

Chapter  
**8**

*Financial Mathematics: Investing Money*

**LEARNING GOALS**

You will be able to develop your number sense in financial applications by

- Understanding and comparing the effects of simple interest and compound interest
- Determining how changes in the variables of an investment affect the return
- Being aware of a variety of different investment instruments
- Comparing different investment strategies

What do you think it means to be financially literate, and how will being financially literate help you achieve your goals?

← What you remember  
What you would like to know

Tom Hamza: Financial Literacy 101

The Agenda with Steve Palkin 2,638 videos 2,865

**8.1 Simple Interest**  
*p444*

**term**  
The contracted duration of an investment or loan.

**interest**  
The amount of money earned on an investment or paid on a loan.

**fixed interest rate**  
An interest rate that is guaranteed not to change during the term of an investment or loan.

**principal**  
The original amount of money invested or loaned

**maturity**  
The contracted end date of an investment or loan, at the end of the term.

**future value**  
The amount,  $A$ , that an investment will be worth after a specified period of time.

**simple interest**  
The amount of interest earned on an investment or paid on a loan based on the original amount (the principal) and the simple interest rate.

**Communication Tip**  
Interest rates are communicated as a percent for a time period. Since most often the time period is per year or per annum (abbreviated as /a), a given percent is assumed to be annual unless otherwise stated. For example, an interest rate of 4% means 4%/a or 4% interest per year.

**GOAL**  
Solve problems that involve simple interest.

**SIMPLE Interest** Based on the **principal** (original amount) that is invested/borrowed. Interest is a certain percentage per **annum** (year). Often used for personal loans and short-term investments. The length of time for the investment/loan is called the **term**.

Interest = Principal x rate x time

$$I = Prt$$

&

$$A = P + I$$

OR

$$A = P + Prt$$

$$A = P(1 + rt)$$

- I - interest earned
- P - principal (original investment/loan)
- r - interest rate as a percent (change to a decimal)
- t - is ALWAYS time in years (how long the money is invested/borrowed)
- A - amount of money including interest



APPLY the Math p. 446

EXAMPLE 1 Solving a simple interest problem

$$2.5\% = 0.025$$

Marty invested in a \$2500 guaranteed investment certificate (GIC) at 2.5% simple interest paid annually with a term of 10 years.

NOTE:  
Means that interest is paid only in yearly increments.

- a) How much interest will accumulate over the term of Marty's investment?
- b) What is the future value of his investment at maturity?

$$\begin{aligned} a) \quad I &= Prt \\ &= 2500(0.025)(10) \\ I &= 625 \end{aligned}$$

$$\begin{aligned} b) \quad A &= I + P \\ &= 625 + 2500 \\ &= \$3125 \end{aligned}$$

EXAMPLE #2:

Betty-Ann's bank offers a simple interest rate of 4% per annum. How much interest would Betty-Ann earn on her investment of \$4000 after 12 months.

$$\begin{aligned} I &= Prt \\ I &= 4000(0.04)(12/12) \\ I &= \$160.00 \end{aligned}$$



Time in years!!

Yearly (annually)	1
Monthly	12
Weekly	52
Days	365.
<hr/>	
Semi Annually	every 6 months
Quarterly	every 3 months (4 times a year)
Bi-Weekly	every 2 weeks (26 times a year)

**rate of return**  
The ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent.

$$ROR = \frac{\text{earn / lost}}{\text{invested}}$$

$$t = \frac{I}{Pr}$$

$$= \frac{3000}{5000(0.08)}$$

**EXAMPLE 3** Determining the duration of a simple interest investment p. 448

Ingrid invested her summer earnings of \$5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday with a girlfriend.

- a) How long will it take for the future value of the investment to grow to \$8000?  
 b) What is Ingrid's rate of return?  $I=3000$   $r=0.08$   $t=?$   
 $P=5000$

**Ingrid's Solution:**  
 a)  $A = P + Prt$   
 $P$  is \$5000.  
 $r$  is 8%, or 0.08.  
 $A$  is \$8000.  
 $8000 = 5000 + (5000)(0.08)t$   
 $3000 = 400t$   
 $7.5 = t$

It will take 8 years for the future value of the investment to be at least \$8000.

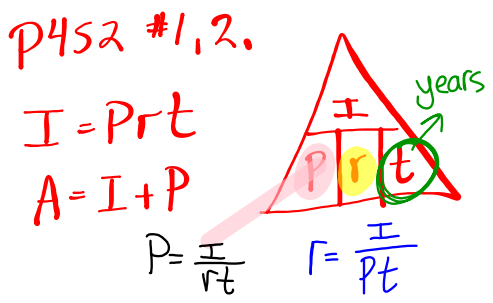
- b) After 8 years:  
 $A = P + Prt$   
 $A = 5000 + (5000)(0.08)(8)$   
 $A = 8200$

At 8 years, the future value will be \$8200.

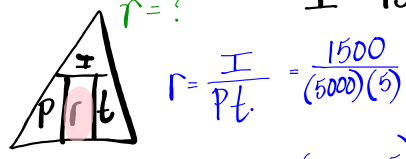
Interest earned:  
 $\$8200 - \$5000 = \$3200$

Rate of return =  $\frac{3200}{5000}$   
 Rate of return = 0.64

The rate of return is 64% over 8 years.



2b)  $P=5000$   $P+I=A$   
 $A=6500$   $I=A-P$   
 $t=5$   $=6500-5000$   
 $r=?$   $I=1500$



b)  $P=5000$   $r=0.06$   
 $t=3$  years  $r=0.06$   
 $I=Prt$   $r=6\%$   
 $=5000(0.06)(3)$   
 $=900$   
 $A=5000+900$   
 $5900$  after  
 $3.25\%$

**EXAMPLE 4** Determining the rate of interest on a simple interest investment p. 450

Grant invested \$25 000 in a simple interest Canada Savings Bond (CSB) that paid interest annually.

- a) If the future value of the CSB is \$29 375 at the end of 5 years, what interest rate does the CSB earn?  
 b) Grant cashed in the bond after 4.5 years because a house he had been admiring came up for sale and he needed a down payment. How much money did he have for the down payment?

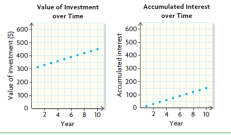
a)  $I=4375$   $A=29375$   
 $P=25000$   $A=P+Prt$   
 $r=?$   $29375=25000+25000(r)(5)$   
 $t=5$   $4375=25000(5)(r)$   
 $r=0.035$   $r=3.5\%$

b)  $t=4.5$   $A=28500$   
 $r=0.035$   $A=P+Prt$   
 $P=25000$   $A=25000+25000(0.035)(4.5)$

**SIMPLE INTEREST...**

**In Summary p. 451**

- Key Ideas**
- Simple interest is determined only on the principal of an investment.
  - The value of an investment that earns simple interest over time is a linear function. The accumulated simple interest earned over time is also a linear function. Since the interest is paid at the end of each period, the growth is not continuous. For example, the following graphs show principal of \$300 invested at 5% interest, paid annually, over a term of 10 years.



**Need to Know**

- The amount of simple interest earned on an investment can be determined using the formula  $I = Prt$  where  $I$  is the interest,  $P$  is the principal,  $r$  is the annual interest rate expressed as a decimal, and  $t$  is the time in years.
- The future value or amount,  $A$ , of an investment that earns simple interest can be determined using the formula  $A = P + Prt$  or  $A = P(1 + rt)$  where  $P$  is the principal,  $r$  is the interest rate expressed as a decimal, and  $t$  is the time in years.
- Unless otherwise stated, an interest rate is assumed to be annual, or per annum.
- Even though interest rates are usually annual, interest can be paid out at different intervals, such as annually, semi-annually, monthly, weekly, and daily.

**HOMEWORK...**

p. 452: #1 - 6, 10, 11

$$I = Prt$$

&

$$A = P + I$$

OR

$$A = P + Prt$$

$$A = P(1 + rt)$$

\$\$\$ Questions...great website for answers!!!



**8.3 Compound Interest: Future Value**

- YOU WILL NEED**
- calculator
  - spreadsheet software
  - financial application on a graphing calculator

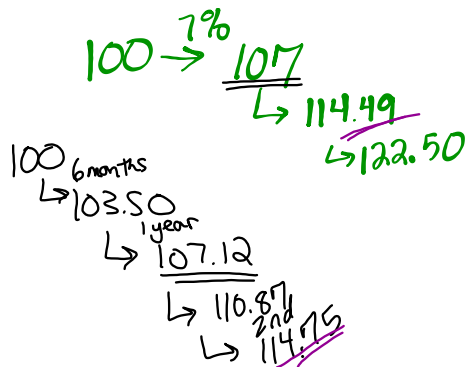
**GOAL**

Determine the future value of an investment that earns compound

p. 463

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$





**COMPOUND Interest** Interest is added to the principal periodically throughout the year. New interest may be paid on the principal plus the interest. The interest rate is stated per annum and is divided by the number of **compounding periods**.

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

$$I = A - P$$

A = final value of the investment ...(principal + interest)  
 P = principal  
 r = annual interest rate  $\rightarrow$  in %  
 n = number of compounding periods in a year  
 t = term of the investment or loan in number of years

**EXAMPLE #1:** If \$1000 is invested at 8 %/a compounded semi-annually for 2 years, how much will the investment be worth?

Using the simple interest formula...

$$I = 1000(0.08)(6/12) = \$40 \text{ (after 1st interest period)}$$

$$\text{New principal} = 1000 + 40 = \$1040$$

$$I = 1040(0.08)(6/12) = \$41.60 \text{ (after 2nd interest period)}$$

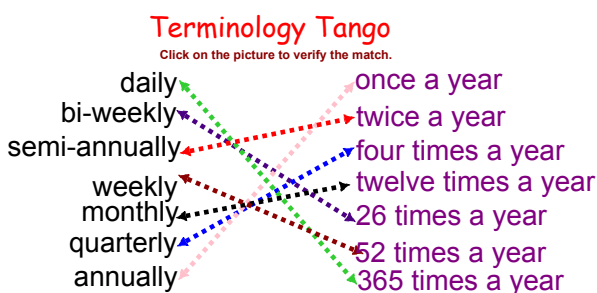
$$\text{New Principal} = 1040 + 41.60 = \$1081.60$$

$$I = 1081.60(0.08)(6/12) = \$43.26 \text{ (after 3rd interest period)}$$

$$\text{New Principal} = 1081.60 + 43.26 = \$1124.86$$

$$I = 1124.86(0.08)(6/12) = \$44.99 \text{ (after 4th interest period)}$$

$$\text{New Principal} = 1124.86 + 44.99 = \$1169.85$$



**Compound Interest Formula...**

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

**SAME ANSWER?**

**EXAMPLE #2:** 4%/a

Calculate the final value of an initial investment of \$6000.00. Interest is paid at 4% per annum, compounded semi-annually, for three years.

