

One of the main food sources for the Arctic fox is the lemming. Suppose the population,  $L$ , of lemmings in the region is modelled by the function

$$L(t) = 5000 \sin \frac{\pi}{12}(t - 12) + 10\,000.$$

- (a) Determine the lemming population during year 15 of this study.
- (b) Determine the second instance that the lemming population reached 8000.

(a)  $L(15) = 5000 \sin \left[ \frac{\pi}{12}(15-12) \right] + 10\,000$  Radians

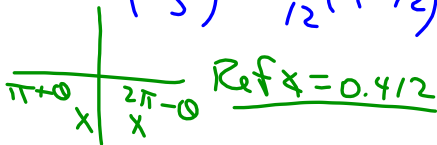
$$= 13\,536 \text{ lemming}$$

(b)  $8000 = 5000 \sin \left[ \frac{\pi}{12}(t-12) \right] + 10\,000$

$$\frac{-2000}{5000} = \frac{5000 \sin \left[ \frac{\pi}{12}(t-12) \right]}{5000}$$

$$-\frac{2}{5} = \sin \left[ \frac{\pi}{12}(t-12) \right]$$

$$\sin^{-1} \left( \frac{2}{5} \right) = \frac{\pi}{12}(t-12)$$



<p><u>Q3</u></p> $\pi + 0.412 = \frac{\pi}{12}(t-12)$ $\frac{12(\pi + 0.412)}{\pi} + 12 = t$ $t = 25.6 \text{ years}$	<p><u>Q4</u></p> $2\pi - 0.412 = \frac{\pi}{12}(t-12)$ $\frac{12(2\pi - 0.412)}{\pi} + 12 = t$ $t = 37.4 \text{ years}$
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What is the Period??

$$\text{Per} = \frac{2\pi}{K} \dots K = \frac{\pi}{12}$$

$$\text{Per} = \frac{2\pi}{\left(\frac{\pi}{12}\right)} = 2\pi \cdot \frac{12}{\pi} = 24$$

$25.6 - 24 = 1.6$   
 $t_1 = 1.6 \text{ years}$

$37.4 - 24 = 13.4$   
 $t_2 = 13.4 \text{ years}$

What about graphs of other  
trigonometric functions ???

### Graph the Tangent Function

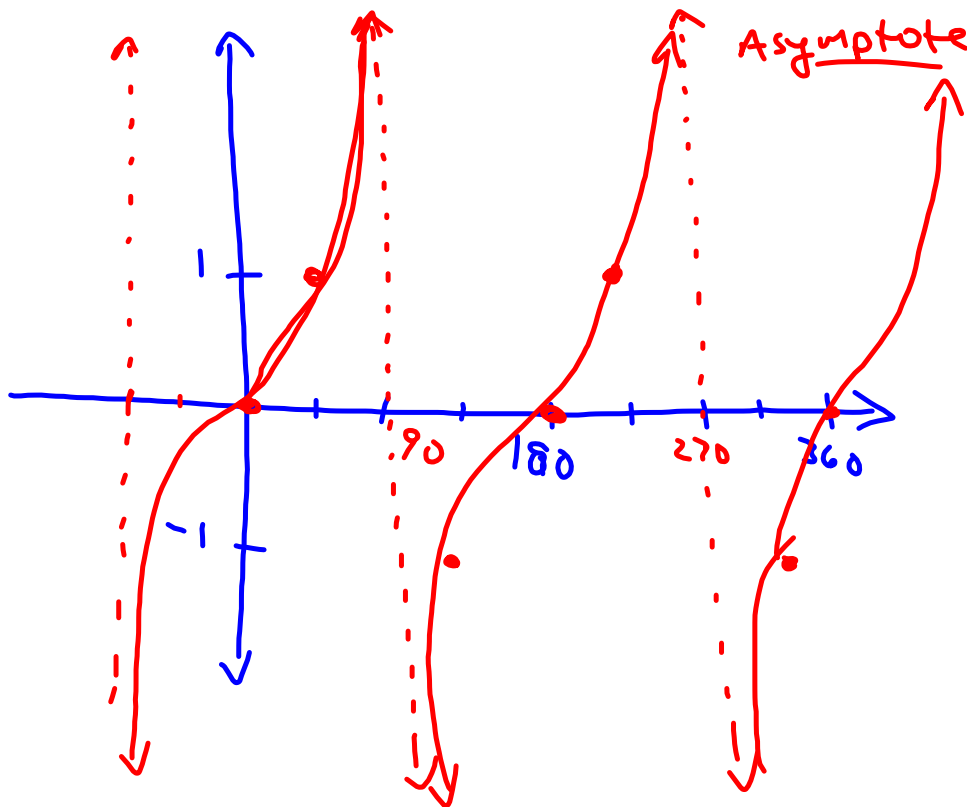
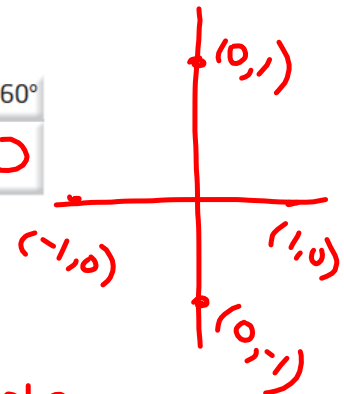
Graph the function  $y = \tan \theta$  for  $-2\pi \leq \theta \leq 2\pi$ . Describe its characteristics.

$$\tan \theta = \frac{y}{x}$$



Angle Measure	0°	45°	90°	135°	180°	225°	270°	315°	360°
y-coordinate on Tangent Line	0	1	<u>undefined</u>	-1	0	1	<u>undefined</u>	-1	0

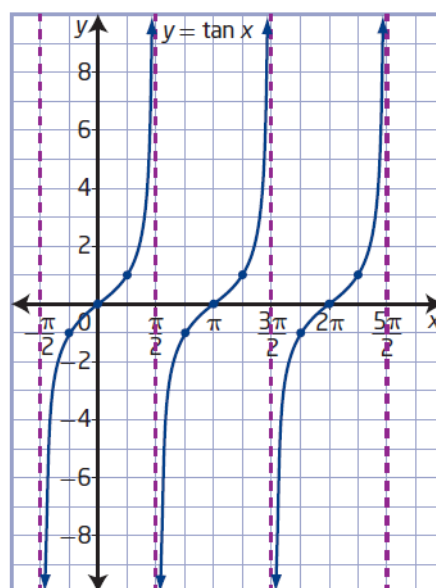
Asymptote line



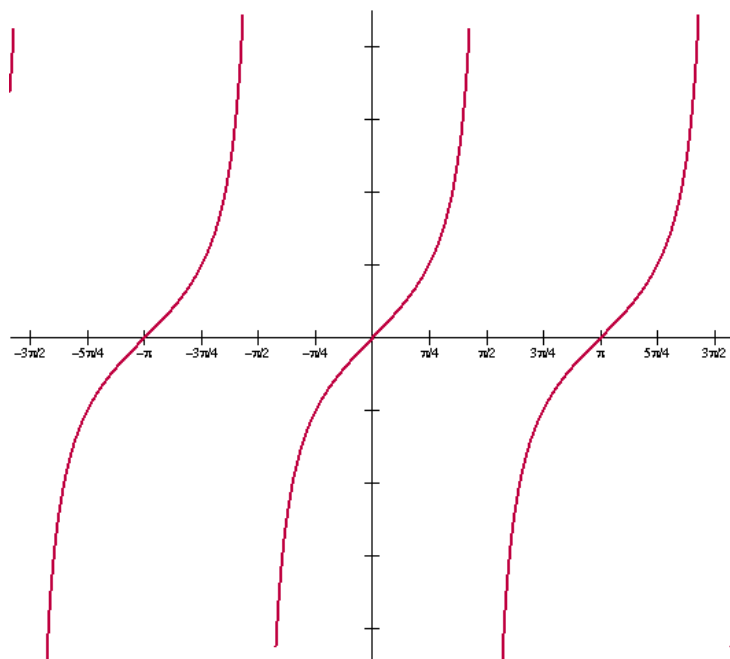
## Key Ideas

- You can use asymptotes and three points to sketch one cycle of a tangent function. To graph  $y = \tan x$ , draw one asymptote; draw the points where  $y = -1$ ,  $y = 0$ , and  $y = 1$ ; and then draw another asymptote.
- The tangent function  $y = \tan x$  has the following characteristics:
  - The period is  $\pi$ .
  - The graph has no maximum or minimum values.
  - The range is  $\{y \mid y \in \mathbb{R}\}$ .
  - Vertical asymptotes occur at  $x = \frac{\pi}{2} + n\pi$ ,  $n \in \mathbb{I}$ .
  - The domain is  $\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in \mathbb{R}, n \in \mathbb{I}\}$ .
  - The  $x$ -intercepts occur at  $x = n\pi$ ,  $n \in \mathbb{I}$ .
  - The  $y$ -intercept is 0.

How can you determine the location of the asymptotes for the function  $y = \tan x$ ?



# y = tan θ



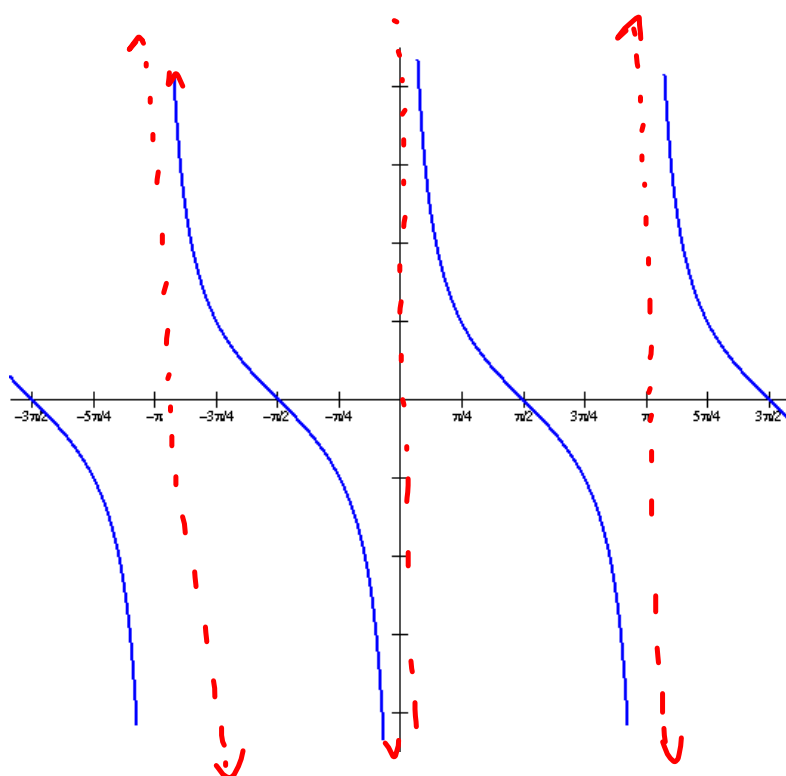
What would the graph of  $\cot \theta$  look like?

**REMEMBER:**

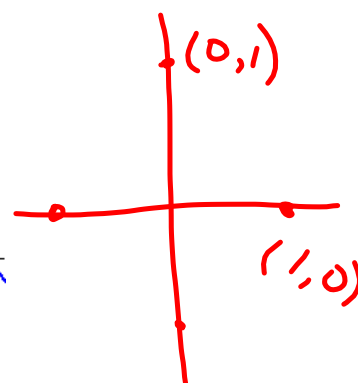
$$\tan x = \frac{1}{\cot x}$$

where  $\tan x = 0$ ,  
 $\cot x$  is undefined

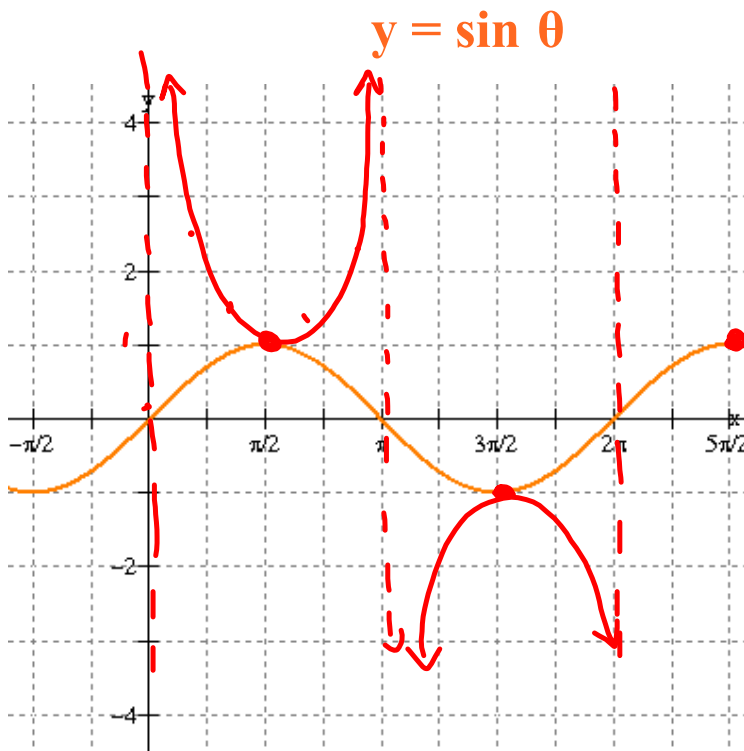
# y = cot θ



$$\cot \theta = \frac{x}{y}$$



# Graphs of Other Trigonometric Functions



What would the graph of  $\csc \theta$  look like?

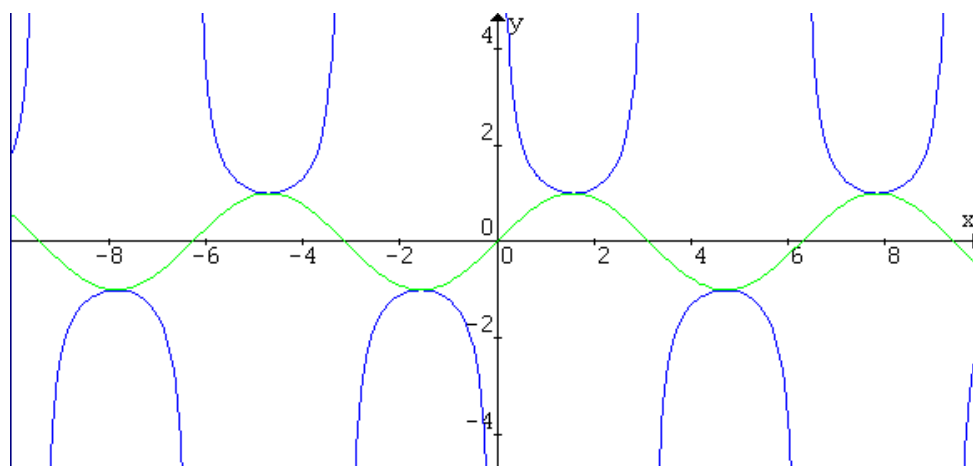
**REMEMBER:**

$$\csc \theta = \frac{1}{\sin \theta}$$

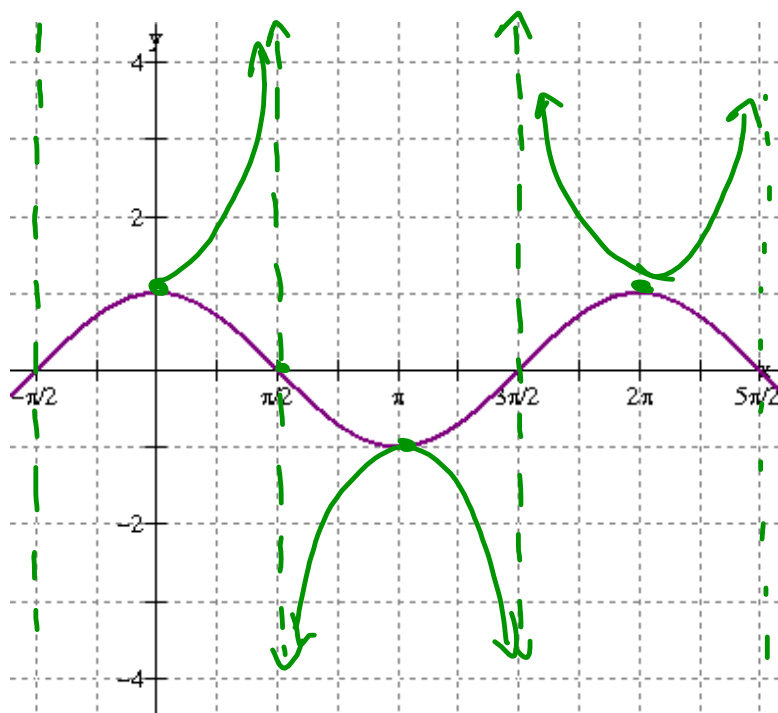
where  $\sin x = 0$ ,  
 $\csc x$  is undefined

$y = \sin x$

$y = \csc x$



$$y = \cos \theta$$



What would the graph of  $\sec \theta$  look like?

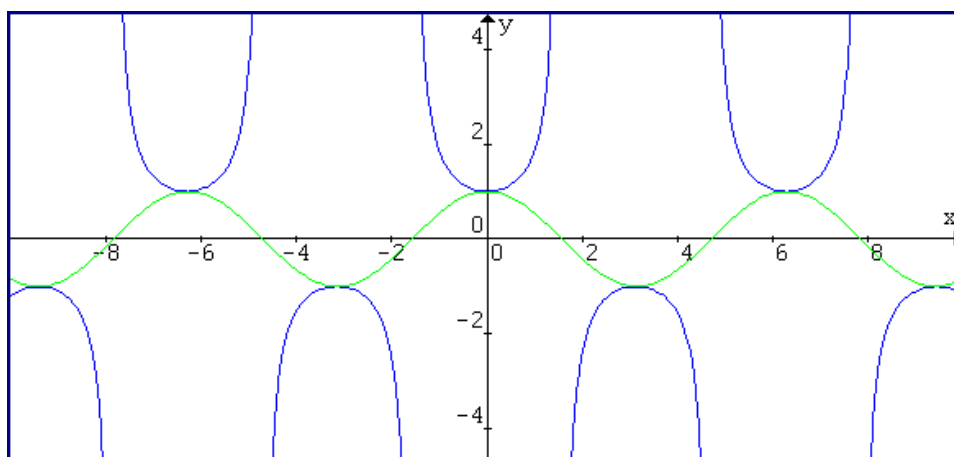
**REMEMBER:**

$$\sec \theta = \frac{1}{\cos \theta}$$

where  $\cos x = 0$ ,  
 $\sec x$  is undefined

$$y = \cos x$$

$$y = \sec x$$



## REVIEW - Sketching Trigonometric Functions

- sinusoidal functions
  - properties: domain/range, amplitude, period, phase shift, vertical translation, eq'n of sinusoidal axis, mapping notation.
  - sketching equation in standard form.
- finding the function (both a sine/cosine) given a graph
- solving trigonometric equations where period is not 360
- applications of sinusoidal functions.
  - sketch
  - develop a function
  - use function to answer question
- sketches of all SIX trigonometric ratios



Textbook Review....

Pg. 282 - 285

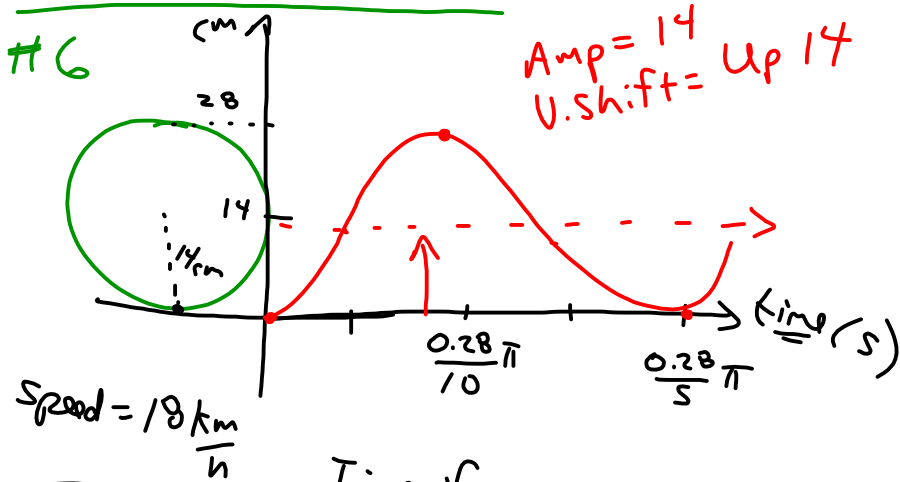
#4, 6, 7, 8, 10, 11, 20, 21, 22, 23, 24

Practice Test: Page 286 - 287

#1 - 7

#11, 12, 14, 15, 16

# Sinusoidal Relations



$d = st$

Time for one Revolution?

