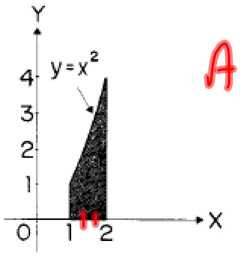
Warm-Up

x	y
1	1
4/3	16/9
5/3	25/9
2	4

$\frac{1}{2}(a+b)h$



$A=\frac{1}{2}\left(\frac{1}{3}\right)\left[\left(1+\frac{16}{9}\right)+\left(\frac{16}{9}+\frac{25}{9}\right)+\left(\frac{25}{9}+4\right)\right]$

$\frac{1}{6}\left(\frac{127}{9}\right)=\frac{127}{54}$

Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x=\frac{4}{3}$ and $x=\frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

$F(x)=(\ln x)^3\left(\frac{1}{x}\right)$

$F(x)=\frac{1}{4}(\ln x)^4+C$

$0=\frac{1}{4}(\ln 1)^4+C$

$0=0+C$

$C=0$

$F(9)=\frac{1}{4}(\ln 9)^4$

Midterm 1:

Wednesday, March 19

Sigma Notation

$$\sum_{k=1}^{n} f(x_k) = f(x_1) + f(x_2) + f(x_3) + \cdots + f(x_n)$$

Example:

Evaluate $\displaystyle\sum_{k=1}^{4} (3k-2)$

$= (3(1)-2) + (3(2)-2) + (3(3)-2) + (3(4)-2)$

$= 1 + 4 + 7 + 10$

$= 22$

What about the following?

$$\sum_{k=1}^{8} (5w) = 5w(8) = 40w$$

$= 5w + 5w + 5w + 5w$

$= 20w$

$$\sum_{k=1}^{8} (-3) = -3(8) = -24$$

$= -3 + -3 + -3$

$= -3(3)$

$= -9$

Summation properties...

$$\sum_{i=1}^{n} C = Cn$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} Ca_i = C\sum_{i=1}^{n} a_i$$

Evaluate the following...
(i) without summation properties
(ii) using summation properties

$$\sum_{i=1}^{3} (2i^2 - 5i + 3)$$

$$(1)\ (2(1)^2-5(1)+3)+(2(2)^2-5(2)+3)+(2(3)^2-5(3)+3)$$

$$= 0 + 1 + 6$$

$$= 7$$

$$2\sum_{i=1}^{3} i^2 - 5\sum_{i=1}^{3} i + \sum_{i=1}^{3} 3$$

$$2(1+4+9)-5(1+2+3)+3(3)$$

$$= 28 - 30 + 9$$

$$= 7$$

Summation Rules

$$\sum_{k=1}^{n} k = 1+2+3+4+\ldots+(n-1)+n$$

$$\sum_{k=1}^{n} k^2 = 1+4+9+16+\ldots+(n-1)^2+n^2$$

$$\sum_{k=1}^{n} k^3 = 1+8+27+64+\ldots+(n-1)^3+n^3$$

Notice that these are INFINITE

$$\sum_{k=1}^{n} k = \frac{n^2}{2}+\frac{n}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^4}{4}+\frac{n^3}{2}+\frac{n^2}{4}$$

Notice that these are FINITE

Let's revisit the example used yesterday...

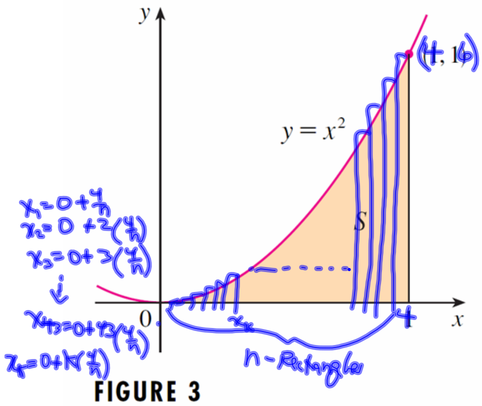


FIGURE 3

We want to find the area below this curve using " n" rectangles.

What will be the width of each rectangle? $\Delta x = \frac{4}{n}$

How will we determine the height of each rectangle? x_k

$x_k = \frac{4k}{n}$ $\quad\quad$ $f(x_k)$

Write out an expression for the area of these "n" rectangles?

$$A = \Delta x \, f(x_k)$$

$$A = \sum_{k=1}^{n} \Delta x \, f(x_k)$$

$$A = \sum_{k=1}^{n} \left(\frac{4}{n}\right) f\left(\frac{4k}{n}\right)$$

$f(x) = x^2$ $\quad A = \left(\frac{4}{n}\right) \sum_{k=1}^{n} f\left(\frac{4k}{n}\right)$

$$\frac{4}{n}\left(\overset{k=1}{f\left(\frac{4}{n}\right)} + \overset{k=2}{f\left(\frac{8}{n}\right)} + \overset{k=3}{f\left(\frac{12}{n}\right)} + \dots + \overset{k=n}{f\left(\frac{4n}{n}\right)}\right)$$

$$\frac{4}{n}\left(\left(\frac{4}{n}\right)^2 + \left(\frac{8}{n}\right)^2 + \left(\frac{12}{n}\right)^2 + \dots + \frac{16n^2}{n^2}\right)$$

$$\frac{4}{n}\left(\frac{16}{n^2} + \frac{64}{n^2} + \frac{144}{n^2} + \dots + \frac{16n^2}{n^2}\right)$$

$$\frac{4}{n}\left(\frac{16}{n^2}\right)\left(1 + 4 + 9 + \dots + n^2\right)$$

$$\sum_{k=1}^{n} k = \frac{n^2}{2} + \frac{n}{2}$$
$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$
$$\sum_{k=1}^{n} k^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

