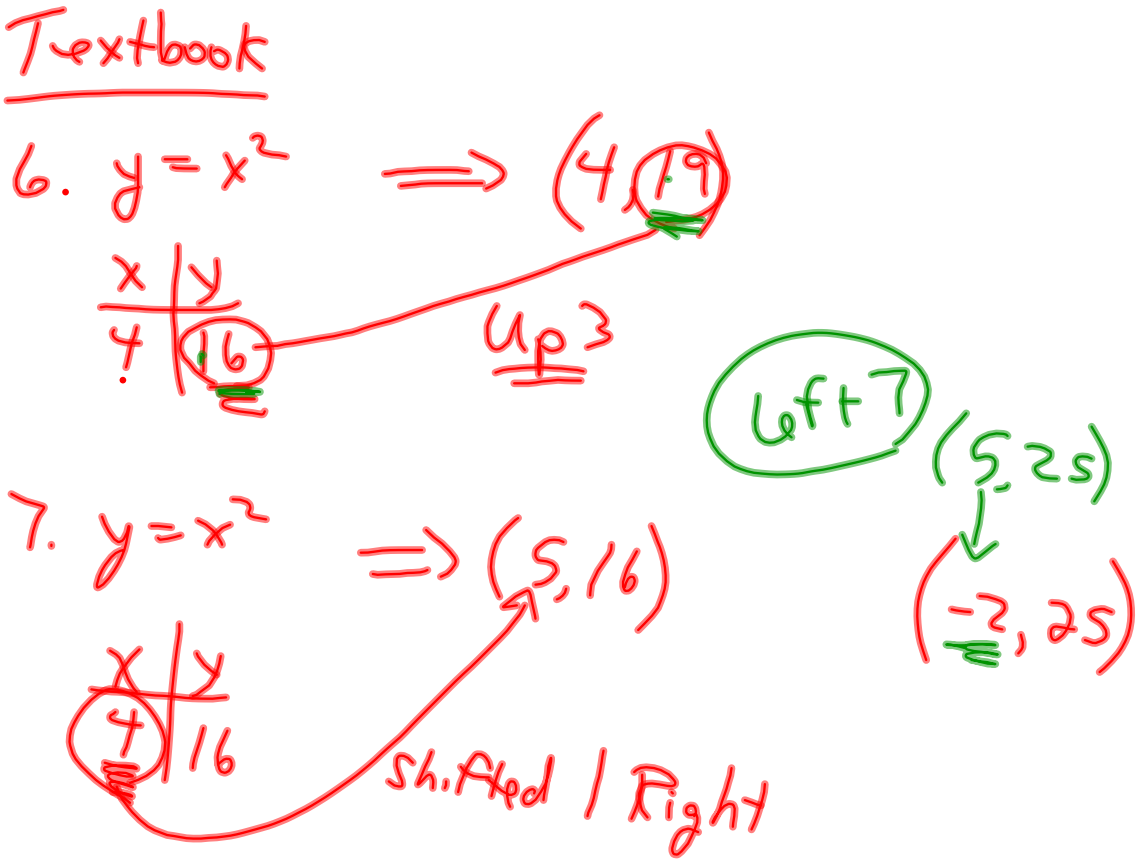
Check-Up... Copy and complete the table.

Translation	Transformed Function	Transformation of Points
~~vertical~~	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
	$y = f(x - 3)$	$(x, y) \rightarrow (x + 3, y)$
	$y = f(x) - 6$	$(x, y) \rightarrow (x, y - 6)$
~~horizontal~~ and vertical	$y + 9 = f(x + 4)$	$(x, y) \rightarrow (x - 4, y - 9)$
~~horizontal~~ and vertical	$y = f(x - 4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$
	$y = f(x + 2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$
~~horizontal~~ and vertical	$y = f(x - h) + k$	$(x, y) \rightarrow (x + h, y + k)$


Textbook
6. $y = x^2$
(4, 19)
x | y
4 | 16
Up 3
Left 7
(5, 25)
(-2, 25)
7. $y = x^2$
(5, 16)
x | y
4 | 16
Shifted / Right

10. $f(x) = |x|$

$$g(x) = f(x-9) + 5$$

(a) $g(x) = |x-9| + 5$

b) Right 9 & Up 5

c) $(x, y) \Rightarrow (x+9, y+5)$

$y = |x|$

x	y
0	0
1	1
-3	3

Image Points

$(0,0) \rightarrow (0+9, 0+5)$

$\rightarrow (9, 5)$

$(1,1) \rightarrow (1+9, 1+5)$

$\rightarrow (10, 6)$

$(-3,3) \rightarrow (-3+9, 3+5)$

$\rightarrow (6, 8)$

11. $y - k = f(x-h)$

(a) $y = f(x-3)$

(b) Right 6, Down 5

$y = f(x-6) - 5$

$y + 5 = f(x-6)$

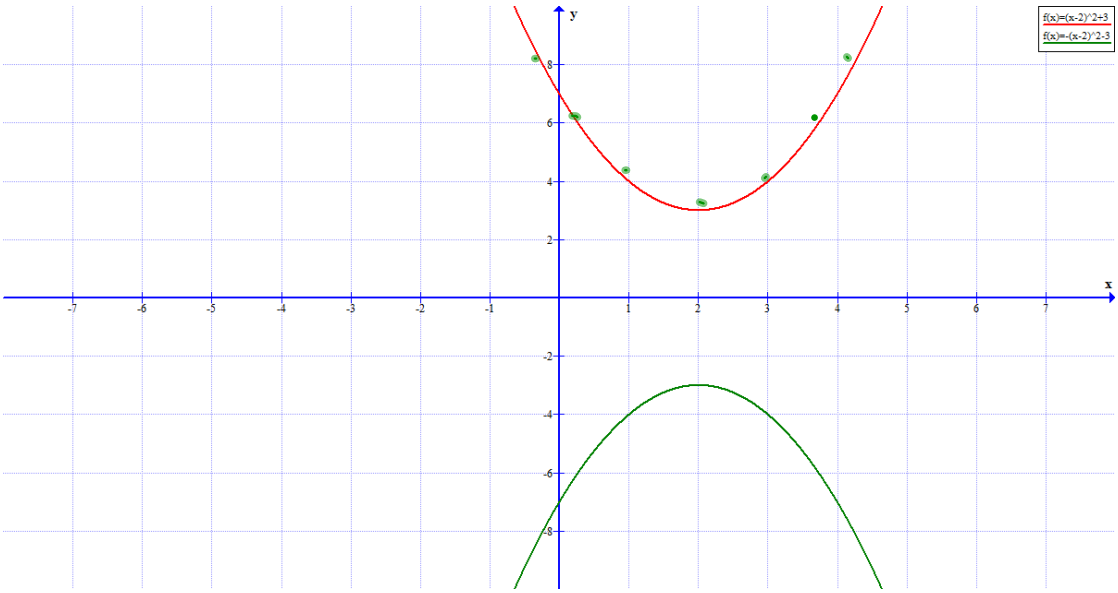
Reflections and Stretches

Focus on...

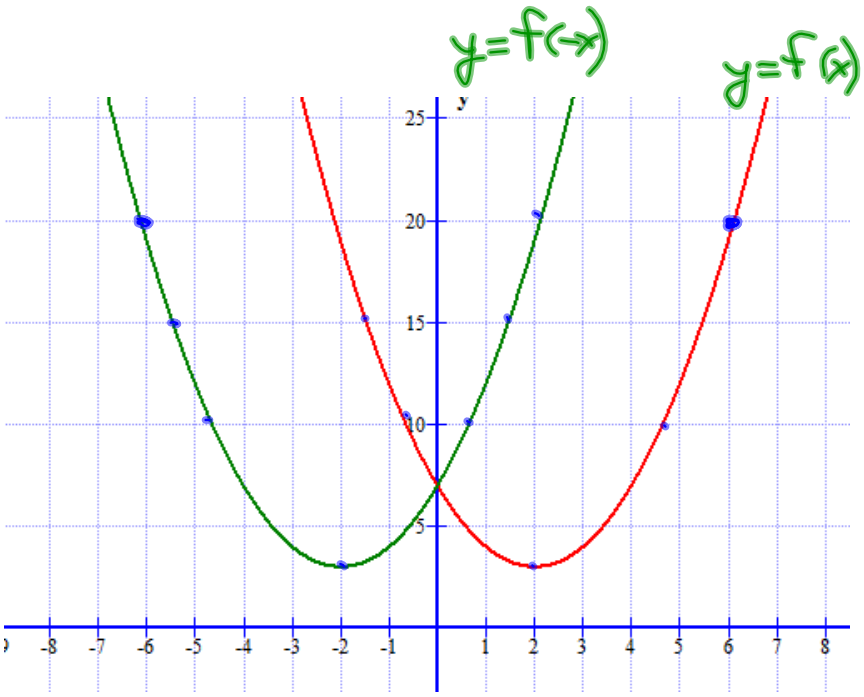
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

