# Warm Up $x^{1/2}$ $\frac{1}{2}x^{-1/2}$

Determine the general antiderivative:

$$f'(x)=\frac{5x}{(7x^2-2)^4}-\frac{\csc\sqrt{x}\cot\sqrt{x}}{\sqrt{x}}+\frac{\sin 2x}{\sqrt{1-\cos^2(2x)}}-x^3e^{5x^4}+\frac{3x-2}{3x^2-4x+1}+\sqrt[5]{x^7}-8\pi$$

$$f'(x)=\frac{5}{14}(7x^2-2)^{-4}14x-2\csc\sqrt{x}\cot\sqrt{x}\,\frac{1}{2}x^{-1/2}\pm\frac{1}{2}\frac{-\sin(2x)(2)}{\sqrt{1-(\cos 2x)^2}}$$

$$-\frac{1}{20}e^{5x^4}20x^3+\frac{1}{2}\frac{6x-4}{3x^2-4x+1}+x^{7/5}-8\pi$$

$$f(x)=-\frac{5}{42}(7x^2-2)^{-3}+2\csc\sqrt{x}\underbrace{-\frac{1}{2}\sin^{-1}(\cos 2x)}_{x}-\frac{1}{20}e^{5x^4}+\frac{1}{2}\ln(3x^2-4x+1)$$

$$+\frac{5}{12}x^{12/5}-8\pi x+C$$

$$+\frac{1}{2}\cos^{-1}(\cos 2x)$$

$$=\frac{1}{2}(2x)=x$$

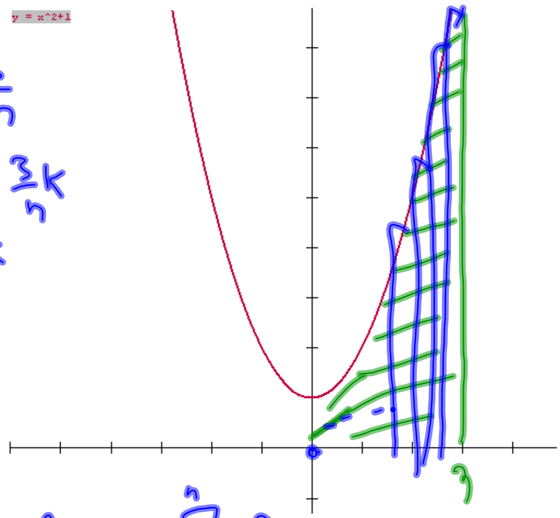
$$\frac{\sin 2x}{\sqrt{1-\cos^2 2x}}=\frac{\sin 2x}{\sqrt{\sin^2 2x}}=\frac{\sin 2x}{\sin 2x}=1$$

Use a Riemann Summation to determine the area below the curve y = x² + 1, between x = 0 and x = 3.

$$\Delta x = \frac{3}{n}$$

$$x_k = 0 + \frac{3}{n}k$$

$$x_k = \frac{3k}{n}$$




$$A = \Delta x \sum_{k=1}^{n} f(x_k)$$

$$A = \frac{3}{n} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right)$$

$$A = \frac{3}{n} \sum_{k=1}^{n} \left[\left(\frac{3k}{n}\right)^2 + 1\right]$$

$$A = \frac{3}{n} \sum_{k=1}^{n} \left(\frac{9k^2}{n^2} + 1\right)$$

$$A = \left(\frac{3}{n}\right)\left[\left(\frac{9}{n^2}\right) \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1\right]$$

$$\sum_{k=1}^{n} k = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

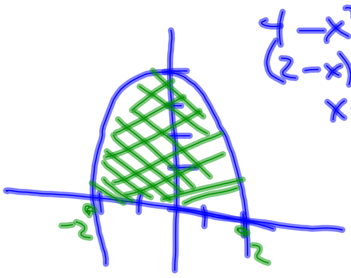
$$A = \frac{27}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + 1n\left(\frac{3}{n}\right)$$

$$A = 9 + \frac{27}{2n} + \frac{9}{2n^2} + 3$$

$$\lim_{n \to \infty}\left(12 + \frac{27}{2n} + \frac{9}{2n^2}\right)$$

$$\underline{A = 12u^2}$$

Use a Riemann sum to determine the area bound by the curve $f(x) = -x^2 + 4$ and the $x$-axis.



$4 - x^2 = 0$

$(2-x)(2+x) = 0$

$x = \pm 2$

$$-\frac{x^3}{3} + 4x \Big|_{-2}^{2}$$

$$\left(-\frac{8}{3}+8\right) - \left(\frac{8}{3}-8\right)$$

$$-\frac{16}{3} + \frac{48}{3} = \frac{32}{3}$$

$\Delta x = \frac{4}{n}$ $\quad x_k = -2 + \frac{4k}{n}$

$f(x) = -x^2 + 4$

$$A = \frac{4}{n}\sum_{k=1}^{n} f\left(-2+\frac{4k}{n}\right)$$

$$A = \frac{4}{n}\sum_{k=1}^{n}\left(-\left(-2+\frac{4k}{n}\right)^2 + 4\right)$$

