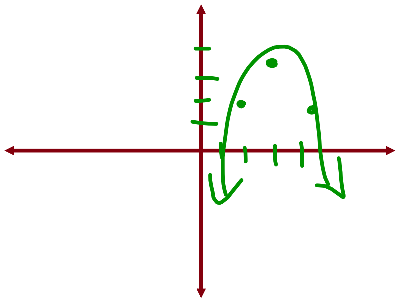
vertical stretch factor

horizontal translation

vertical translation

Vertex $\Rightarrow$ (3,4)

Sketch $\Rightarrow$ 2



Now, let's look at a sinusoidal function...

$$y = -2\sin[3(\theta - 60°)] - 1$$

Sinusoidal Axis

Reflection in x-axis

Amplitude (v. stretch)

Horizontal Stretch (Adjusts Period) *Always Reciprocal $\Rightarrow \frac{1}{3}$ of Period

Phase Shift (h. translation)

Vertical Translation

Amp = 2

Per = $\frac{1}{3}$ of 360° = 120°

Sinusoidal Axis = y = -1

Domain = $\theta \in R$

Range = $-3 \leq y \leq 1$

Phase Shift = 60° Right

Vert. Shift = Down 1

Sketching Sinusoidal Functions using Transformations

Development of a standard form for sinusoidal functions...

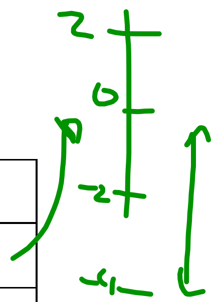
Standard Form ⟶ $f(\theta) = a\sin[k(\theta - c)] + d$

1. Reflection: If a < 0 the graph will be reflected in the *x*-axis.
2. Amplitude: The amplitude of the graph will be equal to | a | .
3. Period: The period of the graph will be equal to $\boxed{\frac{360°}{k} = \text{Per}}$
4. Horizontal Phase Shift: The graph will shift "c" units to the right. (Think Opposite)
5. Vertical Translation: The graph will shift "d" units up.

Mapping Notation: $(x, y) \rightarrow \left(\frac{1}{k}\theta + c, ay + d\right)$

Transformations of Sinusoidal Functions

Example: $f(\theta) = -2\sin 3(\theta + 30°) - 2$



Domain	$\theta \in R$
Range	$-4 \leq y \leq 0$
Reflection	yes, In x-axis
Amplitude	2
Horizontal Phase Shift	30° Left
Vertical Translation	Down 2
Period	$\frac{360°}{3} = 120°$

Sinusoidal Axis: $y = -2$

EXAMPLE #1

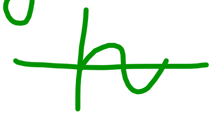
Now let's sketch a graph of $f(\theta) = -2\sin 3(\theta + 30°) - 2$

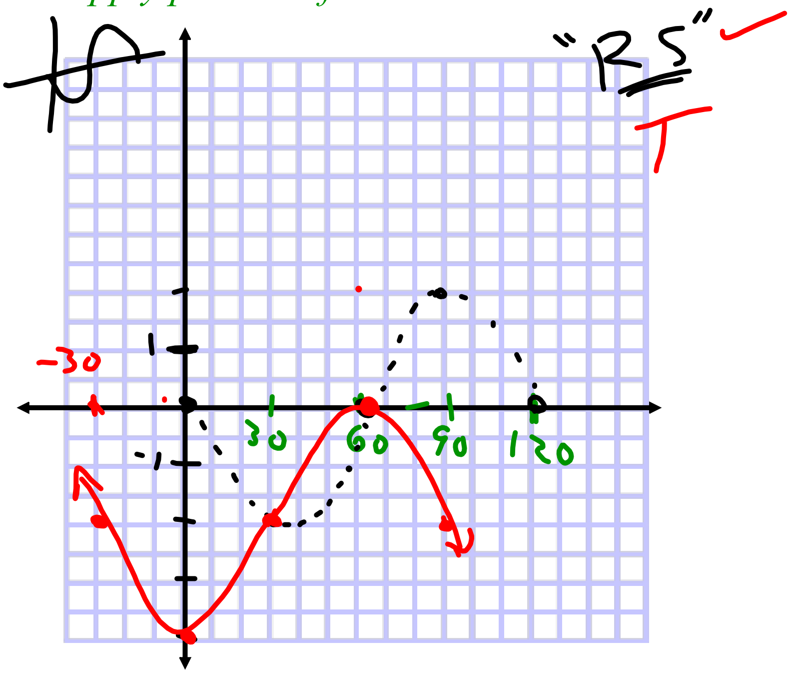
" THINK: RST "