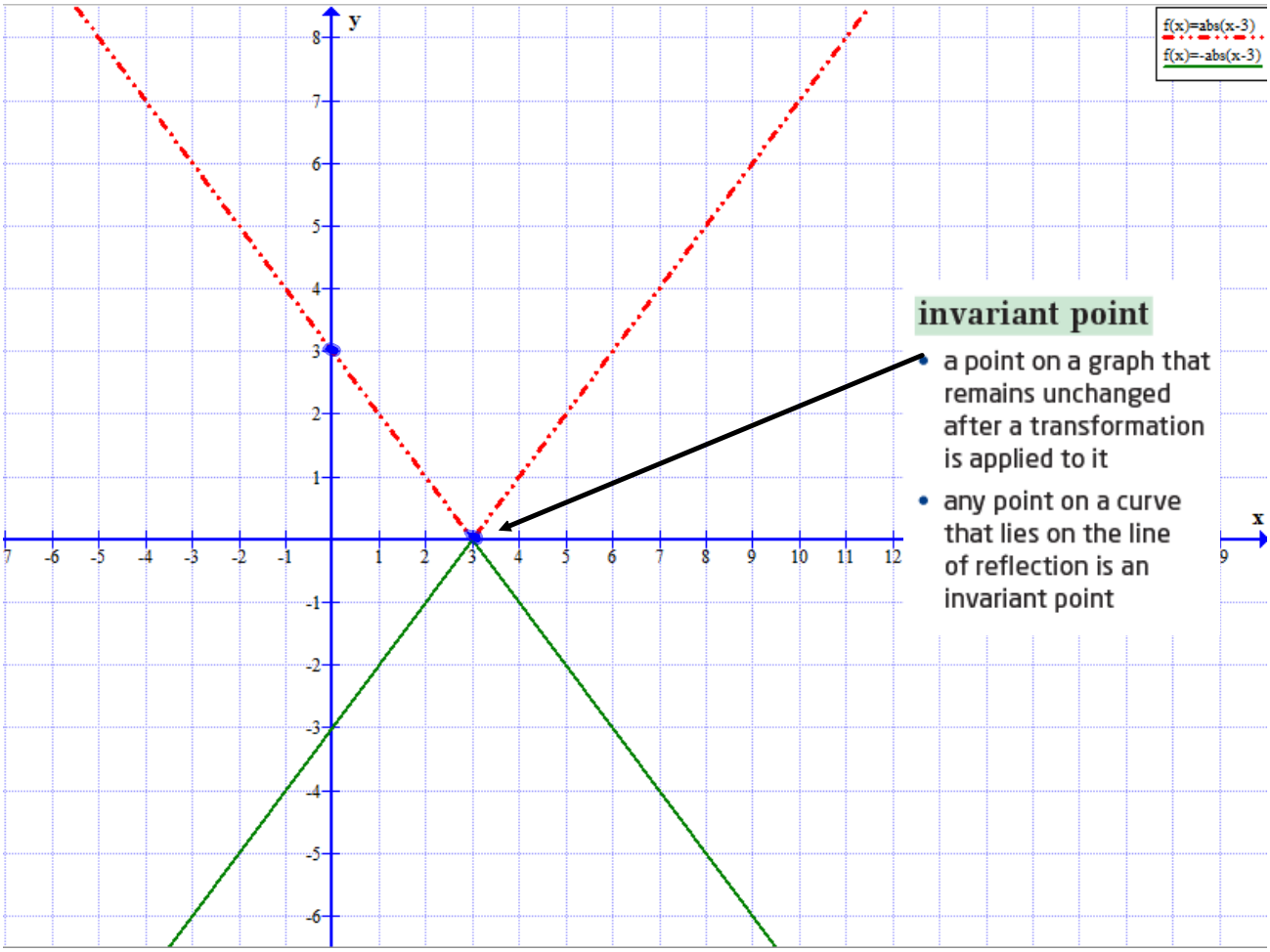
- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.



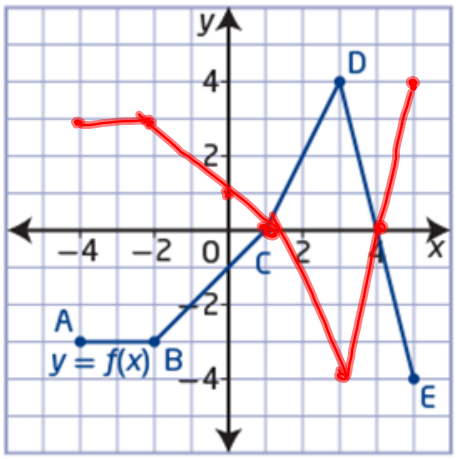
Invariant Point

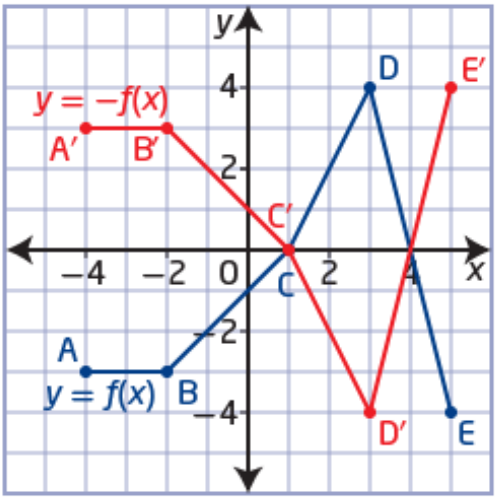


Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below

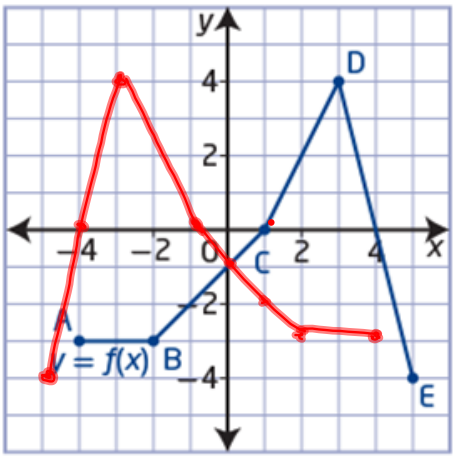


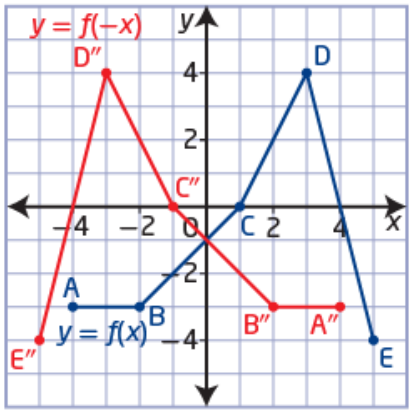


Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below





Stretches and Compressions…

stretch

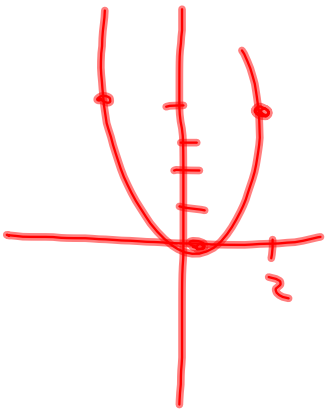
- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor

- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches

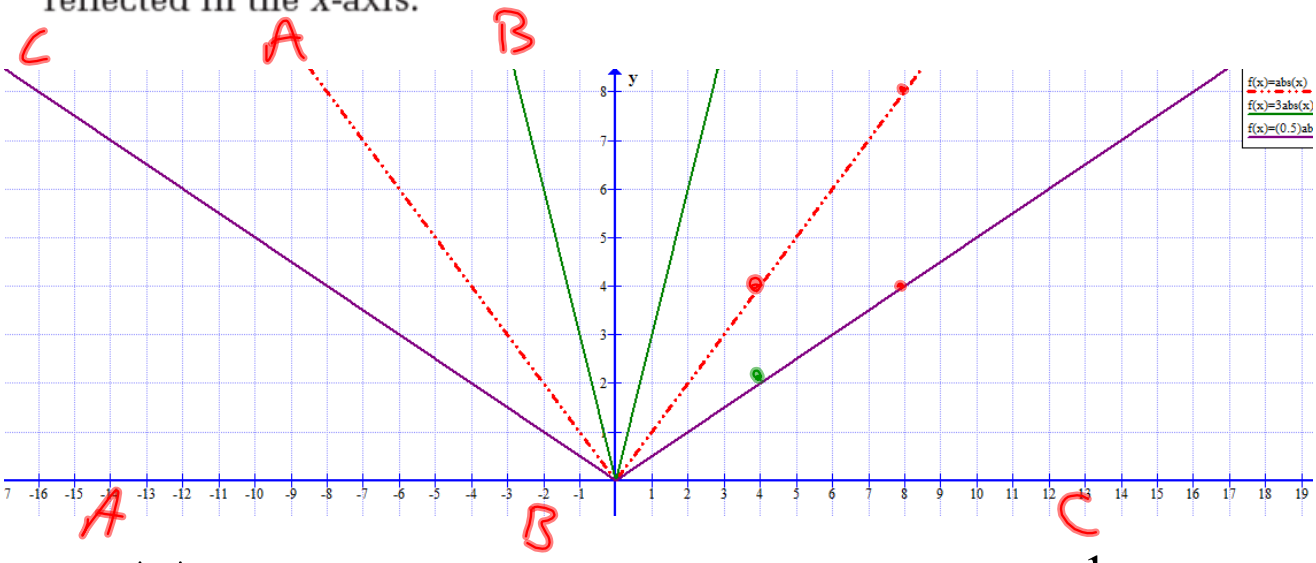
A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

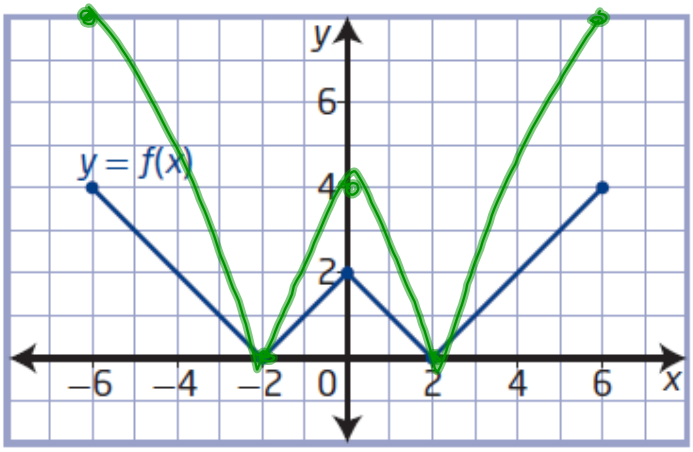


Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.



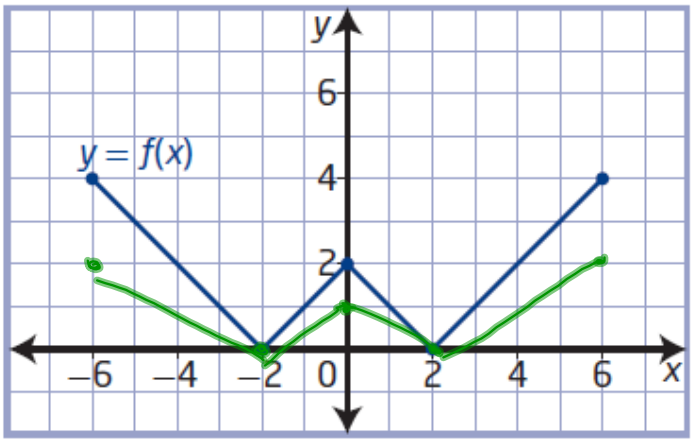
$y = |x|$ $\quad$ $y = 3|x|$ $\quad$ $y = \frac{1}{2}|x|$



Sketch each of the following:

a) $g(x) = 2f(x)$

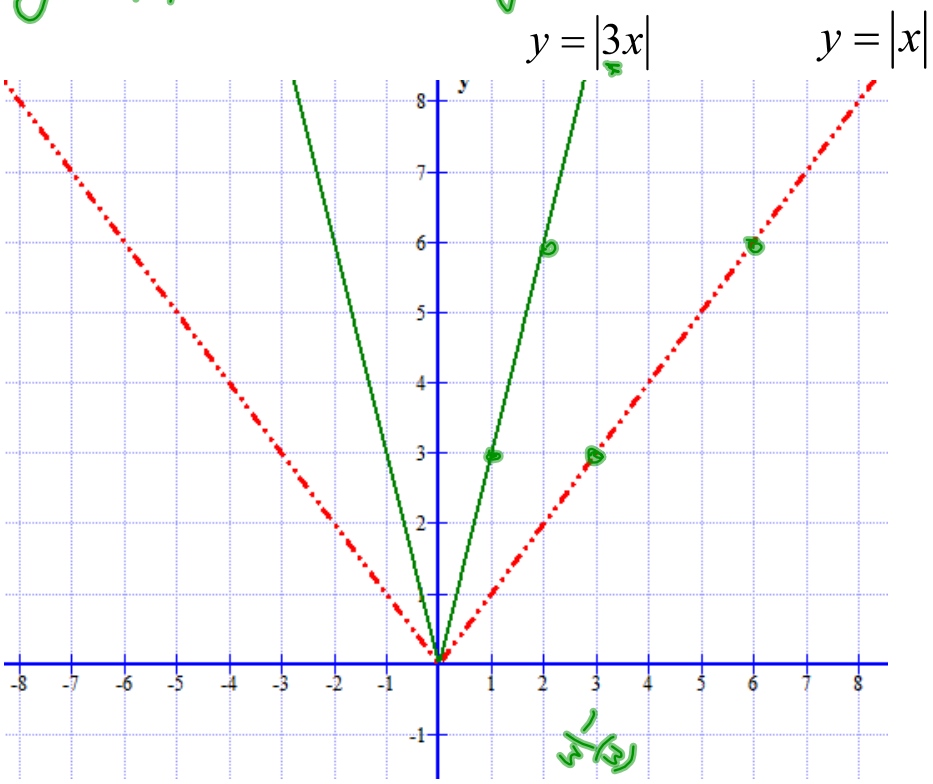
b) $g(x) = \frac{1}{2}f(x)$



Horizontal Stretch or Compression...

$y = 3|x|$

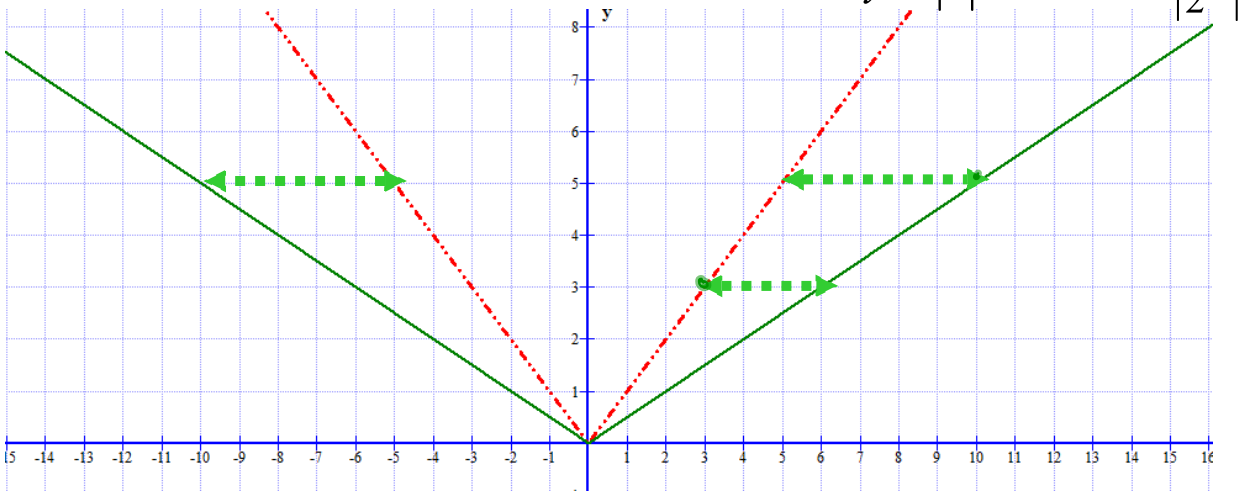
Horizontal stretch by a factor of $\frac{1}{3}$



$y = \frac{1}{2}|x|$

s. factor $= 2$

$y = |x|$ $\quad y = \left|\frac{1}{2}x\right|$

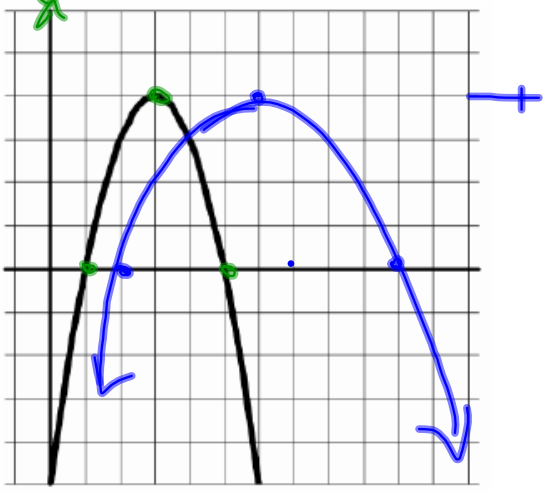


Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Example 1: Apply $f\left(\frac{1}{2}x\right)$ to the graph.

$\frac{1}{2}f(x)$ Horizontal by factor of 2



Apply f(2x) to the given graph.

S.F. $= \frac{1}{2}$