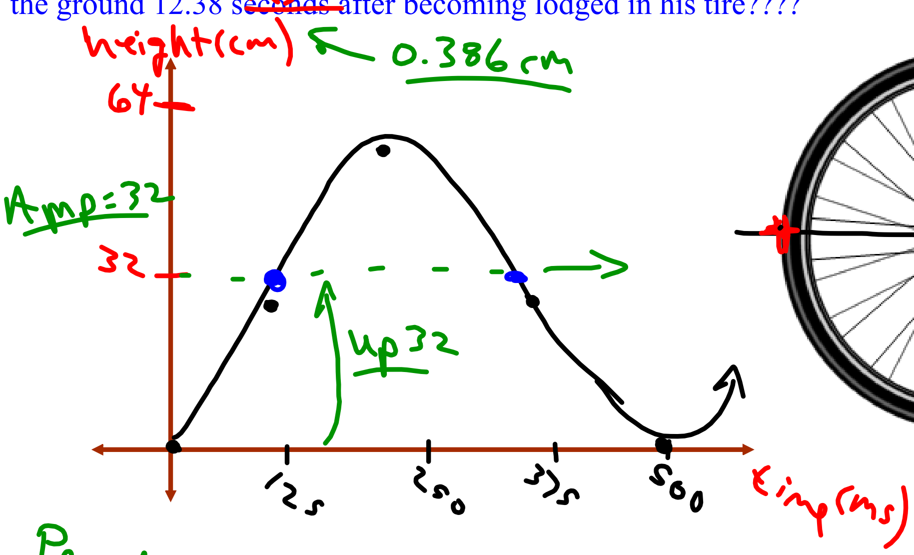
Applications of Sinusoidal Relations

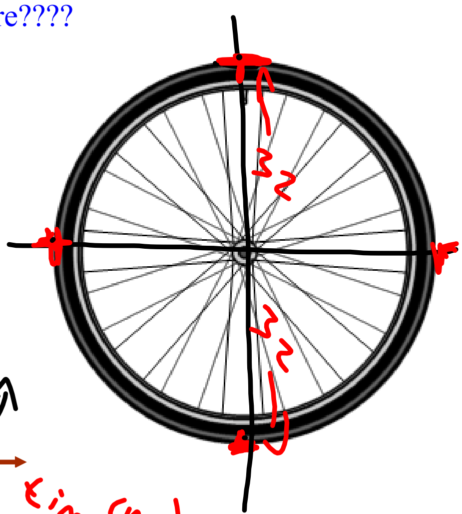
Strategy: (1) Translate ALL key pieces of information from the problem.
(2) Draw a sketch with ALL key points identified.
(3) Develop an equation that models the problem.
(4) Answer the question(s) being asked.

CHECK??? Do the numbers make sense?

EXAMPLE...

Johnny is driving his bike when a tack becomes stuck in his tire. The tire has a radius of 32 cm and makes one complete rotation every 500 ms. How high will the tack be above the ground 12.38 ms ~~seconds~~ after becoming lodged in his tire????





Period = 500

$\frac{360}{k} = 500$

$\frac{360}{500} = \frac{500k}{500}$

$k = 0.72$

$y = -32\cos[0.72t] + 32$

$y = 32\cos[0.72(t-250)] + 32$

$y = 32\sin[0.72(t-125)] + 32$

$y = -32\sin[0.72(t-375)] + 32$

$y = -32\cos(0.72(12.38)) + 32$

$y = 0.386\text{ cm}$

Applications of Sinusoidal Functions

Example: A Ferris Wheel with a radius of 20 m rotates every 40 s. Passengers get on a seat that is 1 m above ground level. How high above the ground would a passenger be situated 3 minutes and 17 seconds after starting this ride?

