Sketch the following piecewise function:

$$f(x)=\begin{cases} -2x+3, & \text{if } x \leq -2 \\ (x+1)^2-2, & \text{if } -2 < x \leq 1 \\ 3x-1, & \text{if } 1 < x < 2 \\ 5, & \text{if } x \geq 2 \end{cases}$$

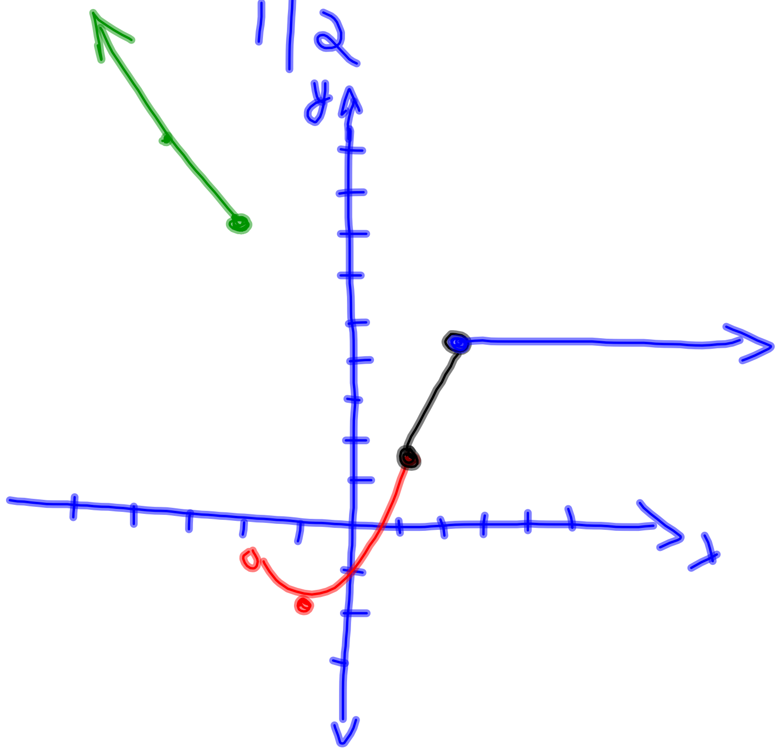
x	y
-2	7
-3	9

Parabala
V(-1,-2)

x	y
-2	-1
1	2

x	y
1	2
2	5

* Horizontal Line



Given the function: $f(x) = -3|4-3x| + 2$

(a) Evaluate *f*(2)

(b) Express *f*(x) as a piecewise function

(a) $f(2) = -3|4-3(2)| + 2$ b)

$= -3|-2| + 2$

$= -3(2) + 2$

$= -4$

$f(x) = -3|4-3x| + 2$

Two Possibilities ....

Case 1: BBP (Between Bars Positive)

If $4-3x > 0$ then.. $f(x) = -3(4-3x) + 2$

$-3x > -4$ $\qquad = -12 + 9x + 2$

$x < \frac{4}{3}$ $\qquad = 9x - 10$

Case 2: BBN

If $4-3x \leq 0$ then $f(x) = -3(\cancel{-}4+3x) + 2$

$-3x \leq -4$ $\qquad f(x) = 12 - 9x + 2$

$x \geq \frac{4}{3}$ $\qquad f(x) = -9x + 14$

$$f(x) = \begin{cases} 9x - 10, & \text{if } x < \frac{4}{3} \\ -9x + 14, & \text{if } x \geq \frac{4}{3} \end{cases}$$

Warm-Up...

Given the function $f(x)=\begin{cases} 2-x^2 & \text{if } x<1 \\ 3 & \text{if } x=1 \\ 2x-1 & \text{if } 1<x\le 3 \\ (x-4)^2 & \text{if } x>3 \end{cases}$

Evaluate the following: f(-1) f(1) f(3) f(2)

