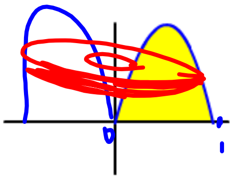
Another means of calculating volume...

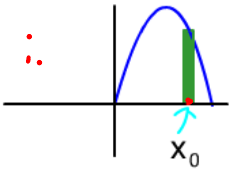
Cylindrical Shell Method:

Consider the area bounded by the graph of the function f(x) = x - x^2 and the x-axis.

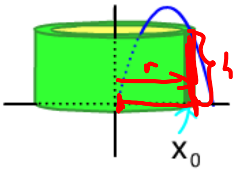


$x - x^2 = 0$
$x(1-x) = 0$
$x = 0, 1$

$y = x - x^2$
$y = -(x^2 - x + \frac{1}{4}) + \frac{1}{4}$
$y = -(x - \frac{1}{2})^2 + \frac{1}{4}$

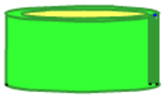


Choose some x_0 between 0 and 1. Draw a rectangle with height $f(x_0)$ and with very small width Δx.

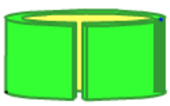


Rotate the rectangle about the y-axis.

The result is a cylinder with a "very small side" like the side of a can:

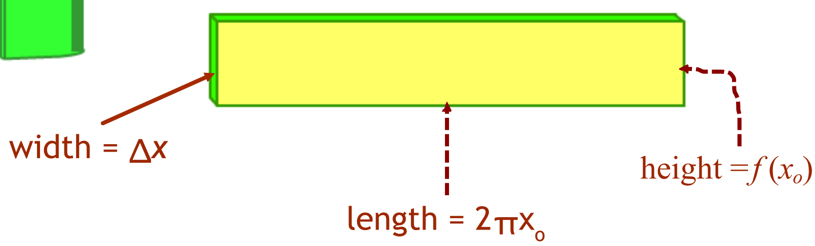


Take this cylinder and cut it vertically as shown:



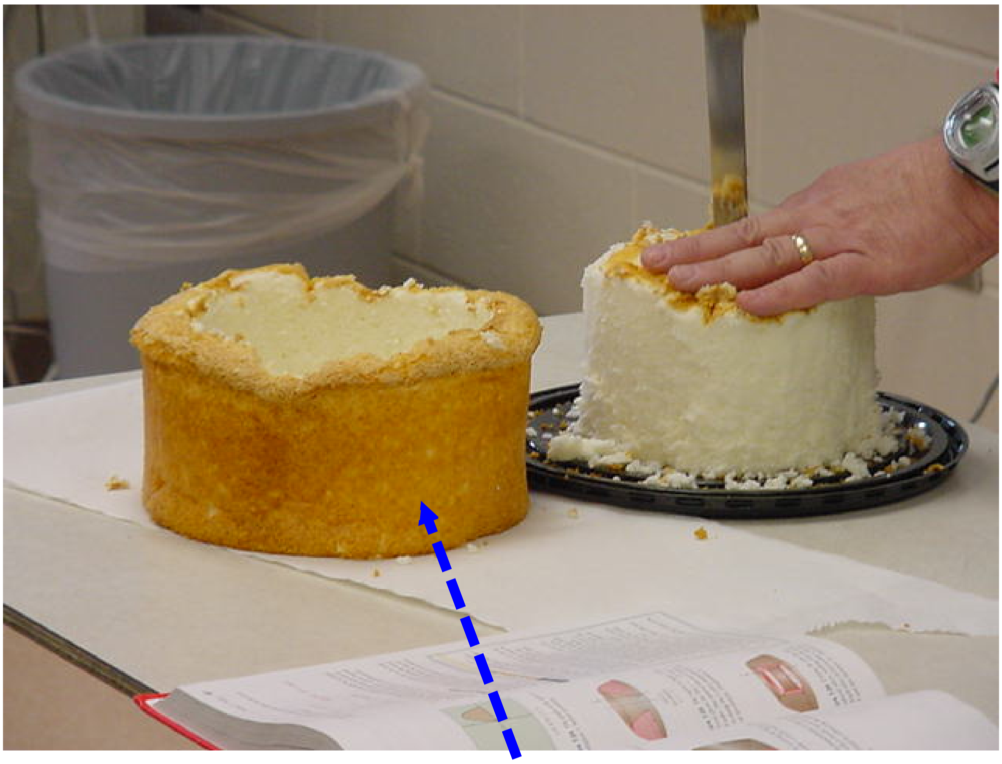
circumference

and stretch it out "flat":