3:17

(197 sec)

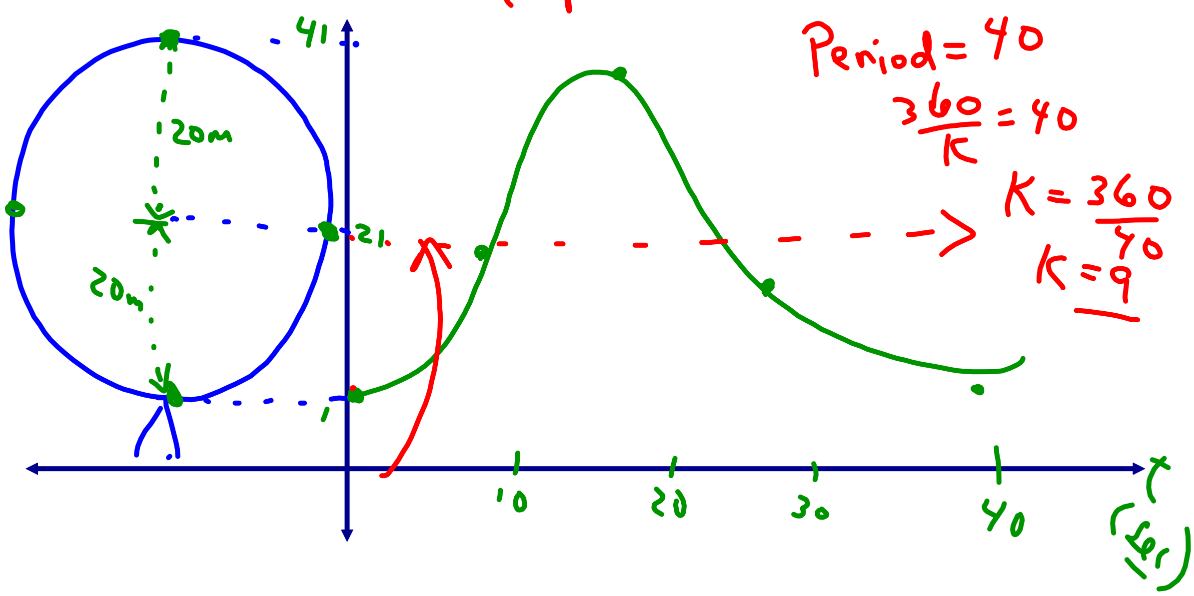
Amp = 20 v. shift = +21

Period = 40

$\frac{360}{k} = 40$

$k = \frac{360}{40}$

$k = 9$



$y = -20\cos[9t] + 21$

$y = -20\cos(9(197)) + 21$

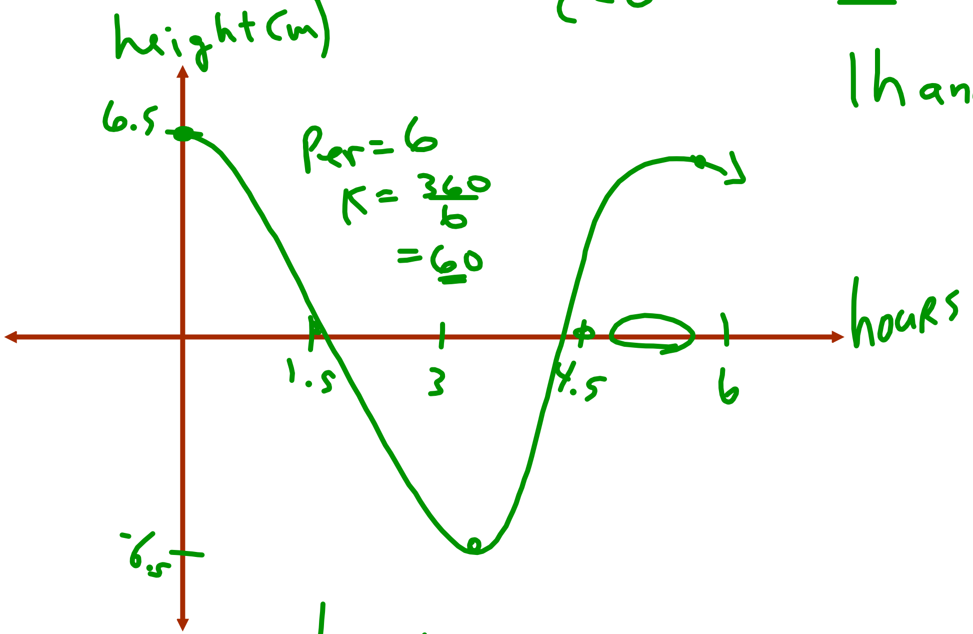
$y = 3.18$ m

Ocean Tides

The alternating half-daily cycles of the rise and fall of the ocean are called tides. Tides in one section of the Bay of Fundy caused the water level to rise 6.5m above mean sea-level and to drop 6.5m below. The tide completes one cycle every 6 h. Assume the height of water with respect to mean sea-level to be modelled by a sinusoidal relationship. If it is high tide at 8:00 AM, determine where the water level would be at 1:47 PM.

$t=0 \Rightarrow 8{:}00$ AM

1h and $\frac{47}{60}$



Per $= 6$

$k = \frac{360}{6}$

$= 60$

$$h = 6.5\cos(60t)$$