- Draw a sketch of this function

$f(-1)=2-(-1)^2$
$=1$

$f(1)=3$

$f(3)=2(3)-1$
$=5$

$f(2)=2(2)-1$
$=3$

① $f(x)=2-x^2$ $(x<1)$
- Parabola $y=-x^2+2$
- $V(0,2)$ Down

x	y
1	1

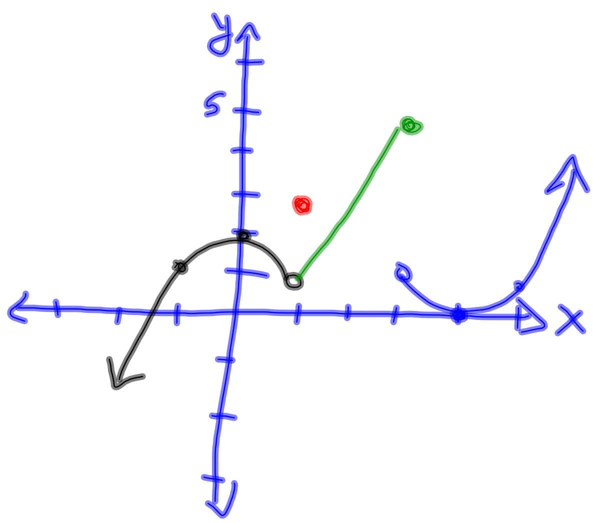
② $f(x)=3$ $x=1$
$(1,3)$

③ $y=2x-1$

x	y
1	1
3	5

④ $y=(x-4)^2$ $(x>3)$
Parabola (up)
$V(4,0)$

x	y
4	0
3	1

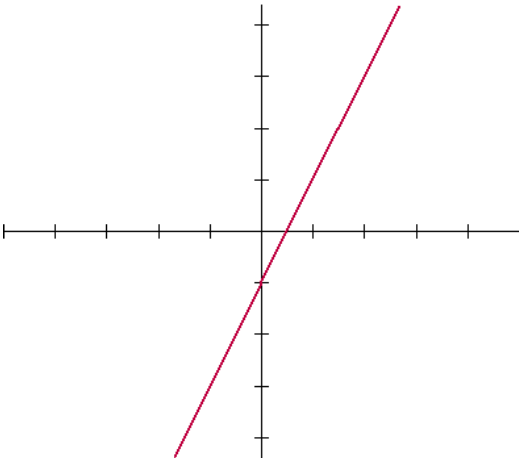


Catalog of Essential Functions

1. Linear

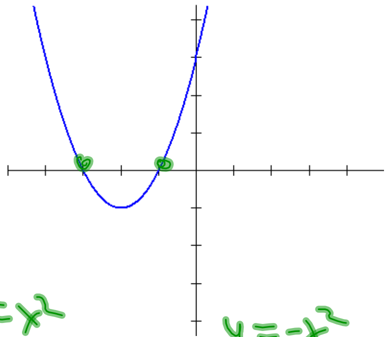
$y = x^{2}x^{3} + x^{7}x^{4}$
5 11

$y = 3x^7 + 7x^4 - x^3$
degree 7

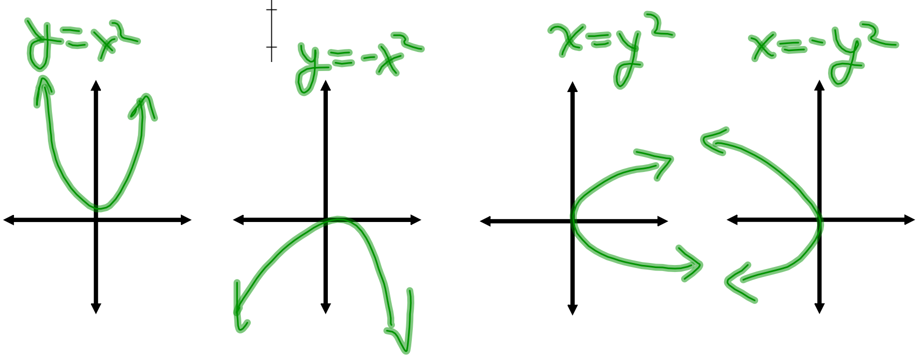


- Straight line
- Equation will be degree one
- Should be able to identify ***slope***, ***intercepts***, and ***equation*** from the graph