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*

$y = \sin\theta$





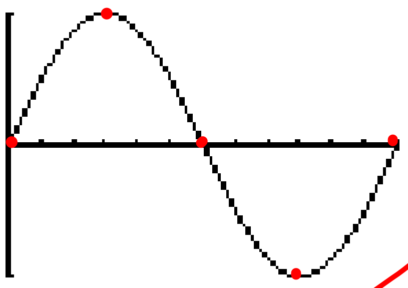
DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

Check our graph using a graphing calculator

This time we will graph the same function using a mapping:

$$f(\theta) = -2\sin 3(\theta + 30°) - 2$$

$y = \sin\theta$



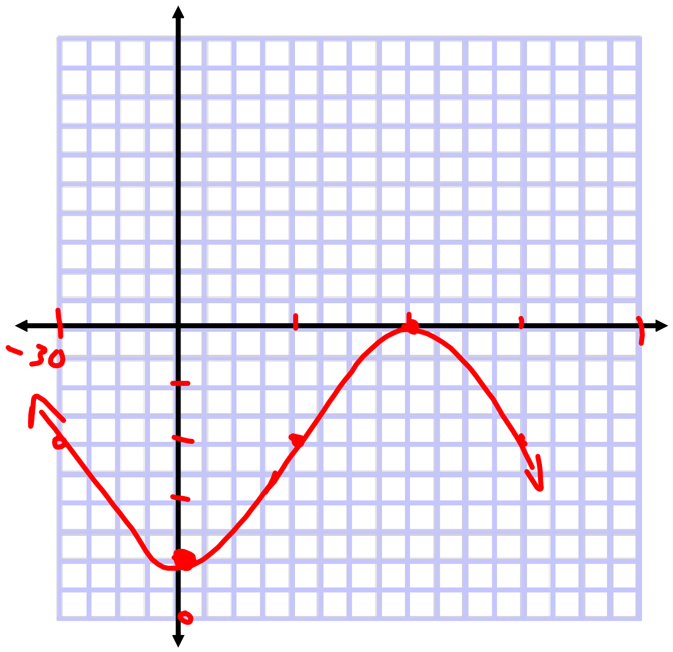
Mapping:

