Warm Up

Prove the following identities…

1. $\sec x - \tan x \sin x = \dfrac{1}{\sec x}$

LS

$\dfrac{1}{\cos x} - \left(\dfrac{\sin x}{\cos x}\right)\dfrac{\sin x}{1}$

$\dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$

$\dfrac{1-\sin^2 x}{\cos x}$

$\dfrac{\cos^2 x}{\cos x}$

$\cos x$

RS

$\cos x$

LS=RS

2. $\dfrac{\sec\theta\sin\theta}{\tan\theta+\cot\theta} = \sin^2\theta$

$\dfrac{\left(\frac{1}{\cos\theta}\right)\sin\theta}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$

$\dfrac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)}$

$\dfrac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\cos\theta\sin\theta}\right)}$

$\dfrac{\sin\theta}{\cancel{\cos\theta}}\cdot\dfrac{\cancel{\cos\theta}\sin\theta}{1}$

$\sin^2\theta$

LS=RS

3. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

LS

$(\csc^2 x - \cot^2 x)(\cot^2 x + \csc^2 x)$

$(1+\cancel{\cot^2 x}-\cancel{\cot^2 x})(\cot^2 x+\csc^2 x)$

$(1)(\cot^2 x+\csc^2 x)$

LS=RS

4. $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$

LS

$\frac{\cos x + 1}{\sin x(\sin^2 x)}$

$\frac{\cos x + 1}{\sin x(1-\cos^2 x)}$

$\frac{\cancel{\cos x + 1}\ 1}{\sin x(1-\cos x)\cancel{(1+\cos x)}}$

$\frac{1}{\sin x(1-\cos x)}$

$\frac{\csc x}{1-\cos x}$

4. $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$

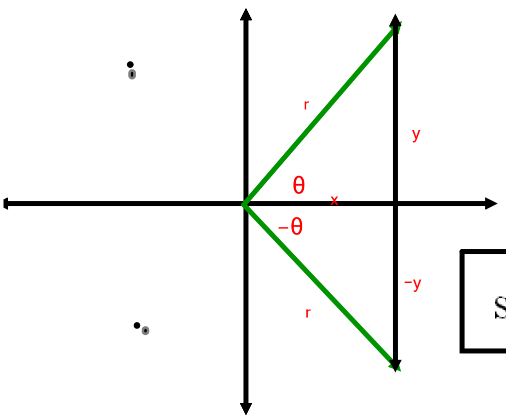
RS

$\frac{\csc x}{1-\cos x}\left(\frac{1+\cos x}{1+\cos x}\right)$ * Multiply by conjugate

$\frac{\csc x(1+\cos x)}{\sin^2 x}$

$\frac{1+\cos x}{\sin^3 x}$

Negative Angles



$\sin\theta = \frac{y}{r}$

$\cos(\theta) =$

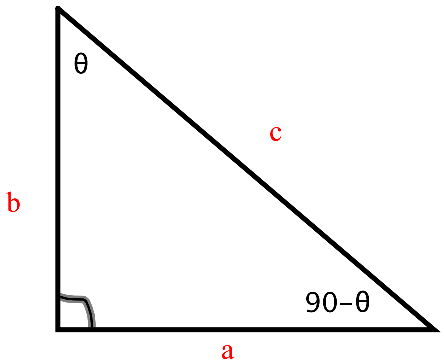
$\sin(-\theta) = \frac{-y}{r}$

$\cos(-\theta) =$

$\boxed{\sin(-\theta) = -\sin\theta}$

$\boxed{\cos(-\theta) = \cos\theta}$

Complimentary Angles



$\sin\theta = \frac{a}{c}$ $\cos(90-\theta) = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$ $\sin(90-\theta) = \frac{b}{c}$

$\boxed{\sin\theta = \cos(90-\theta)}$ $\boxed{\cos\theta = \sin(90-\theta)}$

Sum and Difference Identities

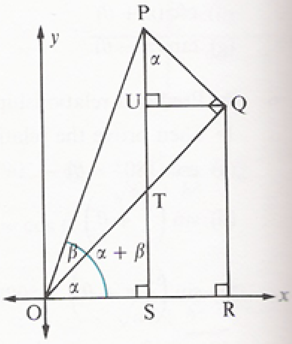
7.4 *Sum and Difference Identities*

Often, your skills in geometry can be combined with your skills in trigonometry, to develop useful relationships. ►PSP

There is a general relationship between $\sin\alpha$, $\sin\beta$, $\cos\alpha$, $\cos\beta$ and $\sin(\alpha+\beta)$ as shown by the following.

From the diagram to the right,

$\angle TSO = \angle PQT = 90°$
$\angle OTS = \angle QTP$ (VOAT)
Thus, $\triangle TOS \sim \triangle TPQ$ and
$\angle TOS = \angle TPQ = \alpha$.



$$\sin(\alpha+\beta) = \frac{PS}{OP}$$
$$= \frac{SU+UP}{OP} \qquad PS = SU + UP$$
$$= \frac{SU}{OP} + \frac{UP}{OP}$$
$$= \frac{RQ}{OP} + \frac{UP}{OP} \qquad SU = RQ$$
$$= \left(\frac{RQ}{OQ}\right)\left(\frac{OQ}{OP}\right) + \left(\frac{UP}{PQ}\right)\left(\frac{PQ}{OP}\right)$$
$$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

* $\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$

$\sin(x+y) \neq \sin x + \sin y$

From this one result, other identities can be found.

$$\begin{aligned}\sin(\alpha-\beta) &= \sin(\alpha+(-\beta))\\ &= \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)\\ &= \sin\alpha\cos\beta + \cos\alpha(-\sin(\beta))\\ &= \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{aligned}$$

$$\boxed{\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta}$$

So far, you have developed the sum and difference identities for the sine function. These results can be extended to develop the sum and difference identities for the cosine function as follows. ►PSP

$$\boxed{\sin(x+y) = \sin x\cos y + \cos x\sin y}$$

Do not distribute through the bracket!

$$\sin(x+y) \neq \sin x + \sin y$$

Sum & Difference Identities

$$\boxed{\sin(x+y)=\sin x\cos y+\cos x\sin y}$$

Finding other identities...

$$\sin(x-y)=\sin(x+(-y))$$

$$\boxed{\sin(x-y)=\sin x\cos y-\cos x\sin y}$$

A - B

$$\cos(x+y)=\sin(90-(x+y))$$

$$\boxed{\cos(x+y)=\cos x\cos y-\sin x\sin y}$$

$$\cos(x-y)=\cos(x+(-y))$$

$$\boxed{\cos(x-y)=\cos x\cos y+\sin x\sin y}$$

New Identities

$$\boxed{\cos\theta = \sin(90-\theta)}$$

$$\boxed{\sin\theta = \cos(90-\theta)}$$

Sum & Difference Identities

$$\boxed{\sin(x+y) = \sin x\cos y + \cos x\sin y}$$

$$\boxed{\sin(x-y) = \sin x\cos y - \cos x\sin y}$$

$$\boxed{\cos(x+y) = \cos x\cos y - \sin x\sin y}$$

$$\boxed{\cos(x-y) = \cos x\cos y + \sin x\sin y}$$

Ex. Prove the following identity:

$$\cos(A + B)\cos(A - B) = \cos^2 A + \cos^2 B - 1$$

LS

$$(\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$$

$$\cos^2 A\cos^2 B - \sin^2 A\sin^2 B$$

$$\cos^2 A\cos^2 B - (1 - \cos^2 A)(1 - \cos^2 B)$$

$$\cancel{\cos^2 A\cos^2 B} - (1 - \cos^2 B - \cos^2 A \cancel{+ \cos^2 A\cos^2 B})$$

$$\cos^2 B + \cos^2 A - 1$$