5 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Example:
Find the critical values of $f(x) = x^{\frac{3}{5}}(4 - x)$ and determine all intervals of increase and decrease as well as any local extrema.

$$f'(x) = \frac{3}{5}x^{-\frac{2}{5}}(4-x) + x^{3/5}(-1)$$

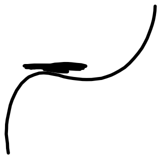
Critical Values:

$$\frac{3}{5}x^{-2/5}(4-x) - x^{3/5} = 0$$

$$x^7 - x^3$$

$$x^3(x^4 - 1)$$

$$x^{-2/5}\left(\frac{3}{5}(4-x) - x\right) = 0$$



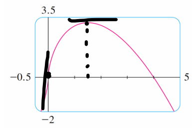


FIGURE 11

$$x^{-2/5}\left(\frac{12}{5} - \frac{3}{5}x - \frac{1}{1}x\right) = 0$$

$$x^{-2/5}\left(-\frac{8}{5}x + \frac{12}{5}\right) = 0$$

$$-\frac{8}{5}x + \frac{12}{5}$$

$$\frac{1}{5}(-8x + 12)$$

$$-\frac{4}{5}(2x - 3)$$

$x^{-2/5} = 0$ or $-\frac{8}{5}x\,(8) + \frac{12}{5}\,(5) = 0\,(5)$

$\emptyset$

$\frac{1}{x^{2/5}} \neq 0$

$-8x + 12 = 0$

$-8x = -12$

$x = \frac{12}{8}$

$x = \frac{3}{2}$

Critical Value

$x = 0$ $(\sqrt[5]{x})^2$

$\frac{1}{x^{2/5}}$

	$x^{-2/5}$	$\frac{1}{5}$	$-8x+12$	f'	f
$(-\infty, 0)$	+	+	+	+	Inc
$(0, 3/2)$	+	+	+	+	Inc
$(3/2, \infty)$	+	+	−	−	Dec

LOCAL Min: None LOCAL MAX. $\left(\frac{3}{2}, 3.18\right)$

$f\left(\frac{3}{2}\right) \doteq 3.18$

How do we determine absolute maximum and minmum values?

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Example:

Given the function... $[-1, 4]$

$$f(x) = 3x^4 - 16x^3 + 18x^2 \quad , \; -1 \le x \le 4$$

Determine the absolute maximum and minimum values of the function.

Solution

$f'(x) = 12x^3 - 48x^2 + 36x$

$0 = 12x(x^2 - 4x + 3)$

$0 = 12x(x-3)(x-1)$

$x = 0, \; x = 3, \; x = 1 \leftarrow$ Critical Values

$f(-1) = 3(-1)^4 - 16(-1)^3 + 18(-1)^2$
$= 3 + 16 + 18$
$= 37$

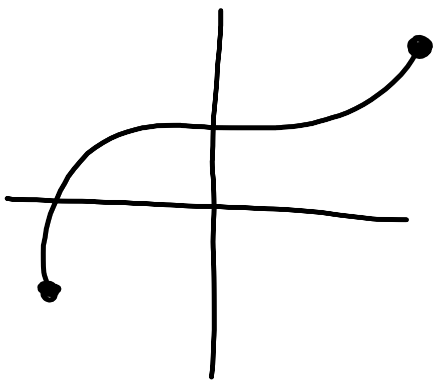
x	y
-1	37
0	0
1	5
3	-27
4	32

endpoint (x = -1); critical values (x = 0, 1, 3); endpoint (x = 4)

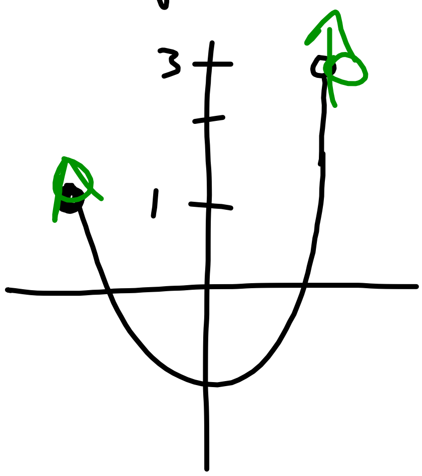
Absolute Maximum $\Rightarrow 37$

Absolute Minimum $\Rightarrow -27$

Closed



Open



Example 2:

Using Calculus methods determine the absolute maximum and minimum values of the function given below:

$$f(x) = x^3 + 2x^2 + x - 1 \quad \text{over the interval } [-1\ ,\ 1]$$

$f'(x) = 3x^2 + 4x + 1$

$0 = 3x^2 + 4x + 1$

$0 = 3x^2 + 3x + x + 1$

$0 = 3x(x+1) + 1(x+1)$

$0 = (x+1)(3x+1)$

$x = -1, -\frac{1}{3}$

Critical Values

x	y
-1	-1
$-\frac{1}{3}$	$-\frac{31}{27}$
1	3

$-\frac{1}{27} + \frac{2}{9} - \frac{1}{3} - \frac{1}{1}$

$\frac{-1 + 6 - 9 - 27}{27}$

$= -\frac{31}{27}$

Abs. Min. $= -\frac{31}{27}$

Abs. Max $= 3$

endpoints $[-3, 4]$

Critical Values: $x = -1, 0, 1, \cancel{4}$ Not within Domain

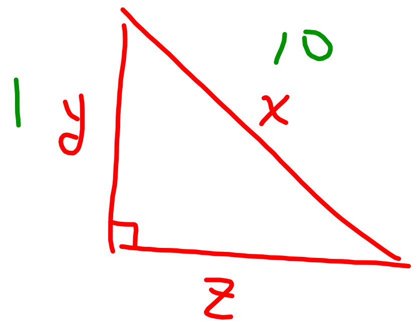
x	y
-3	
-1	
0	
1	

Local max/min.

Pg. 182 #2

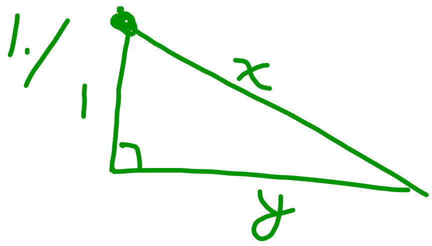
Homework

Page 177
#3 d, f, l
#4 d, f, h, i
#6
#7



$z^2 = 10^2 - 1^2$
$= 99$
$z = \sqrt{99}$

1/



$y^2 + 1^2 = x^2$

$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

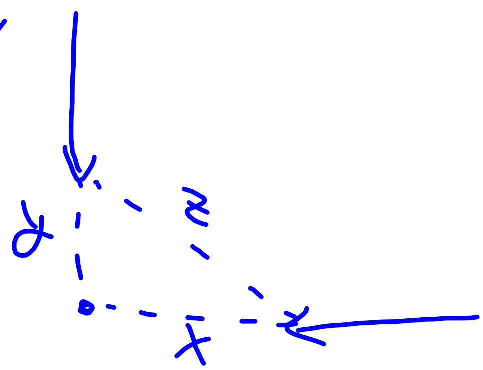
2/ $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right)$

$5 = \frac{4}{\cancel{3}}\pi(\cancel{3})(2)^2\frac{dr}{\cancel{d}t}$

$\frac{dr}{dt} = 0.0995$ m/min.

#3/



Attachments

coord geom review.doc