$(x, y) \rightarrow \left(\frac{1}{3}\theta - 30°, -2y - 2\right)$

θ	y
0	0
90	1
180	0
270	-1
360	0

New points after mapping

θ	y
-30	-2
0	-4
30	-2
60	0
90	-2

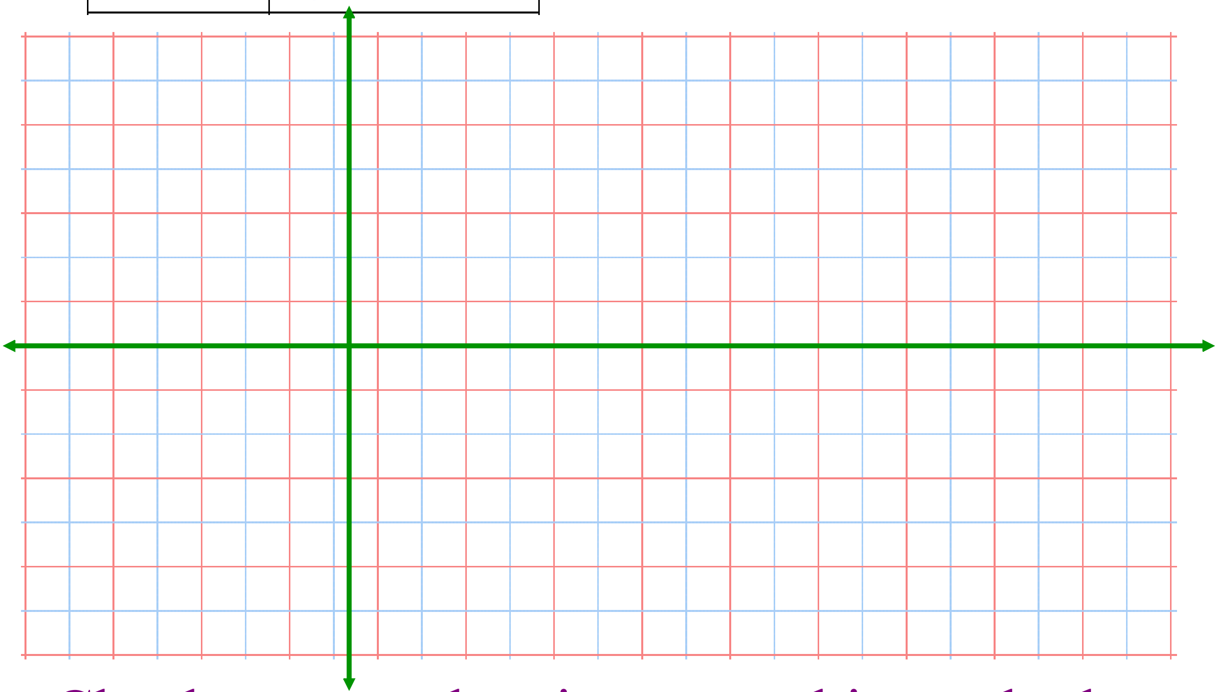


EXAMPLE #2

Now let's sketch a graph of $y = 3\cos[2(\theta - 135^\circ)] + 2$

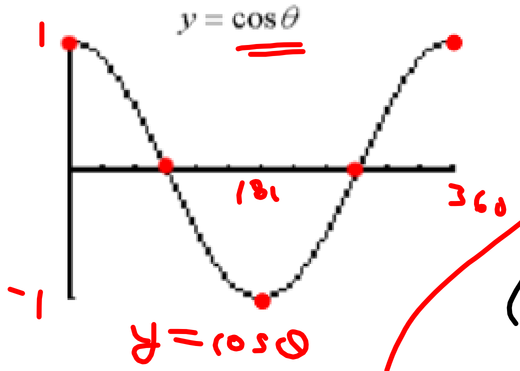
DOMAIN	$\theta \in R$
RANGE	$-1 \leq y \leq 5$
AMPLITUDE	3
PERIOD	180°
PHASE SHIFT	135° Rt.
VERTICAL TRANSLATION	Up 2
EQUATION OF SINUSOIDAL AXIS	$y = 2$

$Per = \frac{360^\circ}{K} = \frac{360^\circ}{2}$



Check our graph using a graphing calculator

$$y = 3\cos[2(\theta - 135^\circ)] + 2$$



Mapping:

