Check-Up Time...

Evaluate each of the following:

1. $\log_2 8\sqrt{32} = x$

$\left(x^{\frac{2}{5}}\right)^{\frac{5}{2}}$

x^1

$2^x = 8\sqrt{32}$

$2^x = 2^3 (2^5)^{1/2}$

$2^x = 2^{3/1} \cdot 2^{5/2}$

$2^x = 2^{11/2}$

$x = \frac{11}{2}$

2. $-\frac{2}{3} = \log_x 81$

$\left(x^{-2/3}\right)^{-3/2} = (81)^{-3/2}$

$x^1 = \left(\sqrt{81}\right)^{-3}$

$x = \frac{1}{9^3} = \frac{1}{729}$

3. $\log_5 \frac{1}{125} = x$

$5^x = \frac{1}{125}$

$5^x = \frac{1}{5^3}$

$5^x = 5^{-3}$

$x = -3$

4. $\log_{\sqrt{6}} 36 = x$

$\sqrt{6}^x = 36$

$\left(6^{\frac{1}{2}}\right)^x = 6^2$

$6^{\frac{1}{2}x} = 6^2$

$\frac{1}{2}x = 2$

$x = 4$

What is the value of any logarithm with an argument of 1? Why?

$\log_7 49 =$ "7 Raised to what exponent gives 49?"

General Properties of Logarithms:

If $a > 0$ and $a \neq 1$, then...
(i) $\log_a 1 = 0$
(ii) $\log_a a^x = x$
(iii) $a^{\log_a x} = x$

$3^{\log_3 7} = y$

$\Downarrow$

$\log_3 y = \log_3 7$

$\therefore y = 7$

$A^B = C \Leftrightarrow \log_A C = B$

$7^{\log_7 10} = 10$

Laws of Logarithms: If $a > 0$, $M > 0$, $N > 0$ *and* $n \varepsilon R$ *then…*

1) Product Law → the logarithm of a product is equal to the sum of the logarithms of the factors.

PROOF: Let $\log_a M = b$ and $\log_a N = c$

so $a^b = M$ $a^c = N$

then,

$$\log_a(MN) = \log_a(a^b \bullet a^c)$$
$$= \log_a(a^{b+c})$$
$$= b + c$$

Product Law

$$\therefore \boxed{\log_a(MN) = \log_a M + \log_a N}$$

$\Longleftrightarrow$

examples: a) $\log_{10}(6 \times 9)$

$= \log_{10} 6 + \log_{10} 9$

b) $\log_2 12 + \log_2 7$

$\log_2(12 \times 7)$

$\log_2 84$

$\log_b(17xy) = \log_b 17 + \log_b x + \log_b y$

$\log_b(x^2 - 1)$ Factoring

$\log_b[(x+1)(x-1)] = \log_b(x+1) + \log_b(x-1)$

2) Quotient Law → the logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator.

PROOF: Let $\log_a M = b$ and $\log_a N = c$

so $a^b = M$ $a^c = N$

then,

$$\log_a\left(\frac{M}{N}\right) = \log_a\left(\frac{a^b}{a^c}\right)$$
$$= \log_a(a^{b-c})$$
$$= b - c$$

$$\therefore \boxed{\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N}$$

examples: a) $\log_5\left(\frac{97}{62}\right)$ b) $\log_2 15 - \log_2 3$ c) $\log_{10}\left(\frac{1}{9}\right)$

(a) $\log_5 97 - \log_5 62$

(b) $\log_2\left(\frac{15}{3}\right) = \log_2 5$

c) ?? $= 0$

$\log 1 - \log 9$

$= 0 - \log 9$

3) Law of Logarithms for Powers → the logarithm of a power of a number is equal to the exponent multiplied by the logarithm of the number

PROOF: Let $\log_a M = b$

so $a^b = M$

then,

$$\log_a M^p = \log_a (a^b)^p$$
$$= \log_a (a^{b \times p})$$
$$= b \times p$$
$$\therefore \boxed{\log_a M^p = p \times \log_a M}$$

examples: a) $\log_{10} 8^9$ b) $2\log_3 5$ c) $\log_5 \sqrt{125}$

a) $= 9\log 8$

b) $\log_3 5^2$

c) $\log_5 125^{\frac{1}{2}}$

$\frac{1}{2} \cdot \log_5 125$

$\frac{1}{2}(3)$

$= \frac{3}{2}$

$(\log_3 7)^2$ Not Power Law!!

3 Properties of Logarithms:

$$\boxed{\begin{array}{c}\log_m b = x \\ \Updownarrow \\ m^x = b\end{array}}$$

① $\log_b 1 = 0$

② $\log_a a^x = x$

③ $b^{\log_b x} = x$

$\log_{\uparrow} W$
Base 10

3 Laws of Logarithms

Product: $\log_b(MN) = \log_b M + \log_b N$

Quotient: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Power: $\log_b M^N = N \log_b M$

$\Longleftrightarrow$

Example 1

Use the Laws of Logarithms to Expand Expressions

Write each expression in terms of individual logarithms of x, y, and z.

a) $\log_6 \frac{x}{y}$

b) $\log_5 \sqrt{xy}$

c) $\log_3 \frac{9}{\sqrt[3]{x^2}}$

d) $\log_7 \frac{x^5 y}{\sqrt{z}}$

(a) $\log_6 x - \log_6 y$

(b) $\log_5 (xy)^{1/2}$ OR $\log_5 x^{1/2} y^{1/2}$

$\frac{1}{2} \log_5 (xy)$

$= \frac{1}{2}(\log_5 x + \log_5 y)$

$= \frac{1}{2}\log_5 x + \frac{1}{2}\log_5 y$

$\sqrt[n]{x^w} = x^{\frac{w}{n}}$

(c) $\log_3 \frac{9}{\sqrt[3]{x^2}} = \log_3 9 - \log_3 \sqrt[3]{x^2}$

$= 2 - \log_3 x^{2/3}$

$= 2 - \frac{2}{3}\log_3 x$

d) $\log_7 \frac{x^5 y}{\sqrt{z}} = \log_7 (x^5 y) - \log_7 \sqrt{z}$

$= \log_7 x^5 + \log_7 y - \log_7 z^{1/2}$

$= 5\log_7 x + \log_7 y - \frac{1}{2}\log_7 z$

Example 2

Use the Laws of Logarithms to Evaluate Expressions

Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_3(9\sqrt{3})$

b) $\log_5 1000 - \log_5 4 - \log_5 2$

c) $2 \log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

(a) $\log_3 9 + \log_3\sqrt{3}$

$2 + \log_3 3^{1/2}$

$2 + \frac{1}{2}$

$= \frac{5}{2}$

$\log_3 (3)^2(3)^{1/2}$

$\log_3 3^{5/2}$

$= \frac{5}{2}$

b) $\log_5 1000 - \log_5 4 - \log_5 2$

$\log_5\left(\frac{1000}{4}\right) - \log_5 2$

$\log_5 250 - \log_5 2$

$\log_5\left(\frac{250}{2}\right)$

$\log_5 125$

$= 3$

c) $2 \log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

$\log_3 6^2 - \log_3 64^{1/2} + \log_3 2$

$\log_3 36 - \log_3 8 + \log_3 2$

$\log_3\left(\frac{36}{8}\right) + \log_3 2$

$\log_3\left[\frac{36}{8} \times 2\right]$

$\log_3 9$

$= 2$

Example 3

Use the Laws of Logarithms to Simplify Expressions

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a) $4 \log_3 x - \frac{1}{2}(\log_3 x + 5 \log_3 x)$

b) $\log_2 (x^2 - 9) - \log_2 (x^2 - x - 6)$

a) $4\log_3 x - \frac{1}{2}\log_3 x - \frac{5}{2}\log_3 x$

$\log_3 x^4 - \log_3 x^{1/2} - \log_3 x^{5/2}$

$\log_3\left(\frac{x^4}{x^{1/2}}\right) - \log_3 x^{5/2}$

$\log_3 x^{7/2} - \log_3 x^{5/2}$

$\log_3\left(\frac{x^{7/2}}{x^{5/2}}\right)$

$= \log_3 x^1$

OR

$\log_3 x^4 - \frac{1}{2}\log_3 x^6$

$\log_3 x^4 - \log_3 x^3$

$\log_3\left(\frac{x^4}{x^3}\right)$

$\log_3 x$

OR

$4\log_3 x - 3\log_3 x$

4 ☺ − 3 ☺

$= 1\log_3 x$

b) $\log_2(x^2-9) - \log_2(x^2-x-6)$

$\log_2\left(\frac{x^2-9}{x^2-x-6}\right)$

$\log_2\left(\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+2)}\right)$

$\log_2\left(\frac{x+3}{x+2}\right)$

Key Ideas

- Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

Do I really understand??...

a) Express the following as a single logarithm... $2\log_2 3^2 + \log_2 6 - 3\log_2 3$

$\log_2 81 + \log_2 6 - \log_2 27$

$\log_2\left(\frac{81}{27}\right) + \log_2 6 = \log_2 18$

b) Evaluate the following... $\log_2 (32)^{\frac{1}{3}}$

$\frac{1}{3}\log_2 32 = \frac{1}{3}(5)$

$\frac{1}{3}(5) = \frac{5}{3}$

c) Express the following as a single logarithm... $\frac{1}{2}[(\log_5 a + 2\log_5 b) - 3\log_5 c]$

$\log_5\left(\frac{\sqrt{a}\,b}{c^{3/2}}\right)$

OR

$\log_5\sqrt{\frac{ab^2}{c^3}}$

$\frac{1}{2}[(\log_5 a + \log_5 b^2) - \log_5 c^3]$

$\frac{1}{2}[\log_5(ab^2) - \log_5 c^3]$

$\frac{1}{2}\log_5\left(\frac{ab^2}{c^3}\right)$

$\log_5\left(\frac{ab^2}{c^3}\right)^{1/2}$

ex.

$\log_3 x^8 - \log_3 w^4 - \log_3 a^7 + \log_3 y + \log_3 c^7$

$\log_3\left(\frac{x^8 y c^7}{w^4 a^7}\right)$

Negatives end up as factors in denominator

d) Express as a single logarithm in simplest form...

$$\frac{3}{4}\left[12(\log_b x^2 - 2\log_b x) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7}\right]$$

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Attachments

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