A = final value of the investment ... (principal + interest)  
 P = principal  
 r = annual interest rate  
 n = number of compounding periods in a year  
 t = term of the investment or loan in number of years

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 6000 \left(1 + \frac{0.04}{2}\right)^{6}$$

$$A = 6756.97$$

**EX #3:** Maggie invests \$30 000 at 10% /a compounded quarterly for 20 years. Determine...

- a) How much will this investment be worth?
- b) How much interest did you earn?

$P = 30\,000$   
 $r = 10\% \rightarrow 0.10$   
 $n = 4$   
 $t = 20$

$$A = 30000 \left(1 + \frac{0.10}{4}\right)^{80}$$

$$A = 216287.03$$

b)  $I = A - P$   
 $216287.03 - 30000$   
 $186287.03$

**EXAMPLE #4...**

A keen MVHS student wants to save some money from their summer employment. They decide to take out a Canada Savings Bond which pays 2.5% interest per year compounded monthly. If the student invests \$850 into the bond, how much interest will they earn if they don't touch the money for 3 years?

$r = 2.5\%$   
 $n = 12$   
 $P = 850$   
 $t = 3$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 850 \left(1 + \frac{0.025}{12}\right)^{36}$$

$$A = 916.13$$

$A = P + I$   
 $I = A - P$   
 $= 916.13 - 850$   
 $I = 66.13$

**HOMEWORK...**

p. 457: #1, 2

p. 468: #2, 6

Simple	Compound
$I = Prt$	$A = P \left(1 + \frac{r}{n}\right)^{nt}$
$A = P + I$	$I = A - P$
$A = P + Prt$	
$A = P(1 + rt)$	

#6.  $P = 250\,000$   
 $r = 3.8$   
 $n = 2$   
 $t = 1$   
 $A = 250000 \left(1 + \frac{0.038}{2}\right)^2$

$$I = A - P$$

$$= 259590.25 - 250\,000$$

$$= 9590.25$$

More Practice With Compound Interest...

Worksheet - Introduction to Compound Interest.doc

Worksheet Solutions - Compound Interest.pdf

HW QUESTIONS???

How to make money???



? What do you think it means to be financially literate, and how will being financially literate help you achieve your goals?



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EXAMPLE 4 p. 463 Comparing interest on investments with different compounding periods

Celine wants to invest \$3000 so that in 5 years, Celine has the following investment:

- A. 4.8% compounded annually
- B. 4.8% compounded semi-annually
- C. 4.8% compounded monthly
- D. 4.8% compounded weekly
- E. 4.8% compounded daily

[B] [C]  
 $3000(1 + \frac{0.048}{2})^{10}$   
 3729.52



[A] [D] [E]  
 $3000(1 + \frac{0.048}{12})^{60}$   
 3811.22156

$3000(1 + \frac{0.048}{52})^{260}$   
 3813.325288

$3000(1 + \frac{0.048}{365})^{1825}$   
 3813.687273

**Rule of 72**  
 A simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years.  
 The Rule of 72 is most accurate when the interest is compounded annually.

$$\text{Rule of 72} = \frac{72}{\text{Rate}}$$

p. 465 **PH**  
**EXAMPLE 5** Estimating doubling times for investments

Both Berta and Kris invested \$5000 by purchasing Canada Savings Bonds. Berta's CSB earns 8%, compounded annually, while Kris's CSB earns 9%, compounded annually.

a) Estimate the doubling time for each CSB.

Berta's  $\frac{72}{8} = 9$  years

Kris'  $\frac{72}{9} = 8$  years

$$A = 5000 \left(1 + \frac{0.08}{1}\right)^9$$

$$A = 9995.02$$

$$A = 5000 \left(1 + \frac{0.09}{1}\right)^8$$

$$A = 9963.81$$

## 8.4

### Compound Interest: Present Value

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

**GOAL**

Determine the principal or present value of an investment, given its future value and compound interest rate.

**EXAMPLE 2** p. 475 Determining the present value of an investment that is compounded quarterly

Agnes and Bill are musicians. They have researched the costs to set up a small recording studio. They estimate that \$40 000 will pay for the soundproofing, recording equipment, and computer hardware and software that they need. They plan to set up the studio in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.



- How much money should they have invested?
- How much interest will they earn over the term of their investment?

$A = 40000$   
 $t = 3$   
 $r = 0.096$   
 $n = 4$   
 $P = ?$   
 $I = A - P$   
 $9907.34$

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

$$P = \frac{40000}{\left(1 + \frac{0.096}{4}\right)^{12}}$$

$$P = 30092.66$$

$A = \checkmark$   
 $P = ?$   
 $r = \checkmark$   
 $n = \checkmark$   
 $t = \checkmark$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

### HOMEWORK...

p. 468: **Rule of 72...**

#3 (only estimate the doubling time)

#5 & #8

**Compound Interest (Future Value)**

#10 & #12

p. 478: **Compound Interest (Present Value)**

#4, #6, #7, & #9

# 8.5

## Investments Involving Regular Payments

**GOAL**

Determine the future value of an investment that earns compound interest involving regular payments.

**EXAMPLE 1**  
p. 485

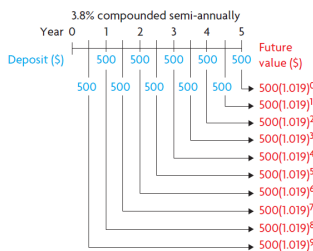
Determining the future value of an investment involving regular deposits

Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits \$500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?



### SOLUTION BY HAND...

I drew a timeline to show the future value of each of the \$500 deposits that I made at the end of each 6-month period for 5 years.



I could see that I needed to do 10 calculations and then determine the sum.

Notes - TVM Solver.pdf

### INSTRUCTIONS on using the TVM-Solver...

- On the TI-83, press 2<sup>nd</sup>, then FINANCE, then select 1:TVM Solver. On the TI-83 plus and TI-84, press APPS, then 1:FINANCE, then 1:TVM Solver. You should see the screen below:  
 $N=$   
 $I\% = 0$   
 $PV = 0$   
 $PMT = 0$   
 $FV = 0$   
 $P/Y = 1$   
 $C/Y = 1$   
 $PMT: END$  BEGIN
- Now, suppose you are taking out a 5-year loan on \$25000 at 6% annual interest compounded monthly and you want to know the monthly payment. Fill in the values on the TVM Solver screen as shown:  
 $N=60$   
 $I\% = 6$   
 $PV = 25000$   
 $PMT =$   
 $FV = 0$   
 $P/Y = 12$   
 $C/Y = 12$   
 $PMT: END$  BEGIN
- Now, move the cursor to PMT, press the green ALPHA key, then ENTER. Your payment will show up as a negative number:  
 $N=60$   
 $I\% = 6$   
 $PV = 25000$   
 $PMT = -483.32003...$   
 $FV = 0$   
 $P/Y = 12$   
 $C/Y = 12$   
 $PMT: END$  BEGIN
- Suppose you know you can afford a \$250 per month payment on a 60 month loan at 6% annual interest compounded monthly. Fill out the TVM Solver screen as shown:  
 $N=60$   
 $I\% = 6$   
 $PV =$   
 $PMT = -250$   
 $FV = 0$   
 $P/Y = 12$   
 $C/Y = 12$   
 $PMT: END$  BEGIN
- To find how much you can afford to borrow, move the cursor to PV, press the green ALPHA key, then ENTER. The amount you can afford to borrow is shown:  
 $N=60$   
 $I\% = 6$   
 $PV = 3981.39819$   
 $PMT = -250$   
 $FV = 0$   
 $P/Y = 12$   
 $C/Y = 12$   
 $PMT: END$  BEGIN

**NOTE:** a negative number means that the money is coming 'out of your pocket'

### SOLUTION WITH TI-84 (Finance APP)...

$N =$	Total number of payments
$I\% =$	Yearly interest rate (as a percent)
$PV =$	Present Value (money invested/borrowed)
$PMT =$	Payment
$FV =$	Future Value (money at the end of the term)
$P/Y =$	Number of payments/year
$C/Y =$	Number of times interest gets compounded/year

$PMT: \text{END}$  BEGIN  
 Payment is given at the beginning/end of pay period

**EXAMPLE 1**  
p. 485

Determining the future value of an investment involving regular deposits

Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits \$500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?



SOLUTION WITH TVM-Solver...

SOLUTION by hand...

**EXAMPLE 2** Comparing a regular payment investment with a single payment investment  
p. 487

Adam made a \$200 payment at the end of each year into an investment that earned 5%, compounded annually. Blake made a single investment at 5%, compounded annually. At the end of 5 years, their future values were equal.

- a) What was their future value?
- b) What principal amount did Blake invest 5 years ago?
- c) Who earned more interest? Why?

**Adam**

$$N = 5$$

$$I\% = 5$$

$$PV = 0$$

$$PMT = -200$$

$$FV = 1105.13 *$$

$$P/Y = 1$$

$$C/Y = 1$$
  

$$200(5) = 1000$$

$$1105.13 - 1000 = I$$

$$I = 105.13$$
  

$$A = P(1 + \frac{r}{n})^{nt}$$

$$P = \frac{A}{(1 + \frac{r}{n})^{nt}}$$

$$P = \frac{1105.13}{(1 + \frac{0.05}{1})^5}$$

$$P = 865.90$$

$$I = 1105.13 - 865.90$$

$$I = 239.23$$

**EXAMPLE 3** Determining the interest rate of a regular payment investment  
p. 489

Jeremiah deposits \$750 into an investment account at the end of every 3 months. Interest is compounded quarterly, the term is 3 years, and the future value is \$10 059.07. What annual rate of interest does Jeremiah's investment earn?

$$N = 12$$

$$I\% = 8.0 *$$

$$PV = 0$$

$$PMT = -750$$

$$FV = 10059.07$$

$$P/Y = 4$$

$$C/Y = 4$$

**EXAMPLE 4** Determining the regular payment amount of an investment  
p. 490

Celia wants to have \$300 000 in 20 years so that she can retire. Celia has found a trust account that earns a fixed rate of 10.8%, compounded annually.

- a) What regular payments must Celia make at the end of each year to meet her goal of \$300 000?
- b) How much interest will she earn over the 20 years?

$$N = 20$$

$$I\% = 10.8$$

$$PV = 0$$

$$PMT = -4781.01 *$$

$$FV = 300000$$

$$P/Y = 1$$

$$C/Y = 1$$
  

$$20(4781.01) = 95620.20$$

$$I = 300000 - 95620.20$$

$$I = 204379.80$$

**EXAMPLE 5** Determining the term of a regular payment investment  
p. 491

On Luis's 20th birthday, he started making regular \$1000 payments into an investment account at the end of every 6 months. He wants to save for a down payment on a home. His investment earns 3.5%, compounded semi-annually. At what age will he have more than \$18 000?

**HOMEWORK...**

p. 493: #3, 5, 6, & 9

**NOTE:** When using the TI-84...

Each question must have the following completed for homework AND beginning of class tomorrow you will be given time to solve.

