# Warm Up

Evaluate the following integrals:

$$\int \sqrt{x^3 - 3x}\,(x^2 - 1)\,dx \qquad\qquad \int_1^4 \frac{1\,dx}{\sqrt{x}\,(1 + \sqrt{x})^3}$$

$u = x^3 - 3x$

$\dfrac{du}{3} = \dfrac{3x^2 - 3}{3}\,dx$

$\dfrac{du}{3} = (x^2 - 1)\,dx$

$\dfrac{1}{3}\int u^{1/2}\,du$

$\dfrac{1}{3}\left(\dfrac{2}{3}\right) u^{3/2} + C$

$\dfrac{2}{9}\left(x^3 - 3x\right)^{3/2} + C$

$u = 1 + \sqrt{x}$

$du = \dfrac{1}{2} x^{-1/2}\,dx$

$2\,du = \dfrac{1}{\sqrt{x}}\,dx \;(2)$

$2\int_2^3 \dfrac{du}{u^3}$

$\begin{array}{ll} x = 1 & x = 4 \\ u = 1 + \sqrt{1} & u = 1 + \sqrt{4} \\ u = 2 & u = 3 \end{array}$

$= -u^{-2}\Big|_2^3$

$= \left(-\dfrac{1}{(3)^2}\right) - \left(-\dfrac{1}{(2)^2}\right)$

$= -\dfrac{1}{9} - \left(-\dfrac{1}{4}\right)$

$= -\dfrac{1}{9} + \dfrac{1}{4}$

$\dfrac{-4 + 9}{36}$

$= \dfrac{5}{36}$

Midterm

1. $\frac{15}{2}$

$\boxed{\dfrac{du}{\sqrt{1-u^2}}}$

2. $f(x) = 2x^{-\frac{4}{5}} + \dfrac{4}{5}\csc^2 x^5 (5x^4) + \dfrac{2}{12}\left(\dfrac{12x^2}{\sqrt{1-(4x^3)^2}}\right)$

$\boxed{\csc^2 u \cdot du}$    $\boxed{u^n \cdot du}$    $\underbrace{4x^3}$    $\boxed{e^u \cdot du}$

$\boxed{\dfrac{du}{u}} + \dfrac{1}{8}\dfrac{8x^7}{x^8+1} + \dfrac{2}{-4}(1-x^4)^{\frac{1}{2}}(-4x^3) + \dfrac{2}{30}e^{5x^6}(30x^5)$

$-\frac{1}{2} \times \frac{2}{3} = -\frac{1}{3}$

$F(x) = 10x^{\frac{1}{5}} - \cot 5x^4 + \dfrac{1}{6}\sin^{-1}(4x^3) + \dfrac{1}{8}\ln(x^8+1) - \dfrac{1}{3}(1-x^4)^{\frac{3}{2}}$

$+ \dfrac{1}{15}e^{5x^6} + C$

3. $K = \frac{33}{2}$

4. a)    $v(t) = 3t^2 - 12t + 9$

$a(t) = 6t - 12$
$a(4) = 6(4) - 12$
$\qquad = 12 \, u/s^2$

b) $\underline{Stops}$

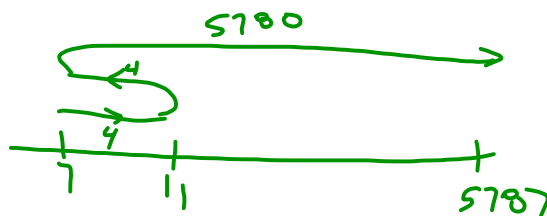$\frac{3t^2}{3} - \frac{12t}{3} + \frac{9}{3} = \frac{0}{3}$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3, 1$

$\left| \int_0^1 v(t) \right| + \left| \int_1^3 v(t) \right| + \left| \int_3^{20} v(t) \right|$   Would work !!

