ex.

$$x^2+y^2=3y \qquad \text{Find } \frac{d^2y}{dx^2}$$

$$2x+2y\frac{dy}{dx}=3\frac{dy}{dx}$$

$$(2y-3)\frac{dy}{dx}=-2x$$

$$\frac{dy}{dx}=-\frac{2x}{2y-3}$$

$$\frac{d^2y}{dx^2}=\frac{(-2)(2y-3)+2x\left(2\frac{dy}{dx}\right)}{(2y-3)^2}$$

$$\frac{d^2y}{dx^2}=\frac{-2(2y-3)+4x\left(\frac{-2x}{2y-3}\right)}{(2y-3)^2} \qquad (2y-3)$$

$$(2y-3)$$

$$=\frac{-2(2y-3)^2-8x^2}{(2y-3)^3}$$

Rectilinear Motion and Derivatives

Any motion along a straight line is called rectilinear motion.

Displacement - Velocity - Acceleration

$\xrightarrow{} f' \xrightarrow{} f''$

If s represents a function that measures displacement, then $\frac{ds}{dt}$ would represent ???

$$velocity = \frac{ds}{dt}$$

The rate of change of the velocity...ie. $\frac{\Delta v}{\Delta t}$

would represent??

$$acceleration = \frac{dv}{dt}$$

So it follows that the second derivative of displacement will give us acceleration:

$$a = \frac{d^2 s}{dt^2}$$ ←-------- Notice the notation

Example

If the displacement (in metres) at time t (in seconds) of an object is given by

$$s = 4t^3 + 7t^2 - 2t,$$

find the acceleration at time $t = 10$.

$S' = 12t^2 + 14t - 2$ (Velocity)

$S'' = 24t + 14$ (acceleration)

$S'' = 24(10) + 14$

$= 254\ m/s^2$

Example:

- The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$, where t is measured in seconds and s in meters.
  a) Find the velocity at time t.
  b) What is the velocity after 2 s? After 4 s?
  c) When is the particle at rest?
  d) When is the particle moving forward (that is, in the positive direction)?
  e) Draw a diagram to represent the motion of the particle.
  f) Find the total distance traveled by the particle during the first five seconds.
  g) Find the acceleration at time t and after 4 s.
  h) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 5$.
  i) When is the particle speeding up? When is it slowing down?

$s = f(t) = t^3 - 6t^2 + 9t$

a) Find the $\underbrace{\text{velocity}}_{s'}$ at time t.

$s' = 3t^2 - 12t + 9$

b) What is the velocity after 2 s? After 4 s?

$s'(2) = 3(2)^2 - 12(2) + 9$
$= -3\ m/s$

$s'(4) = 3(4)^2 - 12(4) + 9$
$= 9\ m/s$

$s = f(t) = t^3 - 6t^2 + 9t$

c) When is the particle at rest?

$s' = 0$ velocity $= 0$

$\frac{3t^2}{3} - \frac{12t}{3} + \frac{9}{3} = \frac{0}{3}$

$t^2 - 4t + 3 = 0$

$(t-1)(t-3) = 0$

$t = 1$ sec & 3 sec

d) When is the particle moving forward (that is, in the positive direction)?

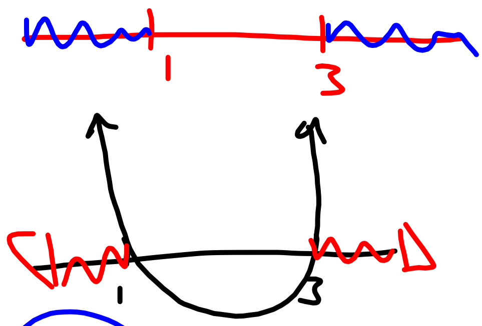
velocity > 0

(+) $3t^2 - 12t + 9 > 0$

① Find zero's (x-Int)

$3t^2 - 12t + 9 = 0$

$t = 1$ and 3



time can not be negative

$t < 1$ OR $t > 3$

$0 \le t < 1$ or $t > 3$

e) Draw a diagram to represent the motion of the particle.

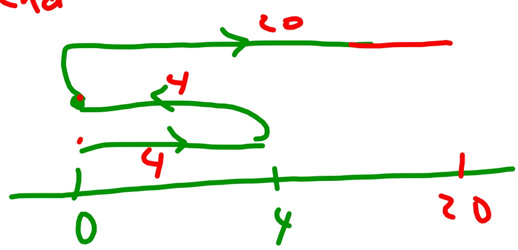
$s = f(t) = t^3 - 6t^2 + 9t$

t	s (Position)	
0	0	← start
1	4	$s' = 0$
3	0	$s' = 0$
5	20	← end

$s(1) = 1 - 6 + 9$

$s(3) = 3^3 - 6(3)^2 + 9(3) = 0$

$s(5$



f) Find the total distance traveled by the particle during the first five seconds.

$s(5) = 5^3 - 6(5)^2 + 9(5)$

$= 125 - 150 + 45$

$= \underline{\underline{20}}$

distance $= 4 + 4 + 20$

$= \underline{\underline{28}}$ m

g) Find the acceleration at time t and after 4 s.

$s = f(t) = t^3 - 6t^2 +$

s''

$s'' = 6t - 12$

$s''(4) = 6(4) - 12$

$= 12 \text{ m/s}^2$

h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.

i) When is the particle speeding up? When is it slowing down?