3.  $N=$   
 $I\% =$   
 $PV =$   
 $PMT =$   
 $FV =$   
 $P/Y =$   
 $C/Y =$   
 $PMT: [ ] [ ] [ ] [ ] BEGIN$

5.  $N=$   
 $I\% =$   
 $PV =$   
 $PMT =$   
 $FV =$   
 $P/Y =$   
 $C/Y =$   
 $PMT: [ ] [ ] [ ] [ ] BEGIN$

6.  $N=$   
 $I\% =$   
 $PV =$   
 $PMT =$   
 $FV =$   
 $P/Y =$   
 $C/Y =$   
 $PMT: [ ] [ ] [ ] [ ] BEGIN$

9.  $N=$   
 $I\% =$   
 $PV =$   
 $PMT =$   
 $FV =$   
 $P/Y =$   
 $C/Y =$   
 $PMT: [ ] [ ] [ ] [ ] BEGIN$

**Quiz Tomorrow...**

**PRACTICE QUESTIONS...**

p. 509:  
 Formulas...#1, 2a, 3, 4, 6, 7a  
 TVM-Solver...#8, 9, 10

- **Simple Interest**  $I = Prt$      $A = P + Prt$   
 $A = P + I$      $A = P(1 + rt)$

- **Compound Interest**  $A = P\left(1 + \frac{r}{n}\right)^{nt}$      $I = A - P$

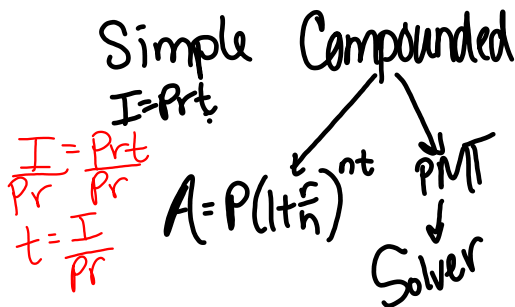
- **Rate of Return**  $ROR = \frac{\text{earn}}{\text{invested}} \times 100\%$

- **Rule of 72**  
 $\text{doubling time} = \frac{72}{\text{rate}}$

- **Present Value**  
 $P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$

- **Regular Payments (TVM-Solver)**

$N=$   
 $I\% =$   
 $PV =$   
 $PMT =$   
 $FV =$   
 $P/Y =$   
 $C/Y =$   
 $PMT: [ ] [ ] [ ] [ ] BEGIN$



**IN CLASS PRACTICE WITH THE TI-84...**



p. 493: #1, 2, 4, 7, 8, 10, 11, 12, 13, 15

**PRACTICE QUESTIONS...**

p. 509:  
 Formulas...#1, 2a, 3, 4, 6, 7a  
 TVM-Solver...#8, 9, 10

### Formula Sheet:

- **Simple Interest**  $I = Prt$   $A = P + Prt$   
 $A = P + I$   $A = P(1 + rt)$

- **Compound Interest**  $A = P\left(1 + \frac{r}{n}\right)^{nt}$   $I = A - P$

- **Rate of Return**  $ROR = \frac{\text{earn}}{\text{invested}} \times 100\%$

- **Present Value**  $P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$

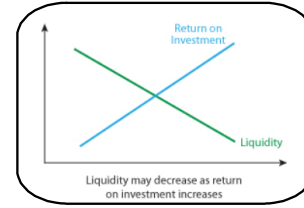
- **Rule of 72**  
 $\text{doubling time} = \frac{72}{\text{rate}}$

- **Regular payments (TVM-Solver)**  
 N=  
 I%=  
 PV=  
 PMT=  
 FV=  
 P/Y=  
 C/Y=  
 PMT: END BEGIN

### Investment Portfolios

As you learn to analyze investment portfolios, don't forget that an investment portfolio must meet the investor's specific needs. A portfolio that works for one person may not be ideal for another, as each person's requirements will be different. In general, our requirements from an investment fall into these three areas:

- **Liquidity:** How quickly and easily the investment can be turned into cash
- **Return on investment:** The increase in value or the money received from an investment
- **Risk:** The probability that an investment will be lost



#### Return on Investment

The purpose of investing is to get back more money than you put in. This increased value or additional cash is your **return on investment** (often referred to as **ROI**). The word "return" can mean different things:

- **The amount of money returned on an investment.** For example, if a \$1,000 investment increases in value to \$1050 in a year, its return is \$50. Generally, people identify the time required for that return.
- **The rate at which an investment grows.** A rate always relates to a specific time—investments generally relate to one year unless a different time is specified. When return on investment is given as a **rate of return (ROR or investment return)**, it is usually written as a per cent that tells how fast an investment grew over a year. For example, the same \$1000 investment that grows to \$1050 in one year has a rate of return of 5%.

Most high-interest-rate savings accounts are currently paying about 3% interest per year. This means that if you put \$1,000 in a savings account today, you will have \$1,030 at the end of one year. Your ROI for the year will be \$30. Your ROR will be 3%.

People use ROI to evaluate, or measure, the performance (success) of an investment or to compare the efficiency of a number of different investments.

#### Risk

Some investments do well, some do not. The probability that an investment will perform poorly is called **risk**.

Some investments—such as bank accounts and Guaranteed Investment Certificates (GICs)—are not risky at all. Their rate of return is guaranteed and predictable. Also, they generally promise a low rate of return.

Some investments—such as shares in a new gold mine—have high risks because the operation being invested in also has a high risk of failure. If the operation fails, investors lose their money. The only way someone will invest in such an operation is for the chance to make a very high return on investment if the operation does well.

Except for guaranteed investments, most investments have some degree of risk. Understanding how the economy is performing, and checking an investment's past performance can help you estimate the investment's risk level. It is important to remember, though, that **an investment's past performance is not a guarantee of its future performance**. History is filled with people who assumed that history predicts the future. Make sure you find out about the risk involved before you choose an investment. Wise investors diversify their investment portfolios to help manage the risk.