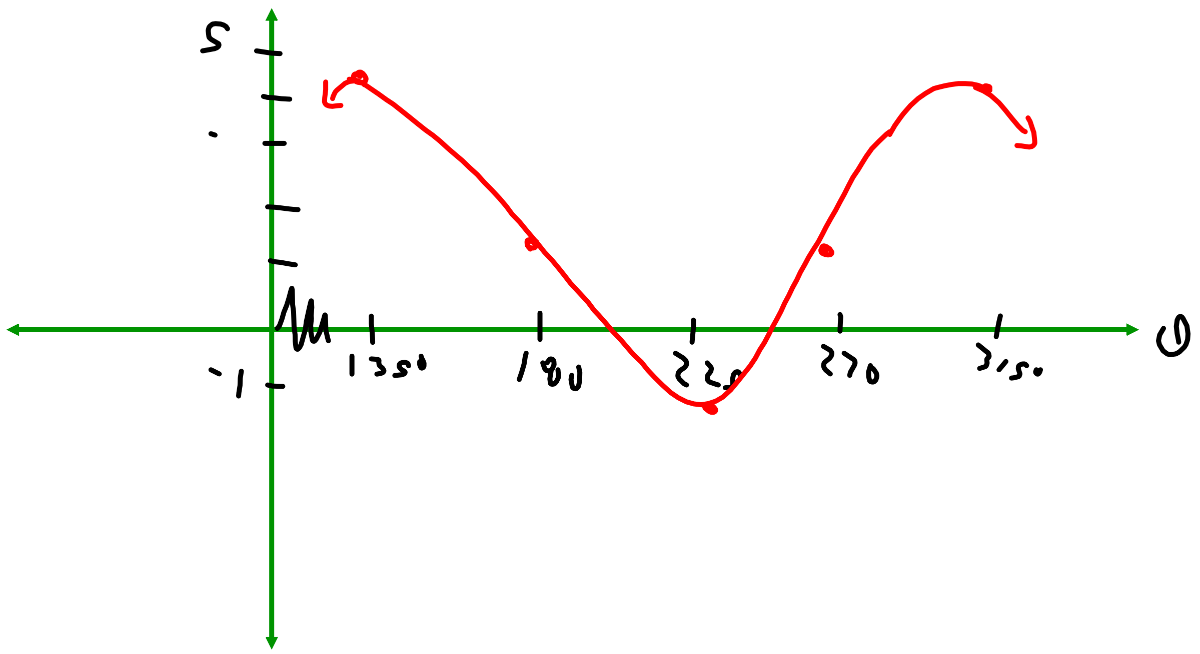
$$(x, y) \rightarrow \left(\frac{1}{2}x + 135^\circ, 3y + 2\right)$$

$$(0, 1) \rightarrow \left(\frac{1}{2}(0) + 135^\circ, 3(1) + 2\right)$$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping →

θ	y
135°	5
180°	2
225°	-1
270	2
315	5



Hopefully you are not too puzzled for this one...

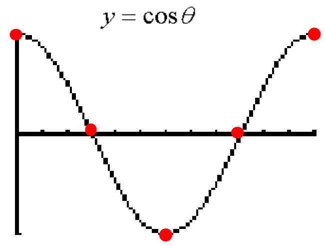
$$(2)\ \frac{1}{\cancel{2}}(y+1) = \overset{(2)}{3}\cos\left(\frac{1}{2}\theta - 90^\circ\right) + 2\ (2)$$

Remember...Put in standard form first!!

$$y + \overset{-1}{1} = 6\cos\left[\frac{1}{2}(\theta - 180^\circ)\right] + 4\ {-1}$$

$$y = 6\cos\left[\frac{1}{2}(\theta - 180^\circ)\right] + 3$$

Remember what the graph of cosine looks like ??



$$y = 6\cos\left[\frac{1}{2}(\theta - 180^\circ)\right] + 3$$

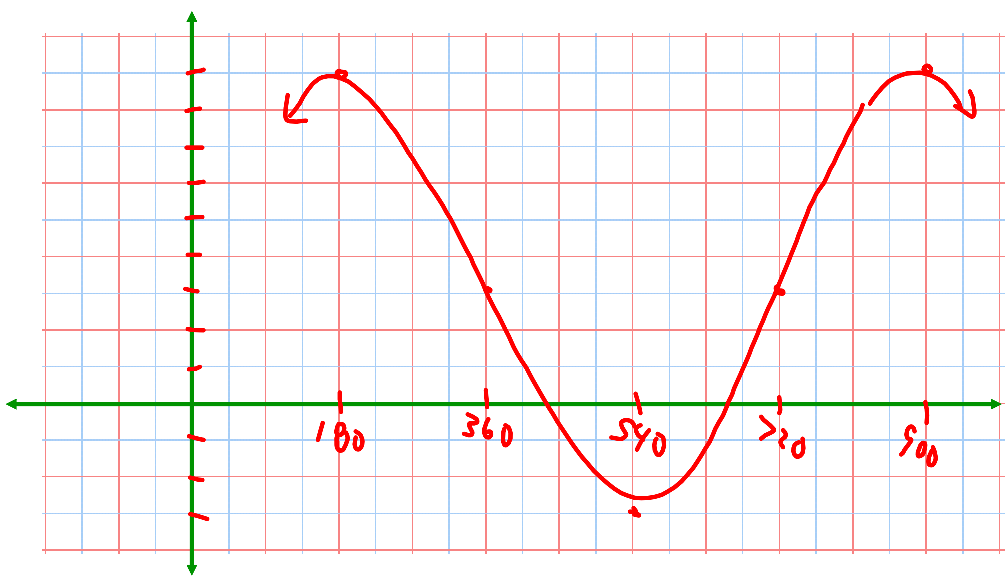
Mapping:
$(x, y) \rightarrow (2x + 180°, 6y + 3)$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
180	9
360	3
540	-3
720	3
900	9

