Compositions of Functions

When the input in a function is another function, the result is called a **composite function.** If

$f(x) = 3x + 2$ and $g(x) = 4x - 5$

then $f[g(x)]$ is a composite function. The statement $f[g(x)]$ is read " f of g of x" or "the composition of f with g." $f[g(x)]$ can also be written as

$(f \circ g)(x)$ or $f \circ g(x)$ $\quad (f \circ g)(x) = f(x)\,g(x)$

The symbol between f and g is a small open circle. When replacing one function with another, be very careful to get the order correct because compositions of functions are not necessarily commutative (as you'll see).

$f(g(x)) = 3(g(x)) + 2$ $\quad f(☺) = 3☺ + 2$

$= 3(4x-5)+2$

can do this algebraically

$= 12x - 15 + 2$

$= 12x - 13$

$f(g(2)) = 12(2) - 13$

$= 11$

OR

$g(2) = 4(2) - 5$

$= 3$

$f(3) = 3(3) + 2$

$= 11$

$(g \circ f)(x)$

$g[f(x)]$

$g(f(x)) = 4(f(x)) - 5$

$= 4(3x+2) - 5$

$= 12x + 8 - 5$

$= 12x + 3$

$(f \circ g \circ g)(x)$

$f[g(g(x))]$

$g(g(x)) = 4(g(x)) - 5$

$= 4(4x-5) - 5$

$= 16x - 20 - 5$

$= 16x - 25$

$f(16x-25) = 3(16x-25) + 2$

$= 48x - 75 + 2$

$= 48x - 73$

$f(x) = 3x + 2$ and $g(x) = 4x - 5$

$(g \circ f \circ f)(-1)$

Method:

$f(-1) = 3(-1) + 2 = -1$

$f(-1) = -1$

$g(-1) = 4(-1) - 5$

$= -9$

Example 1

If $f(x) = 3x + 2$ and $g(x) = 4x - 5$, find each of the following.

1. $f[g(4)]$
2. $g \circ f(4)$
3. $f[g(x)]$
4. $(g \circ f)(x)$

① $g(4) = 16 - 5$
$= 11$
$f(11) = 3(11) + 2$
$= 35$

2) $f(4) = 3(4) + 2$
$= 14$
$g(14) = 4(14) - 5$
$= 51$

③ $f(g(x)) = 3(g(x)) + 2$
$= 3(4x - 5) + 2$
$= 12x - 13$

④ $g(f(x)) = 4(f(x)) - 5$
$= 4(3x + 2) - 5$
$= 12x + 3$

Warm Up

If $f(x)=\dfrac{4}{x-1}$ and $g(x)=2x$, then the solution set of $f(g(x))=g(f(x))$ is

(A) $\left\{\dfrac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1,2\}$ (E) $\left\{\dfrac{1}{3},2\right\}$

$$f(g(x))=\frac{4}{g(x)-1}=\frac{4}{2x-1}$$

$$g(f(x))=2\left(\frac{4}{x-1}\right)=\frac{8}{x-1}$$

Cross-Multiply

$$\frac{4}{2x-1}=\frac{8}{x-1}$$

$$4x-4=16x-8$$

$$\frac{4}{12}=\frac{12x}{12}$$

$$\frac{1}{3}=x$$

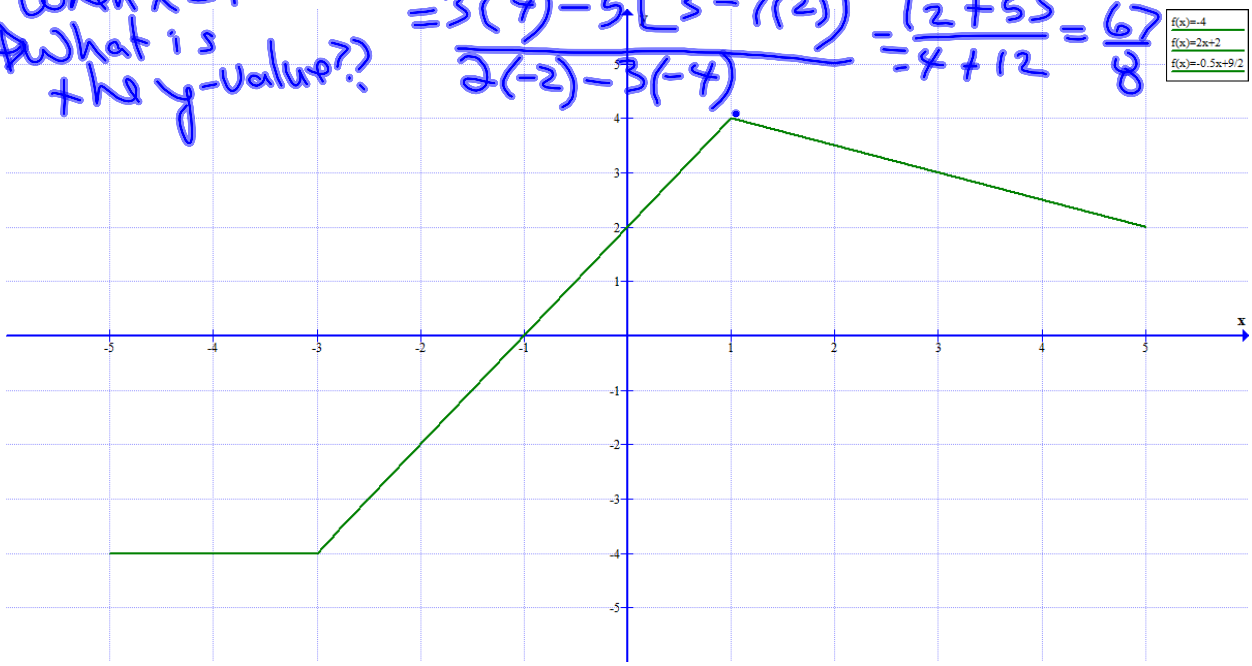
Given the graph of f(x) shown below, evaluate the following:

$$\frac{3f(1)-5\left[f(3)-7f(0)\right]}{2f(-2)-3f(-4)}$$

f(1)

When x=1 what is the y-value??

$$=\frac{3(4)-5[3-7(2)]}{2(-2)-3(-4)}=\frac{12+55}{-4+12}=\frac{67}{8}$$



t	$H(t)$
-1	7.4
3.1	8

t	$G(t)$
0	3.1
7.4	2
8	-1

$$(H \circ G)(8) = 7.4$$

$$(G \circ \overbrace{H)(3.1)}^{8}$$

$$= -1$$

Practice Set:

Pg. 1

1. -19
2. 99
3. 1731
4. 290
5. -3
6. 145
7. 29
8. 730
9. 219

Pg. 2

10. 42
11. -11
12. 13
13. 14
14. 130
15. 110
16. 6
17. 10
18. -81