Warm Up

If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^{x}$

$-1e^{1-x}$

Review: Antiderivatives

Antidifferentiate each of the following:

1. $f'(x) = 4xe^{x^2}$

$f'(x) = \frac{4}{2}e^{x^2}(2x)$

$f(x) = 2e^{x^2} + C$

2. $y' = \frac{2}{(3x-5)^3} + \frac{5x}{x^2+1}$

$y' = \frac{2}{3}(3x-5)^{-3}(3) + \frac{5}{2}\left(\frac{2x}{x^2+1}\right)$

$y = -\frac{1}{3}(3x-5)^{-2} + \frac{5}{2}\ln(x^2+1) + C$

$1\ c)\ \frac{3}{10}\ln(5x^2+2) - \frac{1}{6}(4-x^3)^{-2} + C$

$-\frac{1}{3}(4-x^3)^{-3}(-3x^2)$

$-\frac{1}{6}(4-x^3)^{-2}$

$u^n \cdot du$ $\qquad$ $u^n \cdot du$

$2\ d)\ g(x) = (\ln 5x)^5 \frac{5}{5x} + \frac{5}{3}(2+\tan 3x)^{-1/2} \sec^2 3x(3)$

$G(x) = \frac{1}{6}(\ln 5x)^6 + \frac{10}{3}(2+\tan 3x)^{1/2} + C$

$e^u \cdot du$

$1.\ d)\ g'(x) = \frac{1}{5}\sec^2 5x(5) + \frac{2}{5}e^{\sin 5x}\cos 5x\,(5)$

$G(x) = \frac{1}{5}\tan 5x + \frac{2}{5}e^{\sin 5x} + C$

Identifying a unique solution for an antiderivative

Examples:

Determine the function with the given derivative whose graph satisfies the initial condition provided.

1. $f'(x) = 2x - \cos x + 1, \quad f(0) = 3$

x

y

Initial Condition

y

$f(x) = x^2 - \sin x + x + C$

$3 = 0^2 - \sin(0) + 0 + C$

$3 = C$

$$\boxed{f(x) = x^2 - \sin x + x + 3}$$

2. $f''(x) = 12x^2 + 6x - 4, \quad f(0) = 4 \; and \; f(1) = 1$

$f'(x) = 4x^3 + 3x^2 - 4x + C$

$f(x) = x^4 + x^3 - 2x^2 + Cx + K$

$4 = 0 + 0 - 0 + 0 + K$

$4 = K$

$1 = 1 + 1 - 2 + C(1) + 4$

$1 = 4 + C$

$-3 = C$

$$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$$

$y' = \tan x$

$y' = \dfrac{\tan x \sec x}{\sec x}$

$\ln|\sec x| + C$

Differentiate

$$\text{Position} \underset{\longleftarrow}{\longrightarrow} \text{Velocity} \underset{\longleftarrow}{\longrightarrow} \text{Acceleration}$$

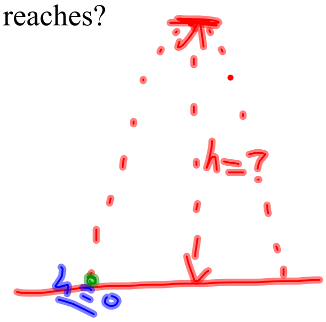
Antidifferentiate

Example 1:
A golfer on the moon (where gravitational acceleration equals 1.67 m/sec²)
hits a ball whose initial velocity in the vertical direction is 30 meters per second.
What is the maximum height the ball reaches?