2. Quadratic

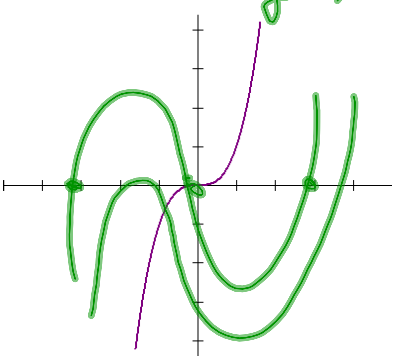


- Parabola (U-Shaped)
- Either *x* or *y* will be squared (Not both!)
- Should know the 4 basic quadratic functions
- Should be able to apply transformations to the basic quadratic functions



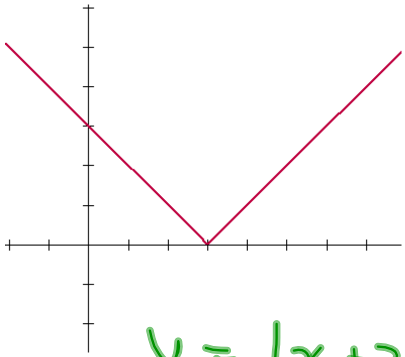
3. Cubic

$y=x^3$



- S-Shaped
- Will work with functions having *x* raised to the third power
- Should be able to apply transformations to the basic ~~quadratic~~ functions

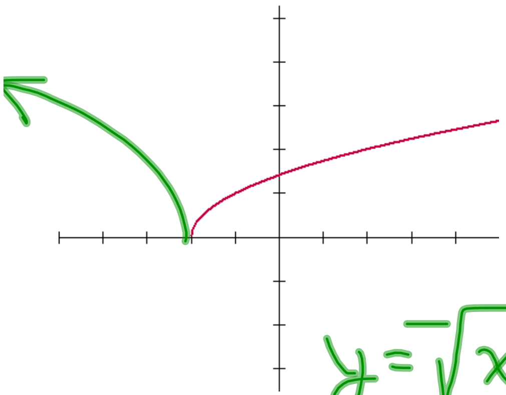
4. Absolute Value



$y = |x+3|$

- V-Shaped
- Equation will have a variable within the absolute value bars
- Should be able to apply transformations to the basic absolute value functions

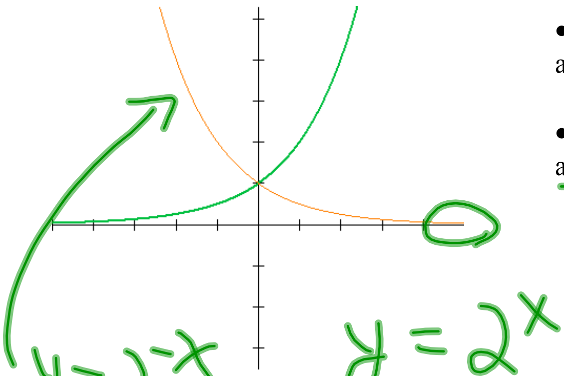
5. Square Root



$y = \sqrt{x+2}$

- Half parabola
- Equation will have a variable under the square root sign
- Should be able to apply transformations to the basic square root function

6. Exponential



- Steadily increasing or decreasing
- Base will be a number and variable will appear in the exponent
- Should be able to identify the horizontal asymptote