DOMAIN	$\theta \in R$
RANGE	$-3 \leq y \leq 9$
AMPLITUDE	6
PERIOD	720°
PHASE SHIFT	180° Right
VERTICAL TRANSLATION	Up 3
EQUATION OF SINUSOIDAL AXIS	$y = 3$



Warm Up

Given the sinusoidal relation $f(\theta) = 5\cos(2\theta + 80°) - 2$

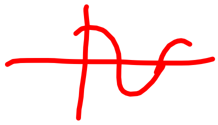
$= 5\cos(2(\theta+40°)) - 2$

Complete the chart shown below:

DOMAIN	$\theta \in R$
RANGE	$-7 \leq y \leq 3$
AMPLITUDE	5
PERIOD	180°
PHASE SHIFT	40° Left
VERTICAL TRANSLATION	Down 2
EQUATION OF SINUSOIDAL AXIS	$y = -2$

Mapping:

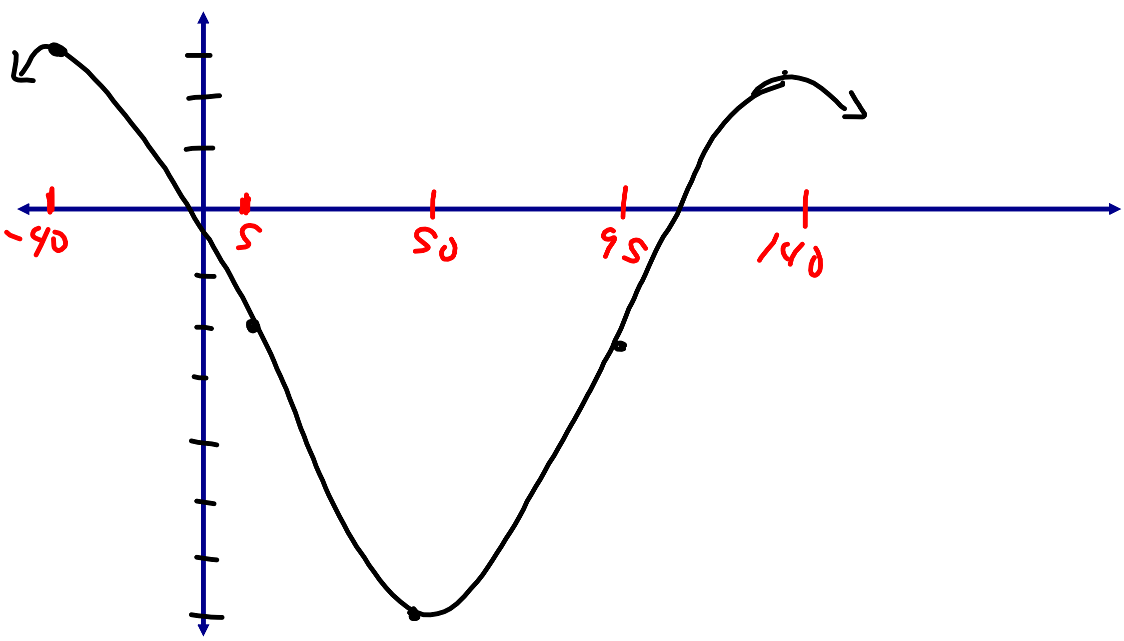
$(x, y) \rightarrow (\frac{1}{2}x - 40°, 5y - 2)$



θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping →

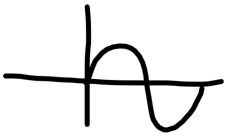
θ	y
-40	3
5	-2
50	-7
95	-2
140	3



Example...

