Warm Up

Determine the general antiderivative for the following:

$$F(x) = -2x^5 + 7\sqrt{x} - \frac{3}{4x^3} + 5\sqrt[4]{x^3} - \sec^2 x + 2\pi^3$$

$$F(x) = -2x^5 + 7x^{1/2} - \frac{3}{4}x^{-3} + 5x^{3/4} - \sec^2 x + 2\pi^3$$

$$f(x) = -\frac{1}{3}x^6 + \frac{14}{3}x^{3/2} + \frac{3}{8}x^{-2} + \frac{20}{7}x^{7/4} - \tan x + 2\pi^3 x + C$$

Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x) = [g(x)]^n$$

$$f'(x) = n[g(x)]^{n-1} g'(x)$$

Examples:

(1) $F(x) = (8x-5)^9$

Chain Rule

$u^n \cdot du$

(2) $F(x) = x^3\sqrt{1-5x^4}$

(3) $F(x) = \dfrac{x^3}{\sqrt[3]{1-x^4}}$

$F(x) = -\frac{1}{4}(1-x^4)^{-\frac{1}{3}}(-4x^3)$

$u^n \cdot du$

$f(x) = -\frac{3}{8}(1-x^4)^{\frac{2}{3}} + C$

(4) $F(x) = \dfrac{(5-3x^{-4})^5}{2x^3}$

$F(x) = \frac{1}{2}\left(\frac{1}{12}\right)(5-3x^{-4})^5 \, 12x^{-5}$

$u^n \cdot du$

$F(x) = \frac{1}{144}(5-3x^{-4})^6 + C$

$$f(x) = 3x\sqrt[7]{1-2x^8}$$

$$f(x) = \frac{3}{-16}(1-2x^8)^{1/2}(-16x^7) \qquad \frac{\cancel{3}}{-\cancel{16}\,8} \times \frac{\cancel{2}}{\cancel{3}} = \frac{-1}{8}$$

$$u^n \cdot du$$

$$F(x) = -\frac{1}{8}(1-2x^8)^{3/2} + C$$

Antiderivatives Involving Trigonometry

Remember the rules to differentiate trigonometric functions...

$\frac{d}{du}(\sin u) = \cos u \bullet du$	$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$
$\frac{d}{du}(\cos u) = -\sin u \bullet du$	$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$
$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$	$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$

$\longleftarrow$

Antiderivatives would just be in the opposite direction

Examples:

Determine the general antiderivative:

(1) $f(x) = \sec^2 x - \cos x + 2\csc x \cot x$

$\sec^2 u \cdot du \quad \cos u \cdot du \quad \csc u \cot u \, du$

$F(x) = \tan x - \sin x - 2\csc x + C$

(2) $f(x) = \cos 5x - x^2 \csc^2 \underbrace{x^3}_{u} + 5x \sin 2x^2$

$\cos u \cdot du$

$F(x) = \frac{1}{5}\cos 5x(5) - \frac{1}{3}\csc^2 x^3 (3x^2) + \frac{5}{4}\sin 2x^2 \, 4x$

$\csc^2 u \cdot du \qquad \sin u \cdot du$

$F(x) = \frac{1}{5}\sin 5x + \frac{1}{3}\cot x^3 - \frac{5}{4}\cos 2x^2 + C$

ex. $f(x) = \sin x^3 (4x^2) - \sec 5x^3 \tan 5x^3 (x^2)$

$f(x) = \frac{4}{3} \sin x^3 3x^2 \ \ - \frac{1}{15} \sec 5x^3 \tan 5x^3 (15x^2)$

$F(x) = -\frac{4}{3} \cos x^3 - \frac{1}{15} \sec 5x^3 + C$

$$\overbrace{\qquad\qquad}^{u^n} \qquad \overbrace{\qquad}^{du}$$

ex. $f(x) = \frac{1}{5}\sqrt{1-\cot x^5}\ (\csc^2 x^5) 5x^4$

$F(x) = \frac{2}{15}(1-\cot x^5)^{3/2} + C$

$(\sec x^2)^3$

2) $f(x) = 5x \underline{\sec^3 x^2} \cdot \sec x^2 \tan x^2$

$f(x) = \frac{5}{2}(\sec x^2)^3 \sec x^2 \tan x^2 2x$

$F(x) = \frac{5}{8}(\sec x^2)^4 + C$

Antiderivatives Involving Inverse Trig Ratios

Remember the rules to differentiate inverse trigonometric functions...

$$\frac{d(\sin^{-1}u)}{du}=\frac{1}{\sqrt{1-u^2}}du \qquad \frac{d(\csc^{-1}u)}{du}=\frac{-1}{u\sqrt{u^2-1}}du$$

$$\frac{d(\cos^{-1}u)}{du}=\frac{-1}{\sqrt{1-u^2}}du \qquad \frac{d(\sec^{-1}u)}{du}=\frac{1}{u\sqrt{u^2-1}}du$$

$$\frac{d(\tan^{-1}u)}{du}=\frac{1}{1+u^2}du \qquad \frac{d(\cot^{-1}u)}{du}=\frac{-1}{u^2+1}du$$

Examples:

Determine the general antiderivative:

(1) $F(x)=\frac{5}{\sqrt{1-9x^2}}$ $\quad \frac{du}{\sqrt{1-u^2}}$

$F(x)=\frac{5}{3}\left(\frac{3}{\sqrt{1-(3x)^2}}\right)$

$u=3x$

$f(x)=\frac{5}{3}\sin^{-1}(3x)+C$

OR

$f(x)=-\frac{5}{3}\cos^{-1}(3x)+C$

(2) $F(x)=\frac{-5x^2}{5x^6+1}$ $\quad \frac{du}{u^2+1}$

$F(x)=\frac{-5}{3\sqrt{5}}\left(\frac{3\sqrt{5}x^2}{(\sqrt{5}x^3)^2+1}\right)$

$f(x)=\frac{-5}{3\sqrt{5}}\tan^{-1}(\sqrt{5}x^3)+C$

$\frac{-5}{3\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)=\frac{-5\sqrt{5}}{15}=\frac{-\sqrt{5}}{3}$

ex.

$$f(x)=\frac{7x^4}{3x^5\sqrt{16x^{10}-1}}$$

$$f(x)=\frac{7\cancel{(4)}}{3\cancel{(20)}_{5}}\left(\frac{20x^4}{4x^5\sqrt{(4x^5)^2-1}}\right) \qquad \frac{du}{u\sqrt{u^2-1}}$$

$$F(x)=\frac{7}{15}\sec^{-1}(4x^5)+C$$