269.46 m

$a = -1.67\ m/s^2$

$t = 0$, $V = 30\ m/s$ $t = 0$, $h = 0$



$V = -1.67t + C$

$30 = 0 + C$

$30 = C$

$\boxed{V = -1.67t + 30} \Longrightarrow h(t) = \frac{-1.67}{2}t^2 + 30t + C$

$t = 0,\ h = 0$

$0 = 0 + 0 + C$

$C = 0$

$$\boxed{h = \frac{-1.67t^2}{2} + 30t}$$

At max height ... $V = 0$

$0 = -1.67t + 30$

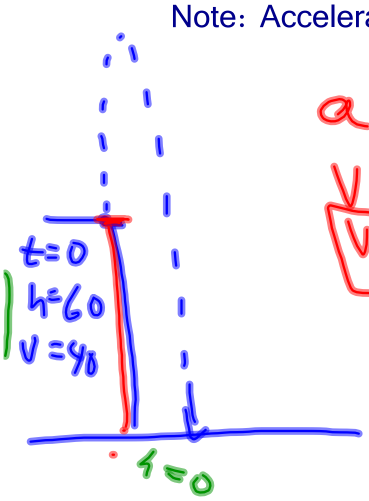
$-30 = -1.67t$

$t = \underline{17.96\ sec} \Longrightarrow h(17.96) = \frac{-1.67}{2}(17.96)^2 + 30(17.96)$

$= \underline{269.46\ m}$

A stone is thrown verticlly upward from the top of a house 60 feet above ground level with an initial velocity of 40 feet per second. How long will it take the stone to reach its maximum height? What is its maximum height? How long will it take the stone to reach the ground? With what velocity will it strike the ground?

Note: Acceleration due to gravity = −32 ft./s^2



$a = -32$

$v = -32t + C$

$\boxed{v = -32t + 40}$

$h(t) = \frac{1}{2}at^2 + v_i t + y$

$C = 60$

$h = -16t^2 + 40t + C$

$h = -16t^2 + 40t + 60$

(i) $V = 0$

$-32t + 40 = 0$

$-32t = -40$

$t = 1.25$ sec

(ii) $h(1.25) = -16(1.25)^2 + 40(1.25) + 60$

$= 85$ feet

(iii) $0 = -16t^2 + 40t + 60$

$$t = \frac{-40 \pm \sqrt{(40)^2 - 4(-16)(60)}}{2(-16)}$$

$t = 3.55$ or $t = \cancel{-1.05}$

3.55 sec after being thrown

(iv) $v = -32(3.55) + 40$

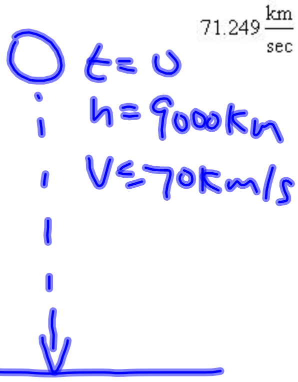
$V = -73.6$ ft./sec

Example:
A meteoroid falling to Earth is discovered when it is at an altitude of 9000 kilometers, traveling at a velocity of 70 kilometers per second. Assuming acceleration due to Earth's gravity is constant, and neglecting air resistance, how fast will the meteoroid be falling when it hits the ground? What will its acceleration be?

$71.249\,\frac{\text{km}}{\text{sec}}$

$a = -9.8\ m/s^2$ $\quad -9.8\frac{m}{s^2} \times \frac{1\,km}{1000\,m}$

$a = -0.0098\ m/s^2$



$V = -0.0098t + C$

$-70 = 0 + C$

$\underline{V = -0.0098t - 70}$

$h = -0.0049t^2 - 70t + C$

$9000 = 0 + 0 + C$

$\boxed{h = -0.0049t^2 - 70t + 9000}$

$-0.0049t^2 - 70t + 9000 = 0$

$$t = \frac{70 \pm \sqrt{(-70)^2 - 4(-0.0049)(9000)}}{2(-0.0049)}$$

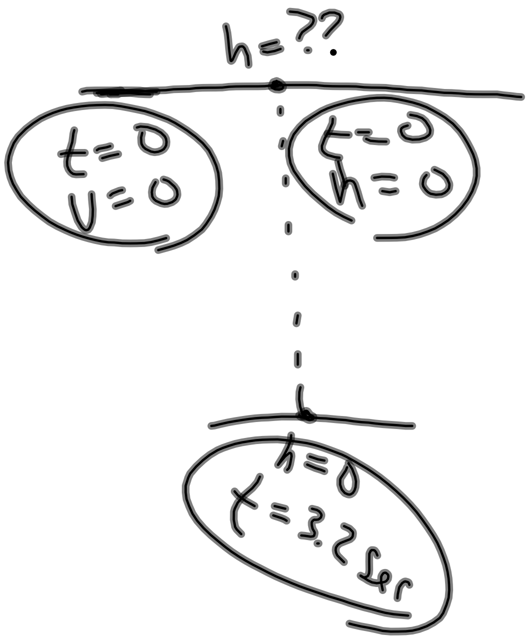
$\underline{t = 127.449\ sec}$ oR $\cancel{t = \ldots}$ $\qquad \frac{70 \pm 71.249}{-0.0098}$

$V = -0.0098(127.449) - 70$

$\underline{V = -71.249\ Km/sec}$

Example:
You drop a rock off Quechee Gorge Bridge and it hits the water below about 3.2 seconds later. Approximately how high is the bridge?

50.176m



$a = -9.8$

$v = -9.8t + c$

$t=0,\ v=0$

$v = -9.8t$

$h = -4.9t^2 + C$

$0 = -4.9(3.2)^2 + C$

$50.176\text{m} = C$

$h = -4.9t^2 + 50.176$

Bridge is <u>50.176m high</u>

Motion Problems:

A cat, walking along the window ledge of a New York apartment, knocks off a flowerpot that falls to the street 122.5m below. How fast is the flowerpot travelling when it hits the street below?

$t = 0$

$h = 122.5$

$v = 0$

$a = -9.8$

$\boxed{v = -9.8t}$

$h = -4.9t^2 + c$

$\boxed{h = -4.9t^2 + 122.5}$

When will $h = 0$ . . .

$-4.9t^2 + 122.5 = 0$

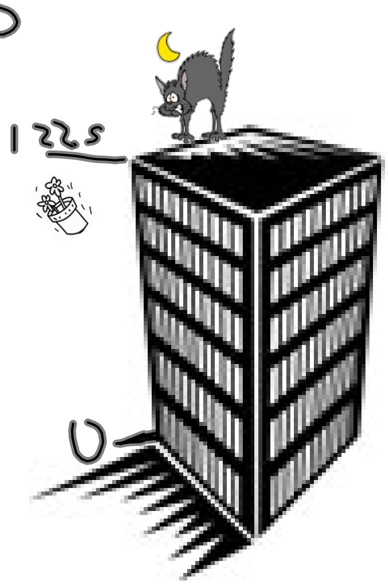
$\frac{-4.9t^2}{-4.9} = \frac{-122.5}{-4.9}$

$t^2 = 25$

$t = 5$

$v(5) = -9.8(5)$

$= \underline{-49 \text{ m/s}}$



Two balls are thrown upward from the top of a cliff 432 feet above the ground. The first ball is thrown with a speed of 48 ft./s and the other is thrown one second later with a speed of 24 ft./s. When will the two balls pass each other, and how high are they above the ground when they pass each other?

Ball 1	Ball 2
$t=0$	* $t=1$
$h=432$	$h=432$
$v=48$	$v=24$

$t=0$
$h=432$
$v=24$

$$\boxed{\begin{aligned} a&=-32\\ V&=-32t+48\\ h&=-16t^2+48t+432\end{aligned}}$$

Ball 2:

$a=-32$

$V=-32t+C$

$24=-32(1)+C$

$56=C$

$V=-32t+56$

$h=-16t^2+56t+C$

$432=-16(1)^2+56(1)+C$

$432=-16+56+C$

$392=C$

$$\boxed{h=-16t^2+56t+392}$$

$-\cancel{16t^2}+56t+392=\cancel{16t^2}+48t+432$

$8t=40$

$t=5$ sec

$h(5)=-16(5)^2+56(5)+392$

$=272$ feet

$$+\frac{3}{2x} \Longrightarrow \frac{3}{2}\left(\frac{1}{x}\right)$$

$$\frac{3}{3}\ln(x)$$

$$\frac{\cancel{7}}{\cancel{7}x} \qquad \frac{3}{2}\left(\frac{2}{2x}\right)$$

$$\frac{1}{x} \qquad \frac{3}{2}\ln(2x)$$