$2(\theta+\frac{\pi}{2})$

Graph the equation $y=-3\sin(2\theta+\pi)+1$ using mapping notation.



$(x,y)\to\left(\frac{1}{2}x-\frac{\pi}{2},\ -3y+1\right)$

Period $=\frac{2\pi}{K}$

θ	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$-\frac{2\pi}{4}$

$-\frac{\pi}{4}$

0

$\frac{\pi}{4}$

$\frac{2\pi}{4}$

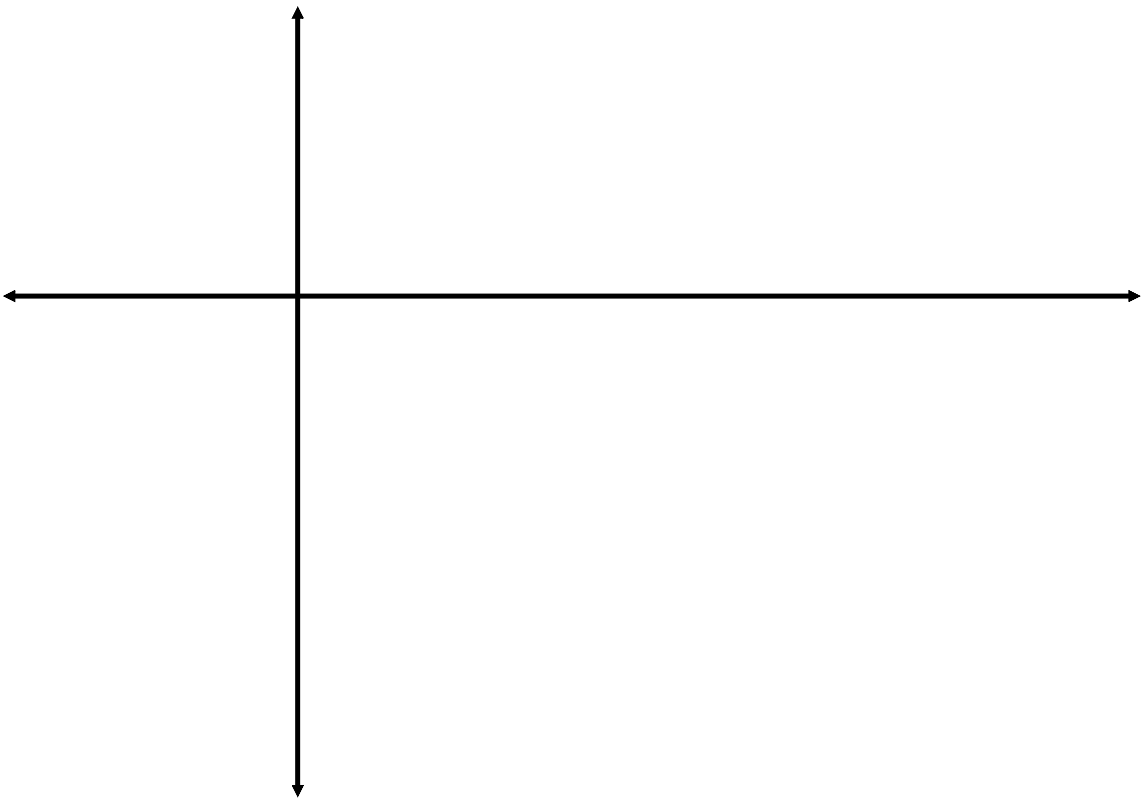
θ	y
$-\frac{\pi}{2}$	1
$-\frac{\pi}{4}$	-2
0	1
$\pi/4$	4
$\pi/2$	1

$\frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{\pi}{2}$

$\frac{\pi}{4}-\frac{1\pi}{2}$

$\frac{\pi}{4}-\frac{2\pi}{4}$

$-\frac{\pi}{4}$



Homework

Worksheet - Sketching Trigonometric Functions.doc

Questions from the homework???

Worksheet Solns - Sketching Sinusoidal Relations.doc

Attachments

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc