Example 1

Use the Laws of Logarithms to Expand Expressions

Write each expression in terms of individual logarithms of x, y, an

a) $\log_6 \frac{x}{y}$

b) $\log_5 \sqrt{xy}$

c) $\log_3 \frac{9}{\sqrt[3]{x^2}}$

d) $\log_7 \frac{x^5 y}{\sqrt{z}}$

(a) $\log_6 x - \log_6 y$ (b) $\log_5 (xy)^{1/2}$ $\log_5 (x^{1/2} y^{1/2})$

$\frac{1}{2} \log_5 (xy)$ $\log_5 x^{1/2} + \log_5 y^{1/2}$

$\frac{1}{2}(\log_5 x + \log_5 y) = \frac{1}{2}\log_5 x + \frac{1}{2}\log_5 y$

c) $\log_3 \frac{9}{\sqrt[3]{x^2}}$

d) $\log_7 \frac{x^5 y}{\sqrt{z}}$

c) $\log_3 \left(\frac{9}{\sqrt[3]{x^2}}\right) =$

$\log_3 9 - \log_3 x^{2/3}$

$= 2 - \frac{2}{3}\log_3 x$

$\log_7 \left(\frac{x^5 y}{\sqrt{z}}\right)$

$\log_7 (x^5 y) - \log_7 z^{\frac{1}{2}}$

$\log_7 x^5 + \log_7 y - \log_7 z^{1/2}$

$5\log_7 x + \log_7 y - \frac{1}{2}\log_7 z$

Example 2

Use the Laws of Logarithms to Evaluate Expressions

Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_3(9\sqrt{3})$

b) $\log_5 1000 - \log_5 4 - \log_5 2$ (3)

c) $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$ (2)

$* \log_b b^M = M$

a) $\log_3 3^2 \cdot 3^{1/2}$ OR $\log_3 9 + \log_3 3^{1/2}$

$\log_3 3^{5/2}$ $2 + \frac{1}{2}\log_3 3$

$= \frac{5}{2}$ $2 + \frac{1}{2}(1)$

$= \frac{5}{2}$

(b) $\log_5\left(\frac{1000}{4}\right) - \log_5 2$ b) $\log_5\left(\frac{1000}{4(2)}\right)$

$\log_5 250 - \log_5 2$ $\log_5 125$

$\log_5\left(\frac{250}{2}\right)$ $= 3$

$= \log_5 125$

$= 3$

c) $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

$\log_3 6^2 - \log_3 64^{1/2} + \log_3 2$

$\log_3 36 - \log_3 8 + \log_3 2$

$\log_3\left(\frac{36}{8}\right) + \log_3 2$

$\log_3\left[\frac{36}{8} \times \frac{2}{1}\right]$

$\log_3\left(\frac{72}{8}\right)$

$\log_3 9$

$= 2$

Example 3

Use the Laws of Logarithms to Simplify Expressions

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a) $4\log_3 x - \frac{1}{2}(\log_3 x + 5\log_3 x)$

b) $\log_2 (x^2 - 9) - \log_2 (x^2 - x - 6)$

a) $4\log_3 x - \frac{1}{2}\log_3 x - \frac{5}{2}\log_3 x$

$\log_3 x^4 + \log_3 x^{-\frac{1}{2}} - \log_3 x^{\frac{5}{2}}$

$\log_3 (x^4 \cdot x^{-\frac{1}{2}}) - \log_3 x^{5/2}$

$\log_3 x^{7/2} - \log_3 x^{5/2}$

$\log_3 \left(\frac{x^{7/2}}{x^{5/2}}\right)$

$\log_3 x^{2/2}$

$= \log_3 x \quad (x > 0)$

a) $4\log_3 x - \frac{1}{2}(\log_3 x + 5\log_3 x)$

$4\log_3 x - \frac{1}{2}(\log_3 x + \log_3 x^5)$

$4\log_3 x - \frac{1}{2}\log_3 x^6$

$\log_3 x^4 - \log_3 (x^6)^{1/2}$

$\log_3 x^4 - \log_3 x^3$

$\log_3 \left(\frac{x^4}{x^3}\right)$

$\log_3 x$

a) $4\log_3 x - \frac{1}{2}(\log_3 x + 5\log_3 x)$

$4\log_3 x - \frac{1}{2}(6\log_3 x)$

$4\log_3 x - 3\log_3 x$

$\log_3 x$

b) $\log_2 (x^2 - 9) - \log_2 (x^2 - x - 6)$

$\log_2 \left(\frac{x^2-9}{x^2-x-6} \right)$

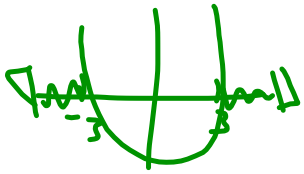
$\log_2 \left(\frac{(x+3)(x-3)}{(x-3)(x+2)} \right)$

$\log_2 \left(\frac{x+3}{x+2} \right)$

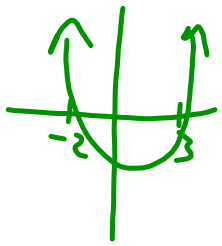
Restrictions

$x^2-9>0$ and $x^2-x-6>0$

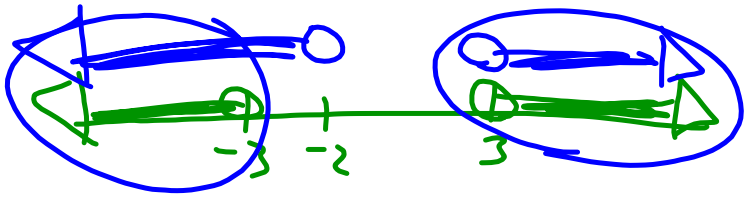
$x=\pm 3$ $\quad$ $x=+3,-2$



$x<-3$ or $x>3$



$x<-2$ or $x>3$



$x<-3$ OR $x>3$

Key Ideas

- Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

Do I really understand??...

a) Express the following as a single logarithm... $2\log_2 3^2 + \log_2 6 - 3\log_2 3$

b) Evaluate the following... $\log_2 (32)^{\frac{1}{3}}$ =

c) Express the following as a single logarithm... $\frac{1}{2}\left[(\log_5 a + 2\log_5 b) - 3\log_5 c\right]$

d) Express as a single logarithm in simplest form...

$$\frac{3}{4}\left[12(\log_b x^2 - 2\log_b x) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7}\right]$$

(a) $\log_2 3^4 + \log_2 6 - \log_2 3^3$

$\log_2 81 + \log_2 6 - \log_2 27$

$\log_2 81 - \log_2 27 + \log_2 6$

$\log_2 \left(\frac{81}{27}\right) + \log_2 6$

$\log_2 3 \cdot 6 + \log_2 6$

$\log_2 18$

(b) $\log_2 (2^5)^{1/3}$

$\log_2 2^{5/3}$

$\frac{5}{3}\log_2 2$

$\frac{5}{3}(1) = \frac{5}{3}$

Practice Problems...

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Attachments

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc