Integral Practice

(substitution, parts, inverse trig, partial fractions)

1) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ $2\int \sin u\,du = \boxed{-2\cos\sqrt{x} + c}$

$u = \sqrt{x}$

$2\,du = \frac{1}{\sqrt{x}}dx$

2) $\int (e^{2x} - e^{-2x})dx = \boxed{\frac{e^{2x}}{2} + \frac{e^{-2x}}{2} + c}$

(Parts) 3) $\int \sqrt{x}\ln x\,dx$

$u = \ln x$ $\int dv = \int \sqrt{x}\,dx$

$du = \frac{1}{x}dx$ $v = \frac{2}{3}x^{3/2}$

$uv - \int v\,du$

$\ln x\left(\frac{2}{3}x^{3/2}\right) - \frac{2}{3}\int x^{3/2} \cdot \frac{1}{x}dx$

$\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}dx$

$\boxed{\frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C}$

4) $\int \frac{dx}{9x^2 + 16}$

$u = 3x$

$\frac{1}{3}du = dx$

$\frac{1}{3}\int \frac{du}{u^2 + 4^2} = \frac{1}{3}\left[\frac{1}{4}\tan^{-1}\left(\frac{u}{4}\right)\right] + C$

$= \boxed{\frac{1}{12}\tan^{-1}\left(\frac{3x}{4}\right) + C}$

5) $\int_0^3 x\sqrt{1+x}\,dx$

$u = 1 + x \rightarrow x = u - 1$

$du = dx$

x	u
3	4
0	1

$\int_1^4 (u-1)\sqrt{u}\,du = \int_1^4 (u^{3/2} - u^{1/2})\,du$

$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big|_1^4 = \left(\frac{64}{5} - \frac{16}{3}\right) - \left(\frac{2}{5} - \frac{2}{3}\right)$

$\frac{62}{5} - \frac{14}{3} = \boxed{\frac{116}{15}}$

6) $\int \tan^5 x \csc^2 x\,dx$

$\frac{\sin^5 x}{\cos^5 x} \cdot \frac{1}{\sin^2 x} = \frac{\sin^3 x}{\cos^5 x} = \tan^3 x \cdot \sec^2 x$

$\int \tan^3 x \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int u^3\,du = \frac{u^4}{4} + c$

$\boxed{\frac{\tan^4 x}{4} + c}$

7) $\int \frac{2x}{x+1} dx$ (use long division to simplify integral)

$$x+1 \overset{2 \quad R^{-}2}{\overline{\big)\, 2x}} \qquad \begin{array}{r} -\;2x + 2 \\ \hline -2 \end{array} \qquad \to 2 - \frac{2}{x+1}$$

$$\int \left[2 - \frac{2}{x+1}\right] dx = \int 2\, dx - 2\int \frac{dx}{x+1}$$

$$= \boxed{2x - 2\ln[x+1| + C}$$

8) $\int \frac{x}{4-x^2} dx$

$$-\frac{1}{2}\int \frac{du}{u} = \boxed{-\frac{1}{2}\ln|4-x^2| + C}$$

$U = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

9) $\int \sqrt[3]{x^2}\, dx$

$$\int x^{2/3} dx = \boxed{\frac{3x^{5/3}}{5} + C}$$

10) $\int \frac{\sin x}{1-\cos x} dx$

$$\int \frac{du}{u} = \ln|u|$$

$$\boxed{\ln|1-\cos x| + C}$$

$U = 1-\cos x$

$du = \sin x\, dx$

11) $\int (2x^2+1)(x^2-3x+2)dx$

$$2x^4 - 6x^3 + 4x^2 + x^2 - 3x + 2$$

$$\int (2x^4 - 6x^3 + 5x^2 - 3x + 2)dx = \boxed{\frac{2x^5}{5} - \frac{3}{2}x^4 + \frac{5x^3}{3} - \frac{3x^2}{2} + 2x + C}$$

12) $\int \frac{\tan(\ln y)}{y} dy$

$$\int \tan u\, du = \int \frac{\sin u}{\cos u} du$$

$V = \cos u$

$dv = -\sin u\, du$

$-dv = \sin u\, du$

$$-\int \frac{dv}{v} = -\ln|v|$$

$$\boxed{-\ln|\cos(\ln y)| + C}$$

$U = \ln y$

$du = \frac{1}{y}dy$

13) $\int x^2 e^{-3x} dx$

(+)	x^2	e^{-3x}
(-)	$2x$	$-\frac{1}{3}e^{-3x}$
(+)	2	$\frac{1}{9}e^{-3x}$
	0	$-\frac{1}{27}e^{-3x}$

$$\boxed{-\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C}$$

$$\frac{A}{x-3} + \frac{B}{x+2}$$

$A(x+2) + B(x-3) = 4x-7$

$x(A+B) + 2A - 3B = 4x-7$

$A + B = 4$

$2A - 3B =$

14) $\int \frac{4x-7}{x^2-x-6}\,dx$

$(x-3)(x+2)$

$$\int \frac{dx}{x-3} + 3\int \frac{dx}{x+2} = \boxed{\ln|x-3| + 3\ln|x+2| + C}$$

15) $\int \tan^{-1} x\,dx$

$u = \tan^{-1} x \qquad \int dv = \int 1\,dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = x$

$$x\tan^{-1}x - \int \frac{x}{1+x^2}\,dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$$x\tan^{-1}x - \frac{1}{2}\int \frac{du}{u}$$

$$\boxed{x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + C}$$