$$\frac{4}{n}\left(\frac{16}{n^2}\right)\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$$

$$A = \frac{64}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$$

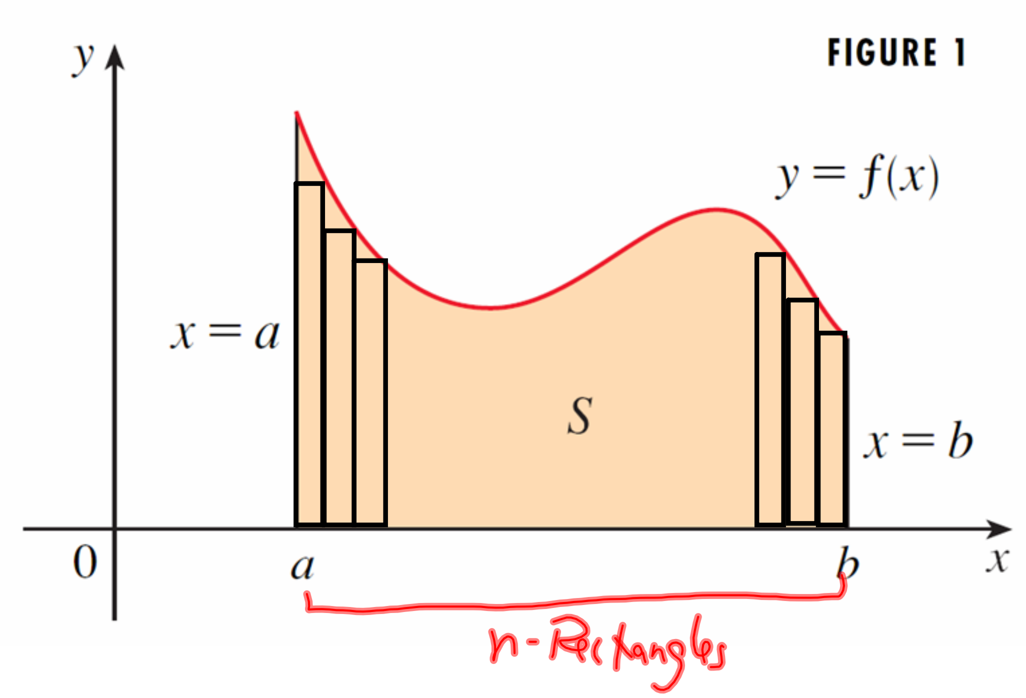
$$A = \frac{64}{3} + \frac{32}{n} + \frac{64}{6n^2}$$ Represents area given "n" Rectangles

$$\lim_{n\to\infty}\left(\frac{64}{3} + \frac{32}{n} + \frac{64}{6n^2}\right)$$

$$= \frac{64}{3} + 0 + 0$$

$$\boxed{A = \frac{64}{3} u^2}$$

Develop a general formula for the area below a curved surface using "n" rectangles.



width Δx

$\boxed{\Delta x = \frac{b-a}{n}}$

height x_k

$f(x_k) \rightarrow x_k = a + (\Delta x)k$

area

$= \Delta x \sum_{k=1}^{n} f(x_k)$

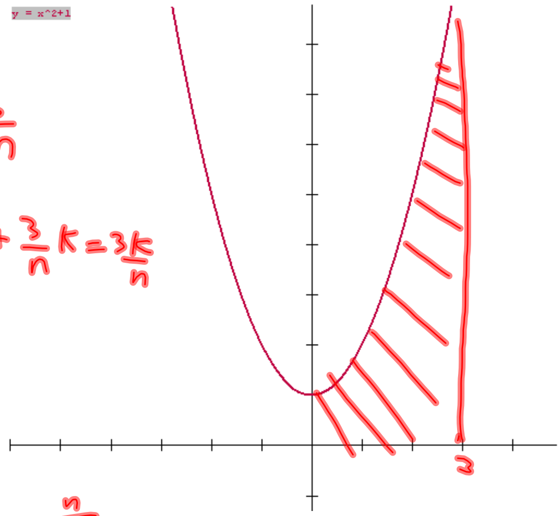
$$A = \Delta x \sum_{k=1}^{n} f(x_k)$$

$$\Delta x = \frac{b-a}{n} \qquad x_k = a + (\Delta x)k$$

Use a Riemann Summation to determine the area below the curve y = x^2 + 1, between x = 0 and x = 3.

$\Delta x = \frac{3}{n}$

$x_k = 0 + \frac{3}{n}k = \frac{3k}{n}$



$A = \frac{3}{n}\sum_{k=1}^{n} f\left(\frac{3k}{n}\right)$ $\qquad f(x) = x^2+1$

$A = \frac{3}{n}\sum_{k=1}^{n}\left[\left(\frac{3k}{n}\right)^2 + 1\right]$

$A = \frac{3}{n}\left[\frac{9}{n^2}\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1\right]$

$A = \frac{3}{n}\left[\frac{9}{n^2}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right) + 1n\right]$

Use Summation Properties

$A = \frac{3}{n}\left(3n + \frac{9}{2} + \frac{3}{2n} + n\right)$

$A = 9 + \frac{27}{2n} + \frac{9}{2n^2} + 3$

$A = 12 + \frac{27}{2n} + \frac{9}{2n^2}$

$\lim_{n\to\infty}\left(12 + \frac{27}{2n} + \frac{9}{2n^2}\right)$

$= 12u^2$

$\int_0^3 (x^2+1)\,dx$

$= \frac{x^3}{3} + x \Big|_0^3$

$= 9 + 3$

$= 12$

Homework

Use a Riemann sum to determine the area bound by the curve $f(x) = -x^2 + 4$ and the x-axis.