SOLUTIONS...

1)


Comparing Investment Portfolios [Investment Portfolio Analysis Question.pdf](#)

Jonathan and Paula are each hoping to buy a house in ten years. They have each chosen an investment portfolio, hoping to save for a large down payment in ten years. Whose portfolio will show the better return?



**Paula's Portfolio**

- \$500 in a tax-free savings account (TFSA) earning 2.2%, compounded monthly.
- Annual end-of-year \$500 purchases of a 10-year Canada Savings Bond (CSB) earning 3.6%, compounded annually.
- Monthly deposits of \$200 to a savings account earning 1.9%, compounded monthly.



**Jonathan's Portfolio**

- 10-year \$2000 guaranteed investment certificate (GIC) earning 4.2%, compounded semi-annually.
- Weekly deposits of \$55 to a savings account earning 1.8%, compounded weekly.
- \$4000 five-year bond earning 3.9%, compounded quarterly and then reinvested in a 4.1% bond.

Using the information provided, answer the following questions for each portfolio. After making an honest effort, click each question to check your work.

1. How much principal do Paula and Jonathan each invest over the ten years? Include both single payment investments and the total of regular payments.

2. What is the future value of each person's portfolio, in ten years? Don't forget that Jonathan reinvests his bond after five years.

3. What rate of return does each person's portfolio have after ten years? Rate of return is the ratio of the amount an investment has increased in value at a given point to the amount invested.

**Paula's Investment:**  
 $5600 + 10(500) + (200)(12)(10)$   
 $= 34600$   
 Paula invested \$34600

**Jonathan's Investment:**  
 $2000 + (55)(52)(10) + 4000$   
 $= 34600$   
 Jonathan invested \$34600

Paula and Jonathan will invest the same amount over the ten years.

2)

Use the financial application on your graphing calculator:

For single-payment investments, enter: Term in years, Present value, Annual interest rate, Compounding frequency

For regular-payment investments, enter: Number of payments, Regular payment amount, Payment frequency, Payments at Beginning or End of compounding period, Annual interest rate, Compounding frequency

Future Value (Paula)	Future Value (Jonathan)
TFSA = \$6976.62	GIC = \$3030.71
CSBs = \$5892.88	Savings Account = <b>\$31 329.72</b>
Savings Account = \$26 007.87	Bond (reinvested at 5 yrs*) = <b>\$5955.45</b>
<b>Portfolio total = \$38 877.37</b>	<b>Portfolio total = \$40 315.88</b>

Total future value = \$38 877.37  
 Total future value = \$40 315.88  
 \* Jonathan's bond has a future value of \$4856.65 after five years, which he reinvests for another five years at 4.1%.

3)

Subtract the amount invested from the future value, then divide by the amount invested.

Rate of Return (Paula)	Rate of Return (Jonathan)
$\frac{38877.37 - 34600}{34600}$	$\frac{40315.88 - 34600}{34600}$
= 0.123	= 0.165

Jonathan's portfolio will have a rate of return of about 17%. This is about 5% higher than the rate of return from Paula's portfolio, which will be about 12%.

8.6

Solving Investment Portfolio Problems

**portfolio**  
 One or more investments held by an individual investor or by a financial organization.

GOAL

Analyze, compare, and design investment portfolios that meet specific financial goals.

p. 497

EXAMPLE 1 Determining the future value and doubling time of an investment portfolio

Phyllis started to build an investment portfolio for her retirement.

- She purchased a \$500 Canada Savings Bond (CSB) at the end of each year for 10 years. The first five CSBs earned a fixed rate of 4.2%, compounded annually. The next five CSBs earned a fixed rate of 4.6% compounded annually.
- Three years ago, she also purchased a \$4000 GIC that earned 6%, compounded monthly.

a) What was the value of Phyllis's portfolio 10 years after she started to invest?  
 b) Phyllis found a savings account that earned 4.9%, compounded semi-annually. She redeemed her portfolio and invested all the money in the savings account. About how long will it take her to double her money?



SOLUTION to PART A:

- First 5 CSB's...
- Other 5 CSB's...
- GIC...

Over first 5 years:

```

N=5
I%=4.2
PV=0
PMT=-500
FV=?
P/Y=1
C/Y=1
PMT:END BEGIN
    
```

Ans+2719.006776

N=5  
 I%=4.6  
 PV=0  
 PMT=-500  
 FV=?  
 P/Y=1  
 C/Y=1  
 PMT:END BEGIN

Ans+2740.837

$4000(1+0.06/12)^{36}$

Ans+4786.722099

Over next 5 years:

```

N=5
I%=4.9
PV=2719.01(1+0.042)^5
PMT=0
FV=?
P/Y=2
C/Y=2
    
```

Ans+3340.822556

The CSB part of the portfolio has a value of \$6080.85.  
 $A = P(1 + i)^n$   
 $A = 4000(1 + \frac{0.06}{12})^{3(12)}$   
 $A = 4786.722...$   
 The GIC part of the portfolio has a value of \$4786.72.  
 Portfolio value = 6080.85 + 4786.72  
 Portfolio value = 10 867.57  
 Phyllis's portfolio is worth \$10 867.57.

TOTAL OF ALL INVESTMENTS...

```

N=5
I%=4.9
PV=2719.01(1+0.042)^5
PMT=0
FV=?
P/Y=2
C/Y=2
    
```

Ans+2740.837  
 Ans+3340.822556  
 Ans+4786.722  
 Ans+10867.57256

SOLUTION to PART B:

b) Use the Rule of 72 to estimate the doubling time.