$y=2^{-x}$ $y=2^x$

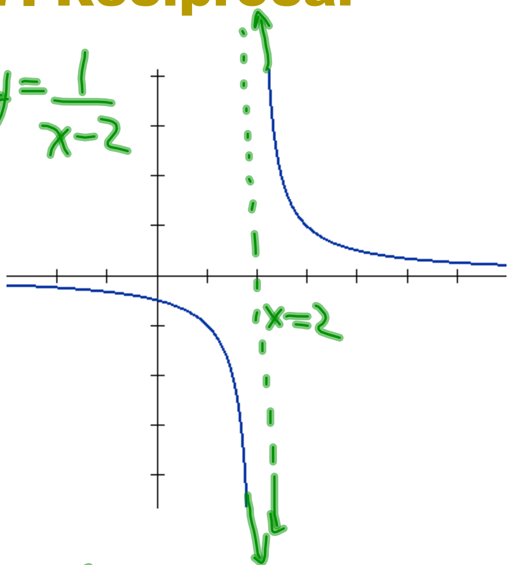
or $y=\left(\frac{1}{2}\right)^x$

$x=-100$

$y=2^{-100}=\frac{1}{2^{100}}$

7. Reciprocal

$y=\frac{1}{x-2}$

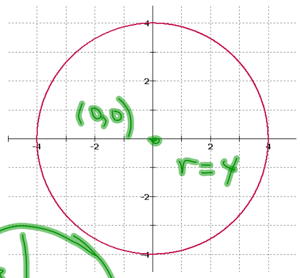


- Will have two branches
- Equation will have a variable within denominator of a rational expression
- Be able to identify the vertical and horizontal asymptotes

$y=\frac{1}{x}$

Asymptote: Graph approaches but DOES NOT Touch this line

8. Circle



- General form: $\boxed{(x-h)^2 + (y-k)^2 = r^2}$

* center: (h, k)
* radius $= r$

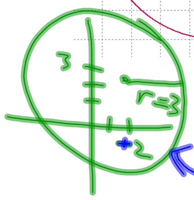
- Be able to identify the function that would describe either just the top or bottom of the circle.

$r^2 = 49$

$(x+6)^2 + (y-2)^2 = 49$

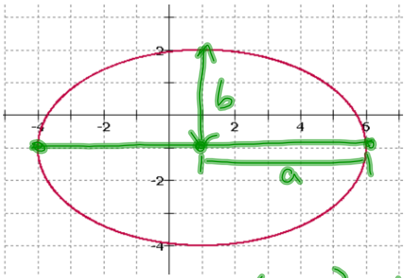
C: $(-6, 2)$
$r = 7$

$x^2 + y^2 = 16$



$$\boxed{(x-2)^2 + (y-3)^2 = 9}$$

9. Ellipse

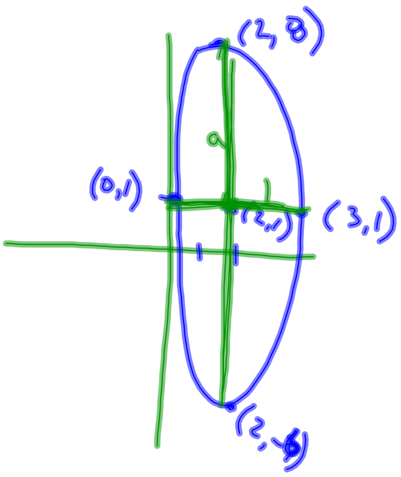


- General form: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

Where…

- Center: (h, k)
- a > b
- If a is the denominator of the "y" term the ellipse will have a vertical major axis.

$$\frac{(x-1)^2}{25} + \frac{(y+1)^2}{9} = 1$$

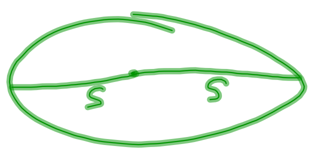


$$\frac{(x-2)^2}{b^2} + \frac{(y-1)^2}{a^2} = 1$$

(1) (49)