$s = f(t) = t^3 - 6t^2 + 9t$

speeding up. OR

vel > 0 and accel > 0

vel < 0 and accel < 0

$0 \le t < 1$ or $t > 3$

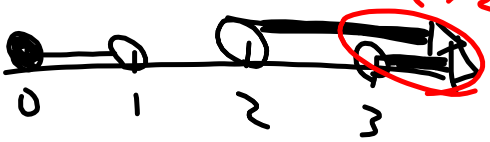
$6t - 12 > 0$

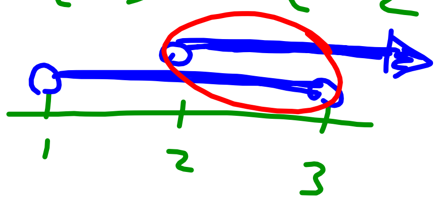
$6t > 12$

$t > 2$

$1 < t < 3$

$t < 2$



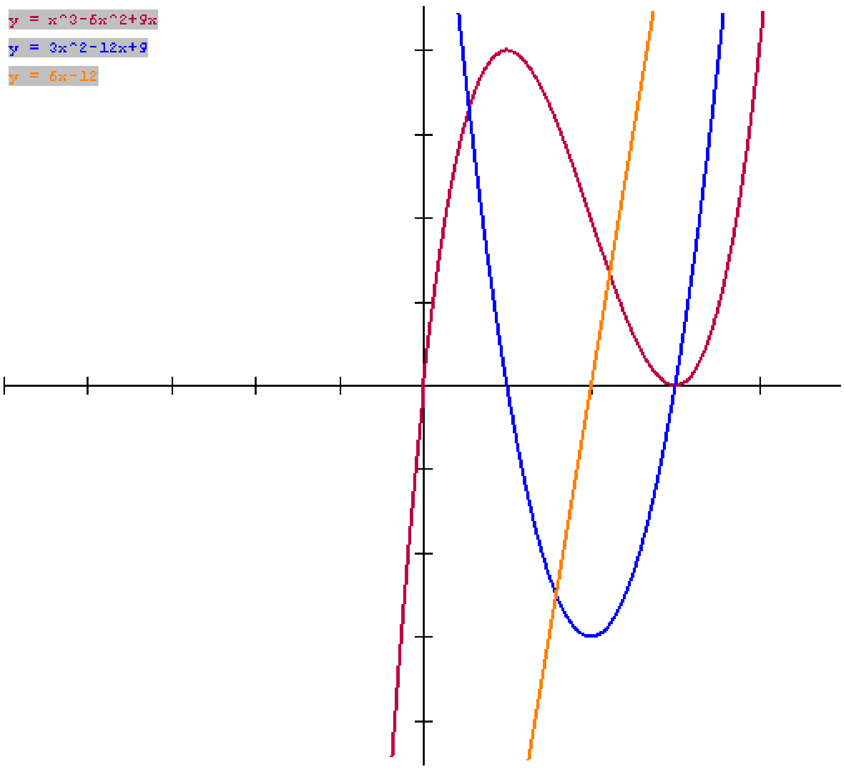


$t > 3$ OR $2 < t < 3$

Slowing Down

$0 \le t < 2$

h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.



i) When is the particle speeding up? When is it slowing down?

Time to check your understanding...

A particle moves according to a law of motion $s(t) = 2t^3 - 9t^2 + 12t + 1,\ t \geq 0$.

(a) Determine the velocity of the particle when it has acceleration 2 units/s^2.
(b) When is this particle moving in a positive direction?
(c) Sketch the path of this particle, and determine how far it has traveled during the first 8 seconds.

(a) $s'(t) = 6t^2 - 18t + 12$

Accel

$s''(t) = 12t - 18$

$2 = 12t - 18$

$\frac{20}{12} = t$

$\frac{5}{3} = t$

$s'\left(\frac{5}{3}\right) = 6\left(\frac{5}{3}\right)^2 - 18\left(\frac{5}{3}\right) + 12$

$= -\frac{4}{3}$ u/sec

(b) Velocity > 0

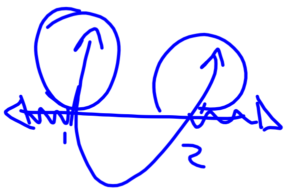
$6t^2 - 18t + 12 > 0$

$\frac{6t^2}{6} - \frac{18t}{6} + \frac{12}{6} = \frac{0}{6}$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1, 2$

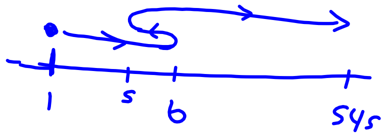


$0 \leq t < 1$ OR $t > 2$

(c)

	t	s
start	0	1
v = 0	1	6
	2	5
End	8	545

substitute into displacement



distance = 5 + 1 + 540

= 546 units

6.c)

t	s
0	-5
4	107
6	103
*35	27015

distance = 112 + 4 + 26912

= 27028 m

Practice exercises...

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#3, 4, 5, 8, 9	#6, 7, 8

Topics to Review:

- Power rule, product rule, quotient rule, chain rule
- Derivatives of trigonometric functions
- Applications of derivatives...
    *slopes of tangent lines
    *rectilinear motion
- Implicit differentiation
- Higher order derivatives

Review Questions...

Page 112 - 114
#1 c, d
#7 b, d
#8 b, d
9 a, b, d, f
#11
#12

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#1 (ii)
#3
#4
#5

Page 154
#2
#3