$\frac{72}{4.9} = 14.694...$   
 The portfolio will take about 14.5 years to double.

EXAMPLE 2 Designing and adjusting an investment portfolio to meet a financial goal

John is an avid sailor and dreams about competing in the Olympics. He wants to buy his own Laser sailboat in 6 years, but, in the meantime, he sails on a friend's boat. The cost of a new Laser is about \$9660, including taxes. John won \$2500 in his most recent race and can save \$50 a month from his part-time job.



- a) What recommendations for a portfolio of two different investments would you make, based on available investments and interest rates? Explain.  
 b) Will the portfolio support the cost of the new Laser in 6 years?  
 c) If not, suggest a solution for John.

Kyla's Solution

- After researching available investments and interest rates, I recommend that John
  - uses the \$2500 cash to buy a 6-year GIC that earns 5.1%, compounded semi-annually, and
  - deposits his monthly savings of \$50 in a high-interest savings account that earns 4.3%, compounded monthly.

b) \$2500 GIC that earns 5.1%, compounded semi-annually, for 6 years:

The present value is \$2500.  
 The annual interest rate is 5.1%.  
 The compounding frequency is 2 times per year.  
 The number of years is 6.  
 The future value is unknown.

Future value = 3381.956...

Deposits of \$50 at the end of each month, earning 4.3%, compounded monthly, for 6 years:

The regular payment amount is \$50.  
 The payment frequency is 12 times per year.  
 The number of payments is 6(12) or 72.  
 The payments are made at the end of each month.  
 The annual interest rate is 4.3%.  
 The compounding frequency is 12 times per year.  
 The future value is unknown.

Future value = 4098.726...  
 Total value = 3381.956... + 4098.726...  
 Total value = 7480.682...

John will have \$7480.68 in 6 years, which is not enough.

c)  $9660 - 7480.68 = 2179.32$

He needs \$2179.32 more.  
 $\frac{2179.32}{72} = 30.268...$

John needs to save about \$30 more each month.  
 $4098.73 + 2179.32 = 6278.05$   
 He needs to have a future value of \$6278.05 in his savings account.  
 A regular payment amount of \$76.60, or \$26.60 more each month, will give him enough money in 6 years to buy the Laser.

I wanted the cash to start earning interest right away at a good interest rate for 6 years. John also needs an account with a high interest rate, where he can deposit his monthly savings.

I used the financial application on my calculator and entered these values to determine the future value of the GIC.

I entered these values into the financial application on my calculator to determine the future value of the savings account.

I divided the additional amount he needs by the number of months in 6 years to estimate how much more he will need to save each month. If he saves \$30 more each month, he will have almost enough money, without counting interest. So, \$30 per month will be enough.

I used the financial application on my calculator and tried different regular payment amounts until I got a future value of \$6278.05. I tried \$75, \$76, \$77, \$76.50, and finally \$76.60.

**In Summary**

**Key Ideas**

- Rate of return is a useful measure for comparing investment portfolios.
- An investment portfolio can be built from different types of investments, such as single payment investments (for example, CSBs and GICs) and investments involving regular payments. Some of these investments, such as CSBs, lock in money for specified periods of time, thus limiting access to the money, but offer higher interest rates. Other investments, such as savings accounts, are accessible at any time but offer lower interest rates. Investments that involve greater principal amounts invested or greater regular payment amounts when contracted tend to offer higher interest rates.
- The factors that contribute to a larger return on an investment are time, interest rate, and compounding frequency. The longer that a sum of money is able to earn interest at a higher rate compounded more often, the more interest will be earned. For investments involving regular payments, the payment frequency is also a factor.

**Need to Know**

- Financial applications on calculators or spreadsheets and online financial tools at banking websites are valuable tools for analyzing and comparing investment portfolios.

**Foundations of Math 11 - Investing Money Formulas**

**Simple Interest**

$$I = Prt \quad A = P + Prt$$

$$A = P + I \quad A = P(1 + rt)$$

**Compound Interest**

$$A = P \left( 1 + \frac{r}{n} \right)^{nt} \quad I = A - P$$

**Rule of 72**

$$\text{doubling time} = \frac{72}{\text{rate}}$$

**Rate of Return**

$$ROR = \frac{\text{earn}}{\text{invested}} \times 100\%$$

**Present Value**

$$P = \frac{A}{\left( 1 + \frac{r}{n} \right)^{nt}}$$

**Regular Payments (TVM-Solver)**

$$\begin{matrix} N= \\ I\% = \\ PV = \\ FV = \\ P/Y = \\ C/Y = \\ PMT = \text{PMT} \text{ BEGIN} \end{matrix}$$

**Homework:**

**Pg. 503 - #4, 5, 8**

**PRACTICE TIME...**

- Mid Chapter Review: Read p. 481 - 482  
Do #1 - 8 on p. 483
- Chapter Review: Read p. 507 - 508  
Do #1 - 12 on p. 509
- Sample Test??? p. 506 #1 - 4

Managed Solutions  
(Class A)

### Select Income Advantage Managed Corporate Class (Class A)

**Fund Facts**  
as of September 30, 2012

Code	Class A	Composite Class
ISIC	061690	062290
ISDC	062090	063090
ISLC	061410	061460

Managed By: CI Investments Inc.  
Advisors: CI Investment Consulting

Assets Under Management\*: \$327.1 million  
Portfolio Manager: Multi-manager  
Asset Class: Diversified Income  
Inception Date: July 2010  
NAV: \$10.80  
Min. Initial Investment: \$25,000  
Subsequent Purchases: \$25,000  
Min. PAC Investment: \$250  
Management Expense Ratio: 2.02%

