Pre-Calculus 12B

Test: Derivatives *Name:*____________________

November 2014

1. Differentiate each of the following: [8]

(a) $f(x) = \tan(3x-5)^3 - \sec^4 2x^5$

$(\cos 5x)^2$

(b) $y = \dfrac{\cos^2 5x - \sqrt{\cot x^4}}{\sin[\tan(3x^2)]}$

$$y' = \frac{\left[2(\cos 5x)^1(-\sin 5x(5)) - \frac{1}{2}(\cot x^4)^{-\frac{1}{2}}\left(-\csc^2 x^4(4x^3)\right)\right](\text{Den}) - \left(\cos^2 5x - \sqrt{\cot x^4}\right)\cos(\tan(3x^2))\sec^2 3x^2(6x)}{\left(\sin(\tan(3x^2))\right)^2}$$

2. (a) A bullet fired straight up from the moon's surface would reach a height of $s = 832t - 2.6t^2$ feet after t seconds. On Earth, in the absence of air, its height would be $s = 832t - 16t^2$ feet after t seconds. How much higher will the bullet rise on the moon than it would on earth? [5]

$v = 0$

$s' = 832 - 5.2t$

$0 = 832 - 5.2t$

$t = 160$ sec

$s(160) = 832(160) - 2.6(160)^2$

$= 66\,560$ feet

$s' = 832 - 32t$

$0 = 832 - 32t$

$t = 26$ sec

$s(26) = 832(26) - 32(26)$

$= 10\,816$ feet

55 744 feet higher on the Moon

3. Determine the equation of the tangent line drawn to the curve $x^2 - 2xy = x^2y - 3x$ at the ordered pair $(-1, 2)$. [5]

$y - y_1 = m(x - x_1)$ $y = mx + b$ $Ax + By + C = 0$

$2x - \left(2y + 2x\frac{dy}{dx}\right) = 2xy + x^2\frac{dy}{dx} - 3$

$\frac{dy}{dx} = \frac{2xy - 3 - 2x + 2y}{-2x - x^2}$

at $(-1, 2)$

$\frac{dy}{dx} = -1$

$y - 2 = -1(x + 1)$

$y - 2 = -x - 1$

$y = -x + 1$ or $x + y - 1 = 0$

4. Find the ***x*-intercept** of the tangent line drawn to the curve $f(x) = \frac{1 - 3x^3}{\sqrt{x+5}}$ at the point where $x = -1$. [6]

$f'(x) = \frac{(-9x^2)(\sqrt{x+5}) - (1 - 3x^3)\frac{1}{2}(x+5)^{-\frac{1}{2}}}{x+5}$ $4^{-\frac{1}{2}}$

$f'(-1) = \frac{(-9)(2) - (4)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{4}$

$m = -\frac{19}{4}$

$y = \frac{4}{2} = 2$

$(-1, 2)$

x-Int

$y = 0$

$y - 2 = -\frac{19}{4}(x + 1)$

$-2 = -\frac{19}{4}(x + 1)$

$x = -\frac{11}{19}$

5. Find the points on the curve $y = \cos x - 2x$, $0 < x < 2\pi$, where a tangent to the curve would be perpendicular to the line $3y - 2x + 6 = 0$. [6]

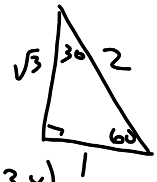
$y' = -\sin x - 2$

$-\frac{3}{2} = -\sin x - 2$

$\frac{1}{2} = -\sin x$

$\sin x = -\frac{1}{2}$

(Ref ∠ $\frac{\pi}{6}$, Q3, 4)



$3y = 2x - 6$

$y = \frac{2}{3}x - 2$

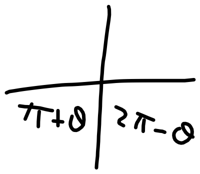
$m = \frac{2}{3}$

$\therefore y' = -\frac{3}{2}$

Q3

$x = \pi + \frac{\pi}{6}$

$= \frac{7\pi}{6}$



Q4

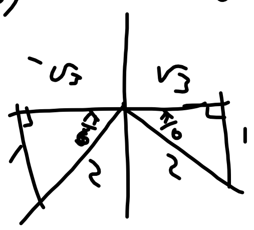
$x = 2\pi - \frac{\pi}{6}$

$x = \frac{11\pi}{6}$

$y = \cos\frac{7\pi}{6} - 2\left(\frac{7\pi}{6}\right)$

$y = -\frac{\sqrt{3}}{2} - \frac{7\pi}{3}$

$\left(\frac{7\pi}{6}, -\frac{\sqrt{3}}{2} - \frac{7\pi}{3}\right)$



$y = \cos\frac{11\pi}{6} - 2\left(\frac{11\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} - \frac{11\pi}{3}$

$\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{2} - \frac{11\pi}{3}\right)$

6. A particle moves along a **vertical** line in such a way that at time t seconds after the start, the particle is located $s = 2t^3 - 21t^2 + 36t + 3$ metres from its starting position, where $t \geq 0$.

(a) What is the velocity of the particle when the acceleration is equal to 18 m/s^2 ? [4]

$s' = 6t^2 - 42t + 36$

$s'' = 12t - 42$

$18 = 12t - 42$

$60 = 12t$

$5 = t$

$s'(5) = 6(5)^2 - 42(5) + 36$

$= -24 \text{ m/s}$

(b) Determine the acceleration of the particle the instant it changes direction for the **second** time. [4]

$v = 0$

$\frac{6t^2}{6} - \frac{42t}{6} + \frac{36}{6} = \frac{0}{6}$

$t^2 - 7t + 6 = 0$

$(t-6)(t-1) = 0$

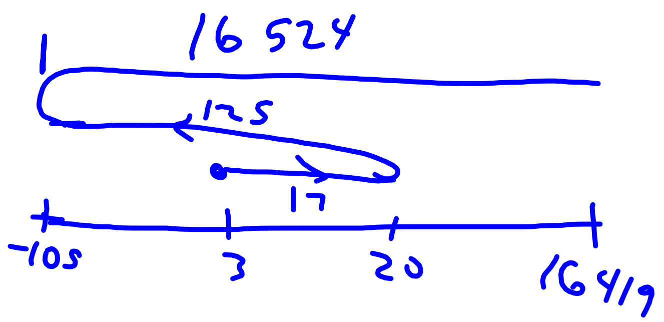
$t = 6, 1$

$s''(6) = 12(6) - 42$

$= 30 \text{ m/s}^2$

(c) What is the total distance traveled by the particle over the first 24 seconds. [4]

t	s
0	3
1	20
6	-105
24	16419



$d = 17 + 125 + 16524$

$= 16666 \text{ m}$