$\underline{displacement}$

$v(t) = 3t^2 - 12t + 9$

$d(t) = t^3 - 6t^2 + 9t + C$

$11 = (4)^3 - 6(4)^2 + 9(4) + C$

$C = 7$

$\boxed{d(t) = t^3 - 6t^2 + 9t + 7}$

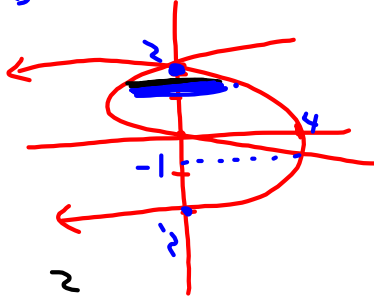| t | d |
|---|---|
| 0 | 7 |
| 1 | 11 |
| 3 | 7 |
| 20 | 5787 |

$d = 4 + 4 + 5780$

$d = 5788 \, units$

5.
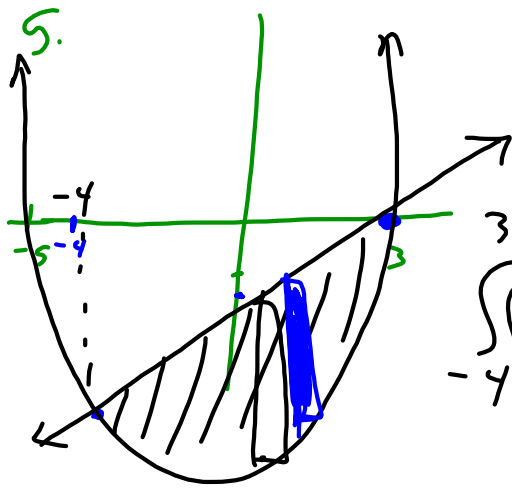


$$\int_{-4}^{3} \left[(x-3)-(x^2+2x-15)\right] dx$$

$$= \frac{343}{6}$$

6. $x = 4-y^2$      $x = y^2-2y$
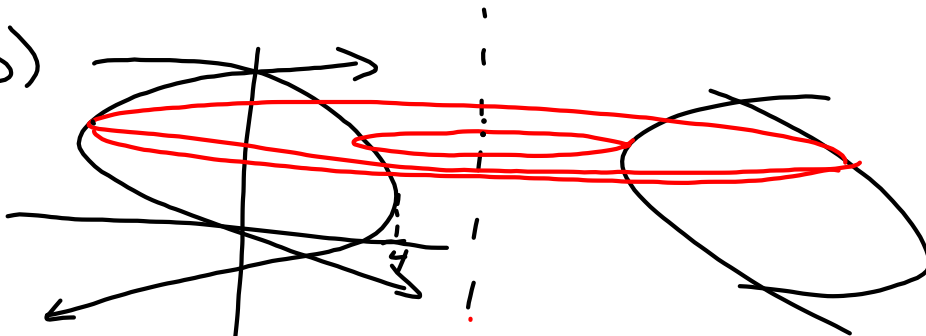
(a)



$y^2-2y = 4-y^2$

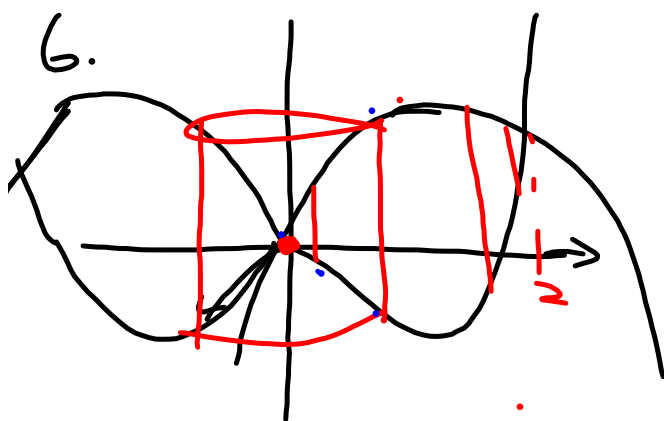$2y^2-2y-4 = 0$

$y^2-y-2 = 0$

$(y-2)(y+1) = 0$

$y = -1, 2$

$$\int_{-1}^{2} (4-y^2)-(y^2-2y)\, dy$$

$$= \boxed{9u^2}$$

b)



$$\pi \int_{-1}^{2} \overbrace{R^2}^{\phantom{x}} - \overbrace{r^2}^{\phantom{x}}$$

$$\pi \int_{-1}^{2} \left[6-(y^2-2y)\right]^2 - \left[6-(4-y^2)\right]^2 dy$$

6.



$$4x - x^2 = x^3 - 2x$$

$$x^3 + x^2 - 6x = 0$$

$$x(x^2 + x - 6) = 0$$

$$x(x+3)(x-2) = 0$$

$$\boxed{x = 0, 2, -3}$$

**Shell**

$$2\pi \int_0^2 x \left(4x - x^2 - (x^3 - 2x)\right) dx$$

$$= \frac{56\pi}{5}$$

$$y = \frac{x^2}{10}$$

$$y = \frac{-x^2}{10}$$

$$\frac{x^2}{10}$$

$$\frac{x^2}{5}$$

$$60°$$

$$\frac{x^2}{5}$$
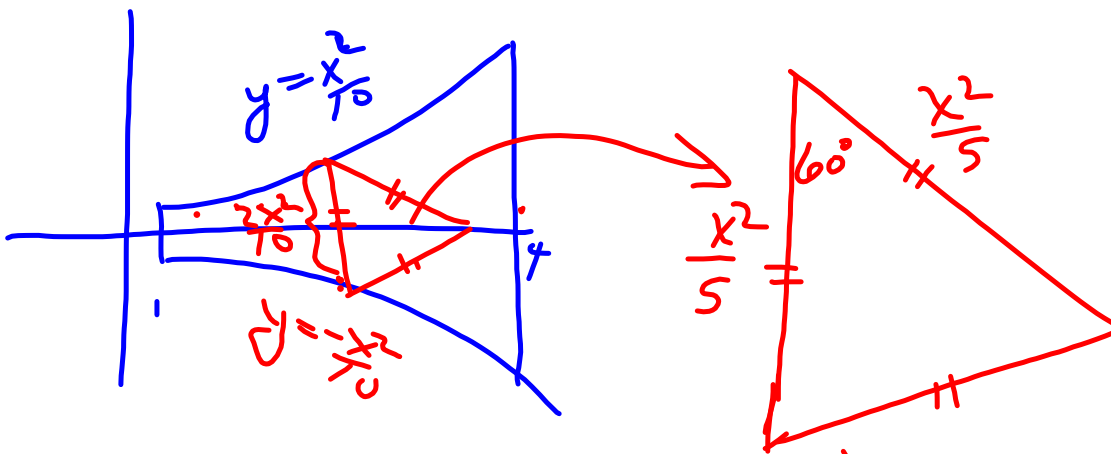
$$A = \frac{1}{2}\left(\frac{x^2}{5}\right)\left(\frac{x^2}{5}\right)\sin 60°$$

$$A = \frac{1}{2}\left(\frac{x^4}{25}\right)\frac{\sqrt{3}}{2}$$

$$A = \frac{\sqrt{3}}{100}x^4$$

$$\frac{\sqrt{3}}{100}\int_{1}^{4} x^4\, dx$$

$$\frac{\sqrt{3}}{100}\left(\frac{x^5}{5}\right)\Big|_{1}^{4}$$

$$\frac{\sqrt{3}}{100}\left(\frac{1024}{5} - \frac{1}{5}\right)$$

$$= \frac{1023\sqrt{3}}{500}\, u^2$$

# Attachments

Tutorial for SMART Response 2013.notebook