$$\frac{(x+7)^2}{25} + \frac{(y-1)^2}{16} = 1$$

ellipse $a=5$
C: $(-7, 1)$



Horizontal Major axis = 10
Minor " =

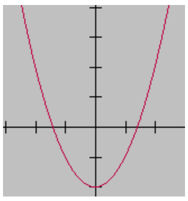
Symmetry

Even

$$f(-x) = f(x)$$

Even functions are line symmetric about the y-axis

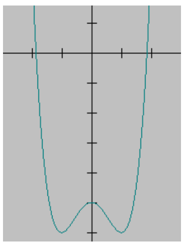
$y = x^2 - 2$



$y = (-x)^2 - 2$

$y = x^2 - 2$

$y = x^4 - 2x^2 - 5$



All even exponents

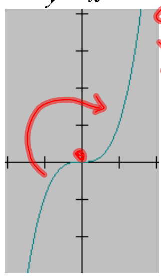
Odd

$$f(-x) = -f(x)$$

Odd functions are point symmetric about the origin

$y = x^3$

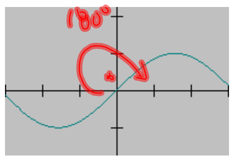
180°



$y = (-x)^3$

$y = -x^3$

$y = \sin(x)$



$f(x) = 3x^3 - 5x^5$

$f(-x) = 3(-x)^3 - 5(-x)^5$

$= -3x^3 + 5x^5$

$= -f(x)$

odd symmetry

ex. $f(x) = 3x^3 - x^2 + 5$

Neither

$f(x) = -(2x^7 - x^3 + 2)$

$f(-x) = 2(-x)^7 - (-x)^3 + 2$

$= -2x^7 + x^3 + 2$

Not odd symmetry

$f(x) = -7x^{11} - x^{13} + 7x - x^3 + 2$

Check-up:

① Express $f(x) = 2|x-4| + 1$ as a piecewise function.

② State the domain...

(a) $f(x) = \dfrac{1}{3x-5}$ (b) $f(x) = \sqrt{3x+2}$

③ $f(x) = 7x - 2$ and $g(x) = x^2 - 3$

find $f(w+3) - 5g(2w+6)$

1) Express $f(x) = 2|x-4|+1$ as a piecewise function.

Case 1: BBP

$x-4>0$

$x>4$

$f(x) = 2(x-4)+1$

$= 2x-7$

Case 2: BBN

$x-4 \leq 0$

$x \leq 4$

$f(x) = 2(-x+4)+1$

$= -2x+9$

$$f(x) = \begin{cases} 2x-7 & \text{if } x>4 \\ -2x+9 & \text{if } x \leq 4 \end{cases}$$

② State the domain...

(a) $f(x) = \dfrac{1}{3x-5}$

$3x-5=0$

$3x=5$

$x=\frac{5}{3}$

$D: \{x \mid x \in \mathbb{R}, x \neq \frac{5}{3}\}$

$(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$

" In Union With "

(b) $f(x) = \sqrt[3]{3x+2}$

$x \in \mathbb{R}$

$3x+2 \geq 0$

$3x \geq -2$

$\{x \mid, x \geq -\frac{2}{3}, x \in \mathbb{R}\}$

or $[-\frac{2}{3}, \infty)$

③ $f(x) = 7x - 2$ and $g(x) = x^2 - 3$

find $f(w+3) - 5g(2w+6)$

$7(w+3) - 2 - 5\left[(2w+6)^2 - 3\right]$

$7w + 21 - 2 - 5(4w^2 + 24w + 36 - 3)$

$7w + 21 - 2 - 20w^2 - 120w - 180 + 15$

$$\boxed{-20w^2 - 113w - 146}$$

New Functions from Old Functions...TRANSFORMATIONS

- Translations
- Stretches
- Reflections

Translation

- To *translate* or *shift* a graph is to move it up, down, left, or right without changing its shape.
- Translation is summarized by the following table and illustration:

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

Base:

$y = x^2$

$f(x) = x^2$

$f(x) + c = x^2 + c$

$f(x+c) = (x+c)^2$

Translations illustrated...

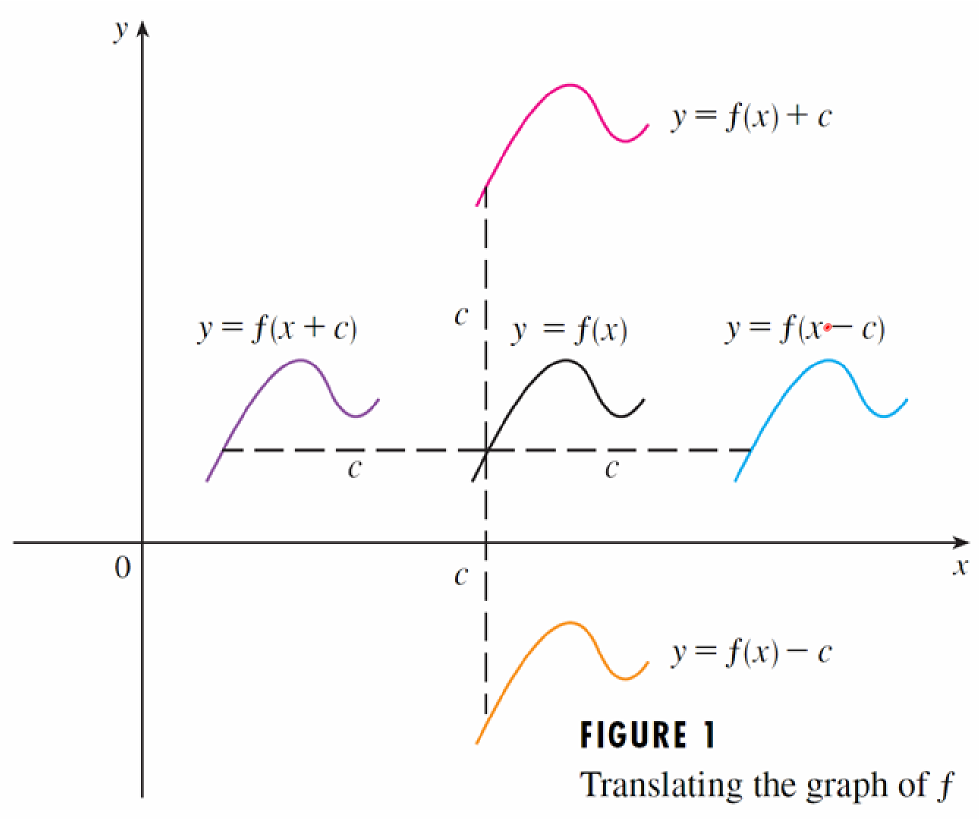
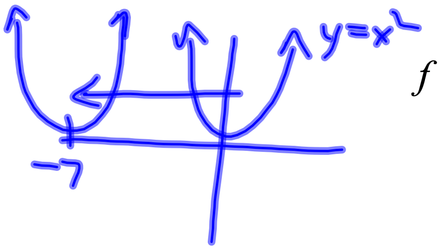


FIGURE 1
Translating the graph of f

Identify the translations for each of the following...

$f(x) = x^2$

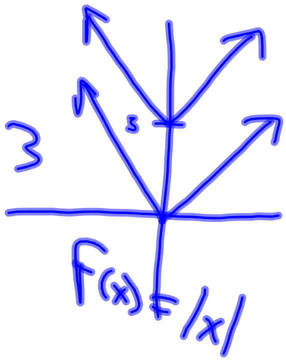
$f(x) = (x+7)^2$

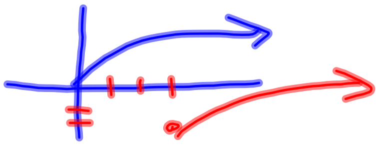


⇒ Left 7

$f(x) = |x| + 3$

⇒ Up 3



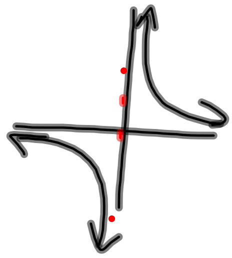


$f(x) = \sqrt{x-3} - 2$

$f(x) = \sqrt{x}$

⇒ 3 units Right
2 units down

$f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x-5} + 7$

⇒ 5 Right
7 up

Using Mapping Notation to Describe Transformations:

*Think of this as a set of instructions to follow to TRANSFORM a graph

$y=x^2$

x	$y=x^2$
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

(x, y)

2 units Up

x	$y=x^2+2$
−3	11
−2	6
−1	3
0	2
1	3
2	6
3	11

$(x, y) \rightarrow (x, y+2)$

gets mapped to ..

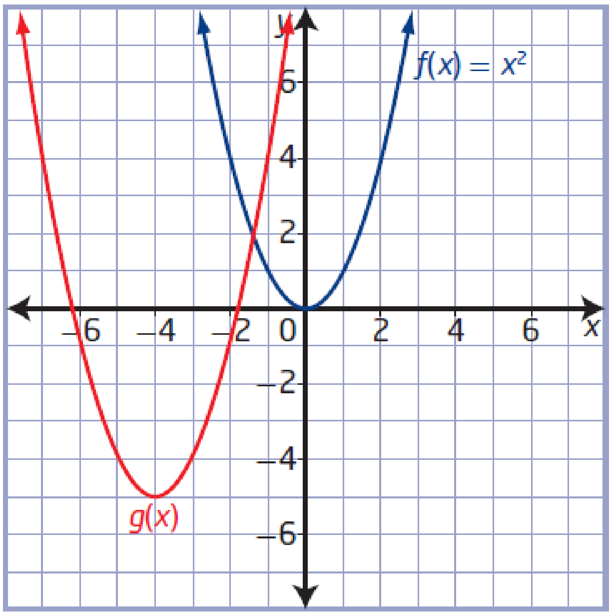
5 units Right

x	$y=(x-5)^2$
2	9
3	4
4	1
5	0
6	1
7	4
8	9

$(x, y) \rightarrow (x+5, y)$)

Determine the Equation of a Translated Function

a)



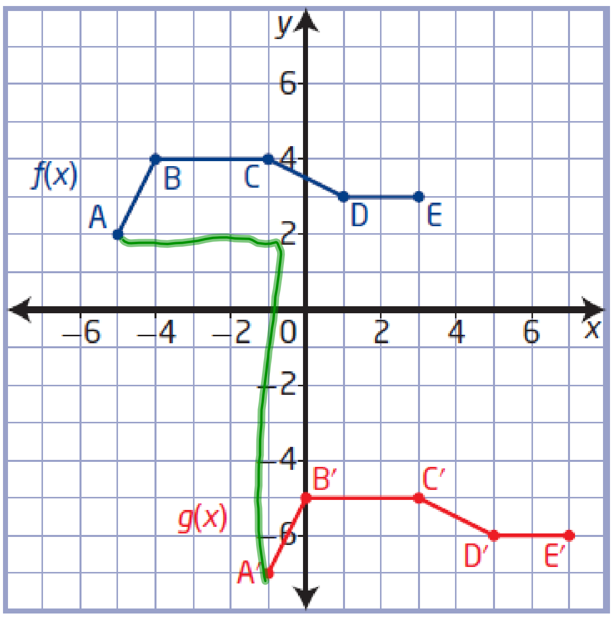
$g(x) = f(x+4) - 5$

Mapping:

$(x, y) \rightarrow (x - 4, y - 5)$

$(5, 25) \rightarrow (5 - 4, 25 - 5)$
$\rightarrow (1, 20)$

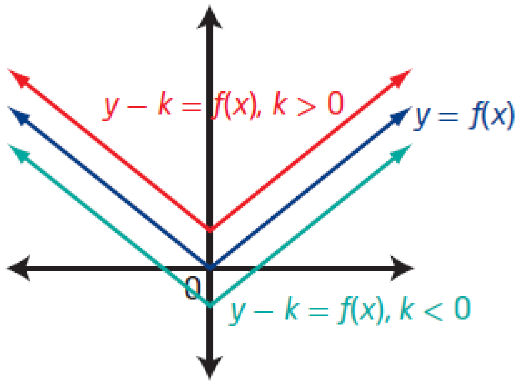
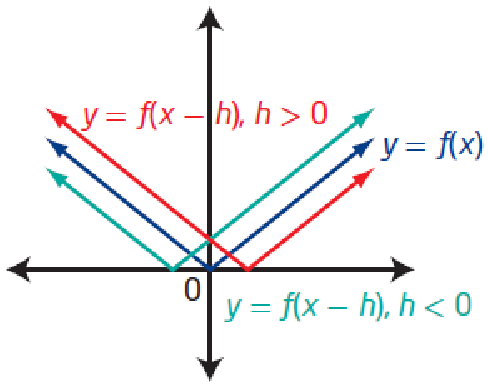
b)



$g(x) = f(x - 4) - 9$

Mapping:

$(x, y) \rightarrow (x + 4, y - 9)$

Transformation from $y = f(x)$	Mapping	Example
A vertical translation If $k > 0$, the translation is up. If $k < 0$, the translation is down.	$(x, y) \rightarrow (x, y + k)$	
A horizontal translation If $h > 0$, the translation is to the right. If $h < 0$, the translation is to the left.	$(x, y) \rightarrow (x + h, y)$	

Practice Problems...

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