$$A = \frac{4}{n}\sum_{k=1}^{n}\left[\left(\cancel{4} - \frac{16k}{n} + \frac{16k^2}{n^2}\right)\cancel{+4}\right]$$

$$A = \frac{4}{n}\sum_{k=1}^{n}\left(\frac{16k}{n} - \frac{16k^2}{n^2}\right)$$

$$A = \frac{4}{n}\left[\frac{16}{n}\sum_{k=1}^{n} k - \frac{16}{n^2}\sum_{k=1}^{n} k^2\right]$$

$$\sum_{k=1}^{n} k = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

$$A = \frac{4}{n}\left[\frac{16}{n}\left(\frac{n^2}{2}+\frac{n}{2}\right) - \frac{16}{n^2}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)\right]$$

$$A = \frac{4}{n}\left(8n + \cancel{8} - \frac{16n}{3} \cancel{- 8} - \frac{8}{3n}\right)$$

$$A = \frac{32}{1} - \frac{64}{3} - \frac{32}{3n^2}$$

$$A = \frac{96}{3} - \frac{64}{3} - \frac{32}{3n^2}$$

$$A = \frac{32}{3} - \frac{32}{3n^2}$$

$$\lim_{n\to\infty}\left(\frac{32}{3} - \frac{32}{3n^2}\right)$$

$$= \frac{32}{3}u^2$$

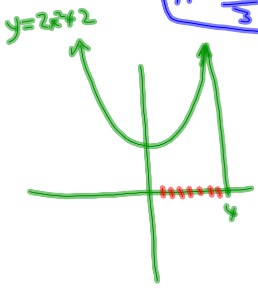
Example:

Suppose $\boldsymbol{R}$ is the area in the first quadrant below the curve
$y = 2x^2 + 2$ between $x = 0$ and $x = 4$ .

*(a) Approximate the area of "R" using 8 rectangles with height determined by the left-hand endpoint* $= 43u^2$

*(b) Determine the exact area of "R" using a Riemann Summation.* $\boxed{A = \frac{152}{3}u^2}$

(a)




$A = \frac{1}{2}(2+2.5+4+6.5+10+14.5+20+26.5)$

$A = \frac{1}{2}(86) = \underline{43u^2}$

(II) $\Delta x = \frac{4}{n}$

$x_k = 0 + \frac{4}{n}k$

$= \frac{4k}{n}$

$A = \Delta x \sum_{k=1}^{n} f(x_k)$

$= \frac{4}{n} \sum_{k=1}^{n} f\left(\frac{4k}{n}\right)$

$= \frac{4}{n} \sum_{k=1}^{n} \left(2\left(\frac{4k}{n}\right)^2 + 2\right)$

$= \frac{4}{n} \sum_{k=1}^{n} \left(\frac{32k^2}{n^2} + 2\right)$

$= \frac{4}{n}\left[\frac{32}{n^2} \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 2\right]$

$= \frac{4}{n}\left[\frac{32}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + 2n\right]$

$= \frac{4}{n}\left(\frac{32n}{3} + 16 + \frac{16}{3n} + 2n\right)$

$= \frac{128}{3} + \frac{64}{n} + \frac{64}{3n^2} + \frac{8}{1}$

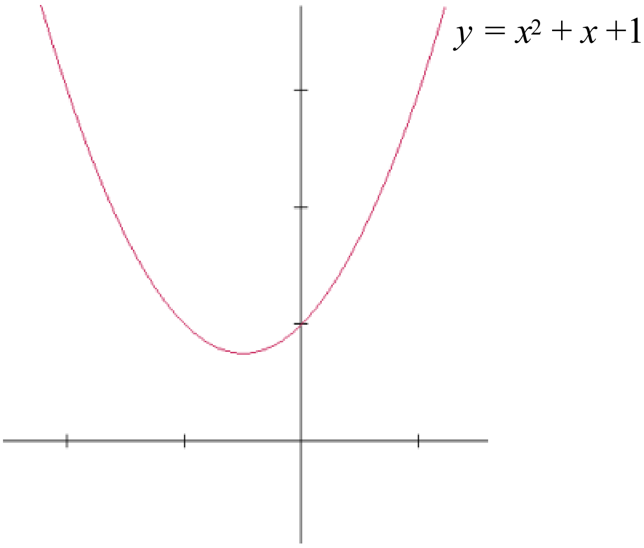
$= \frac{152}{3} + \frac{64}{n} + \frac{64}{3n^2}$

$\lim_{n\to\infty}\left(\frac{152}{3} + \frac{64}{n} + \frac{64}{3n^2}\right)$

$= 152$

Use a Riemann Summation to determine the area below the curve $y = x^2 + x + 1$ and above the x-axis, between x = −2 and x = 1.

$\boxed{A = \frac{9}{2} u^2}$



# Practice Exercises:

Determine each of the following areas using a Riemann Sum with an infinite number of rectangles:

(1) Area below $f(x) = 2x^2 - 1$ between $x = 1$ and $x = 3$. $\frac{46}{3}$

(2) Area below $f(x) = x^2 - 4x + 7$ between $x = -1$ and $x = 2$. $18$

(3) Area below $f(x) = x^3 + 2x^2 + x$ between $x = 0$ and $x = 1$. $\frac{17}{12}$