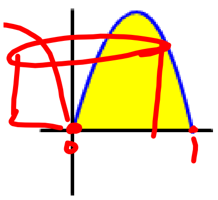


$V = \underbrace{2\pi r}_{\text{length}}\,\underbrace{h}_{\text{thickness}}\,\underbrace{f(x)}_{\text{height}}$

Imagine cutting a cake using this method...



This would be an example of a cylindrical shell



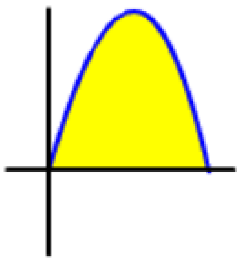
The volume of the solid obtained by rotating this area about the y-axis is:

$$2\pi \int_0^1 x\, f(x)\, dx = 2\pi \int_0^1 x\,(x - x^2)\, dx$$

radius height width

Notice that even though we are rotating about a vertical line, the integral is still in terms of *x*.

What if we had used cylindrical disks?

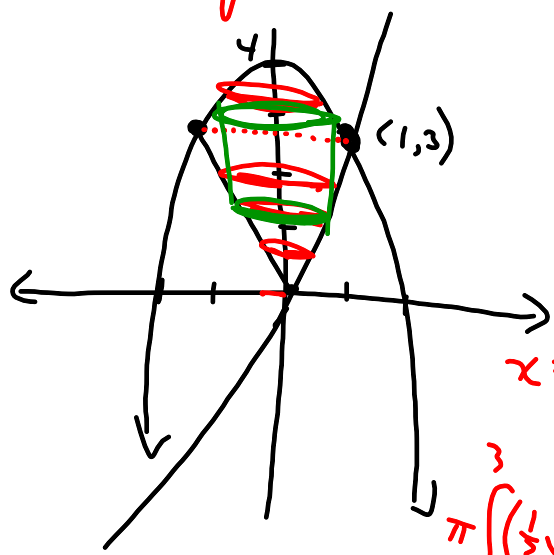


Example 1:

The region bounded by the curve $y = 4 - x^2$, $y = 3x$, and $x = 0$ is revolved about the y-axis to generate a solid. Determine the volume of this solid.

(a) Using Cylidrical Disks

(b) Using Shells



$4 - x^2 = 3x$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4, 1$

(a) Disks

$x = \frac{1}{3}y$ and $\sqrt{4-y} = \sqrt{x^2}$

$\pm\sqrt{4-y} = x$

$$\pi\int_0^3 \left(\tfrac{1}{3}y\right)^2 dy + \pi\int_3^4 \left(\sqrt{4-y}\right)^2 dy$$

$$= \left(\pi\tfrac{1}{9}\right)\int_0^3 y^2\,dy + \pi\int_3^4 (4-y)\,dy$$

$$= \frac{\pi}{9}\left(\frac{y^3}{3}\right)\Big|_0^3 + \pi\left(4y - \frac{y^2}{2}\right)\Big|_3^4$$

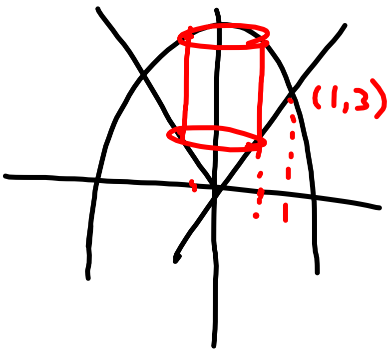
$$= \frac{\pi}{9}\left[(9-0)\right] + \pi\left[(16-8) - \left(12 - \frac{9}{2}\right)\right]$$

$$= \pi + \pi\left(-\frac{4}{1} + \frac{9}{2}\right)$$

$$= \frac{\pi}{1} + \frac{\pi}{2}$$

$$= \frac{3\pi}{2}\,u^3$$

Shells



$$2\pi \int_0^1 x(4 - x^2 - 3x)\,dx$$

$$2\pi \int_0^1 (4x - x^3 - 3x^2)\,dx$$