**Top Holdings as of August 31, 2012**

Gov't of Canada, 2.00%, March 1, 2014	2.03%
Gov't of Canada, 2.75%, June 1, 2022	1.91%
3 Treasury, 3.00%, November 15, 2011	1.89%
Gov't of Canada, 4.00%, June 1, 2041	1.90%
ES Utility Co.	1.46%
Gov't of Canada, 2.25%, August 1, 2014	1.24%
Gov't of Canada, 3.00%, December 1, 2015	1.19%
Province of British Columbia, 3.70%, December 15, 2020	1.15%
iShares IEX Universe Bond	1.14%
Province of Quebec, 4.50%, December 1, 2020	1.10%
<b>Total</b>	<b>14.64%</b>

**Asset Class**

Convertible Bonds	14.0%
Equity	12.0%
Govt Govt Bonds	27.0%
Cash and Equivalents	22.0%
High Yield Bonds	17.0%

**Performance Data**  
This chart shows you the fund's annual performance and how an investment would have changed over time.

**Current Value of a \$10,000 Investment**

**Equity Style and Capitalization Overview**

Large	100%
Mid	0%
Small	0%

Source: CI Investments and The Globe and Mail Inc.

Mutual Funds  
(Class A)

### Signature Corporate Bond Fund (Class A)

**Fund Facts**  
as of September 30, 2012

Code	Class A	Composite Class
ISIC	061610	062290
ISDC	062090	063090
ISLC	061110	061290

Managed By: CI Investments Inc.  
Advisors: Signature Global Advisors  
Chief Investment Officer: Eric Bushnell

Assets Under Management\*: \$345.8 million  
Portfolio Manager: John Shaw and Geoff Marshall  
Asset Class: Global Fixed Income  
Inception Date: December 2001  
NAV: \$9.97  
Min. Initial Investment: \$200  
Subsequent Purchases: \$200  
Min. PAC Investment: \$20  
Management Expense Ratio: 2.12%

**Top Holdings as of August 31, 2012**

Gov't of Canada, 2.75%, June 1, 2022	0.93%
Harvest Operations Corp., 8.88%, October 1, 2017	0.82%
International Lease Finance, 8.75%, March 15, 2017	0.82%
Lincoln National Corp., 7.00%, May 17, 2008	0.74%
Capgem Corp., 7.50%, February 15, 2011	0.72%
Chiron Merger Sub, 10.50%, November 1, 2018	0.70%
National Marine Mart Company, 10.38%, December 15, 2015	0.66%
Sandstorm Data Systems, 10.25%, August 15, 2015	0.60%
Multiplex Inc, 9.88%, September 1, 2015	0.59%
Pacific Rubicon Energy, 7.25%, December 12, 2021	0.57%
<b>Total</b>	<b>7.14%</b>

**Performance Data**  
This chart shows you the fund's annual performance and how an investment would have changed over time.

**Current Value of a \$10,000 Investment**

**Asset Class**

Corporate Debentures	82.5%
Municipal Bonds	6.2%
Other	14.3%
Federal Bonds	2.6%

**Equity Style and Capitalization Overview**

Large	100%
Mid	0%
Small	0%

Source: CI Investments and The Globe and Mail Inc.

Mutual Funds  
(Class A)

### Signature High Income Fund (Class A)

**Fund Facts**  
as of September 30, 2012

Code	Class A	Composite Class
ISIC	061680	062290
ISDC	062090	063090
ISLC	061790	061800

Managed By: CI Investments Inc.  
Advisors: Signature Global Advisors  
Chief Investment Officer: Eric Bushnell

Assets Under Management\*: \$4,188.7 million  
Portfolio Manager: Geoff Marshall, Joe D'Angelo and Ron Fitzgerald  
Asset Class: Canadian Balanced Income  
Inception Date: December 1996  
NAV: \$14.02  
Min. Initial Investment: \$200  
Subsequent Purchases: \$200  
Min. PAC Investment: \$20  
Management Expense Ratio: 1.50%

**Top Holdings as of August 31, 2012**

Inter Pipeline Fund	2.16%
Suncor Energy	2.01%
al Dutch Shell PLC	1.99%
Sumitser REIT	1.93%
Clear Real Estate Investment	1.73%
Allied Properties REIT	1.67%
Brookfield Asset Management	1.62%
Brookfield Renewable Energy	1.32%
Transurban Group	1.30%
H&R Real Estate Invest. Trust	1.24%
<b>Total</b>	<b>16.90%</b>

**Asset Class**

Cash and Equivalents	11.4%
Emerg. Equity	11.2%
Emerg. Bond	21.1%
Canadian Bonds	13.9%
Other	4.4%

**Equity Sectors**

Other	54.8%
Utilities	3.7%
Materials	3.7%
Healthcare	12.6%
Financial	1.6%
Consumer Discretionary	1.6%
Information Services	3.4%

**Geographic Composition**

Australia	4.9%
Luxembourg	1.0%
Canada	3.7%
United States	25.0%
Canada	19.7%
United Kingdom	1.1%
Netherlands	1.1%

**Performance Data**  
This chart shows you the fund's annual performance and how an investment would have changed over time.

**Current Value of a \$10,000 Investment**

Source: CI Investments and The Globe and Mail Inc.

## Attachments

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Worksheet Solutions - Compound Interest.pdf

Worksheet - Introduction to Compound Interest.doc

Notes - TVM Solver.pdf

Investment Portfolio Analysis Question.pdf