$t = \frac{d}{s}$

One Revolution

$C = 2\pi r$

$C = 2\pi(14) \text{ cm}$

$C = 28\pi \text{ cm}$

$C = 0.28\pi \text{ m}$

$18 \frac{\text{km}}{\text{h}} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ sec}}$   
 $= 5 \text{ m/s}$

$t_{\text{me}} = \frac{0.28\pi \text{ m}}{5 \text{ m/s}}$

$t_{\text{ime}} = \frac{0.28}{5} \pi \text{ sec}$

Period =  $\frac{0.28}{5} \pi$

$\frac{2\pi}{k} = \frac{0.28}{5} \pi$

$10 = 0.28k$

$k = \frac{10}{0.28}$

Radians

Radians!!

t... seconds

$h(t) = -14 \cos\left[\frac{10}{0.28}(t)\right] + 14$

5 minutes = 300 sec

$= -14 \cos\left[\frac{10}{0.28}(300)\right] + 14$

$= 12.38 \text{ cm}$

## PRACTICE TIME...

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 Review - Practice Test for Sinusoidal Functions.doc

# Practice Test Solutions

**Part A: Multiple Choice**

- |       |                     |
|-------|---------------------|
| 1. A  | 11. A (second hand) |
| 2. D  | 12. C               |
| 3. A  | 13. A               |
| 4. C  | 14. C               |
| 5. B  | 15. D               |
| 6. D  | 16. D               |
| 7. A  | 17. B               |
| 8. D  | 18. D               |
| 9. B  | 19. A               |
| 10. A | 20. A               |

**Part B: Open Response**

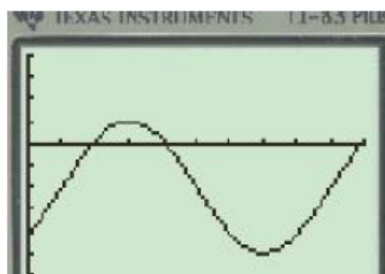
1.  $-\frac{5}{4}$

2. (i)  $y = 3 \sin \frac{3}{2}(x - 160^\circ) - 6$

$y = 3 \cos \frac{3}{2}(x + 20^\circ) - 6$

(ii)  $(x, y) \rightarrow \left( \frac{2}{3}x + 160^\circ, 3y - 6 \right)$

3.



X	Y1
15	-2
45	1
75	-2
105	-5
135	-2
165	1
195	-2

X=195

4. 10.28 m

# MORE PRACTICE???

Omit #7

Review - Trigonometric Functions.doc

## SOLUTIONS

1. (a)  $39^\circ$

(b)  $53^\circ$

2. (a)  $-2$

(b)  $\frac{7-2\sqrt{3}}{4}$

3. (a) II

(b) II

4. (a)  $-1.2799$   
 c)  $1.2690$   
 (e)  $-5$

(b)  $-1.0864$   
 (d)  $39^\circ$   
 (f)  $25^\circ$

5.  $\sin \theta = \frac{-\sqrt{5}}{5}$   
 $\cos \theta = \frac{-2\sqrt{5}}{5}$   
 $\tan \theta = \frac{1}{2}$

$\csc \theta = -\sqrt{5}$   
 $\sec \theta = \frac{\sqrt{5}}{2}$   
 $\cot \theta = 2$

6.  $\frac{-\sqrt{10}}{2}$

8. Amp = 3  
 Period =  $180^\circ$   
 V.T. = Up 2  
 P.S. = none  
 Domain:  $0^\circ \leq \theta \leq 360^\circ$

(b) Amp = 2  
 Period =  $120^\circ$   
 V.T. = Down 2  
 P.S. =  $60^\circ$  left  
 Domain:  $\mathbb{R}$

(c) Amp = 2  
 Period =  $720^\circ$   
 V.T. = Up 5  
 P.S. = none  
 Domain:  $-90 \leq \theta \leq 360^\circ$   
 Range:  $-3 \leq y \leq 7$

(d) Amp = 6  
 Period =  $360^\circ$   
 V.T. = None  
 P.S. =  $90^\circ$  right  
 Domain:  $\mathbb{R}$   
 Range:  $-6 \leq y \leq 6$

10. 11.9 m

11. 46.2 cm

## Attachments

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Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc