

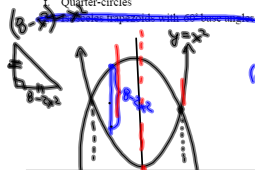
The base of the volume is the region bounded by the curves $y = 8 - x^2$ and $y = x^2$.

The cross sections perpendicular to the x-axis are:

- a. Squares
- b. Equilateral triangles
- c. Isosceles right triangles with leg on the base
- d. Isosceles right triangles with hypotenuse on the base
- e. Semi-circles
- f. Quarter-circles

Intersection

$$\begin{aligned} 8 - x^2 &= x^2 \\ 8 - 2x^2 &= 0 \\ 4 - x^2 &= 0 \\ (2-x)(2+x) &= 0 \end{aligned}$$

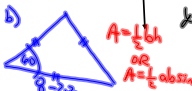
(a) 

$$V = 2 \int_0^2 (8 - 2x^2)^2 dx$$

$$= 2 \int_0^2 (64x - 32x^3 + 4x^6) dx$$

$$= 2 \left(32x^2 - \frac{32x^4}{4} + \frac{4x^7}{7} \right) \Big|_0^2$$

$$= 2 \left(128 - \frac{256}{1} + \frac{128}{7} \right) = \frac{2048}{7} u^3$$

(b) 

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}ab \sin \theta$$

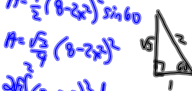
$$A = \frac{1}{2} (8 - 2x^2)^2 \sin 60$$

$$A = \frac{\sqrt{3}}{4} (8 - 2x^2)^2$$

$$V = 2 \int_0^2 \frac{\sqrt{3}}{4} (8 - 2x^2)^2 dx$$

$$= \frac{\sqrt{3}}{2} \int_0^2 (8 - 2x^2)^2 dx$$

$$= \frac{\sqrt{3}}{2} \left(\frac{1024}{15} \right) u^3$$

(c) 

$$A = \frac{1}{2}bh$$


$$A = \frac{1}{2}ab \sin \theta$$

$$A = \frac{1}{2} (8 - 2x^2)^2 \sin 90$$

$$A = \frac{1}{2} (8 - 2x^2)^2$$

$$V = 2 \int_0^2 \frac{1}{2} (8 - 2x^2)^2 dx$$

$$= \frac{1024}{15} u^3$$

(c) 

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}ab \sin \theta$$

$$A = \frac{1}{2} (8 - 2x^2)^2 \sin 45$$


$$A = \frac{1}{2} (8 - 2x^2)^2 \frac{\sqrt{2}}{2}$$

$$A = \frac{\sqrt{2}}{4} (8 - 2x^2)^2$$

$$V = 2 \int_0^2 \frac{\sqrt{2}}{4} (8 - 2x^2)^2 dx$$

$$= \frac{\sqrt{2}}{2} \int_0^2 (8 - 2x^2)^2 dx$$

$$= \frac{1024\sqrt{2}}{15} u^3$$

(d) 

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}ab \sin \theta$$

$$A = \frac{1}{2} \pi r^2$$

$$r = \frac{1}{2} (8 - 2x^2)$$

$$r = 4 - x^2$$


$$A = \frac{1}{2} \pi (4 - x^2)^2$$

$$V = 2 \int_0^2 \frac{1}{2} \pi (4 - x^2)^2 dx$$

$$= \pi \int_0^2 (16 - 8x^2 + x^4) dx$$

$$= \pi \left(16x - \frac{8x^3}{3} + \frac{x^5}{5} \right) \Big|_0^2$$

$$= \pi \left(\frac{32}{1} - \frac{64}{3} + \frac{32}{5} \right) = \frac{512\pi}{15} u^3$$

(e) 

$$A = \frac{1}{2} \pi r^2$$

$$r = \frac{1}{2} (8 - 2x^2)$$

$$r = 4 - x^2$$

$$A = \frac{1}{2} \pi (4 - x^2)^2$$

$$V = 2 \int_0^2 \frac{1}{2} \pi (4 - x^2)^2 dx$$

$$= \pi \int_0^2 (16 - 8x^2 + x^4) dx$$

$$= \pi \left(16x - \frac{8x^3}{3} + \frac{x^5}{5} \right) \Big|_0^2$$

$$= \pi \left(\frac{32}{1} - \frac{64}{3} + \frac{32}{5} \right) = \frac{512\pi}{15} u^3$$