$$h = 6.5\cos\left(60\left(5 + \frac{47}{60}\right)\right)$$

$$h = 6.333\text{ m}$$

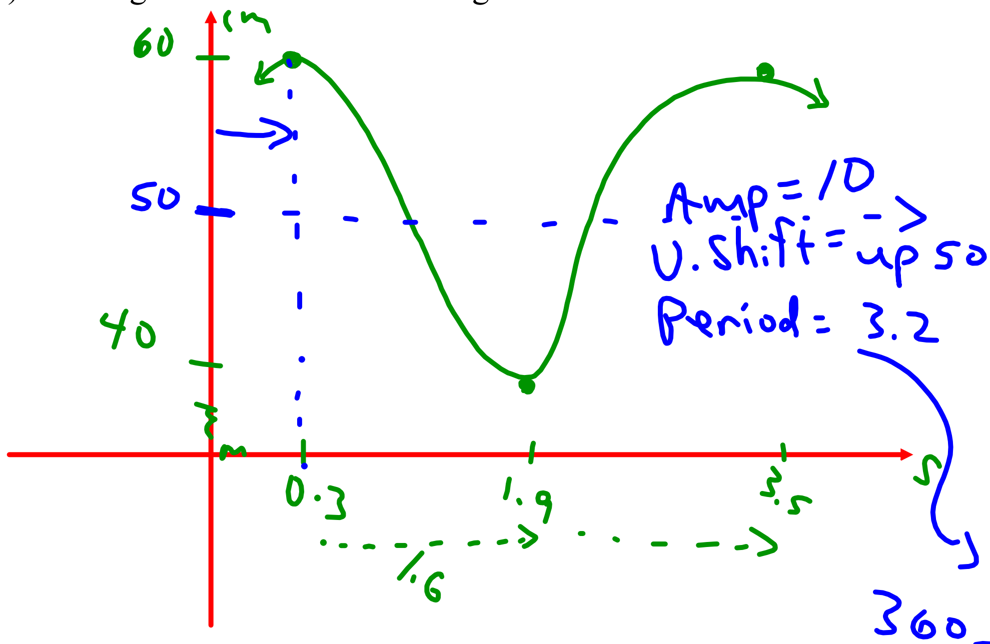
Spring Problem

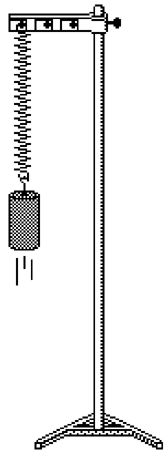
A weight attached to a long spring is being bounced up and down by an electric motor. As it bounces, its distance from the floor varies periodically with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight reaches its first high point 60 cm above the ground. The next low point, 40 cm above the ground, occurs at 1.9 seconds.

a) Sketch a graph of the function.

b) Write an equation expressing the distance above the ground in terms of the numbers of seconds the stopwatch reads.

c) How high is the mass above the ground after 17.2 seconds?





$$y = 10\cos[11.25(t-0.3)] + 50$$

$$\frac{360}{k} = 3.2$$

$$k = \frac{360}{3.2}$$

$$k = 11.25$$

(b) $y = 10\cos[11.25(17.2-0.3)] + 50$

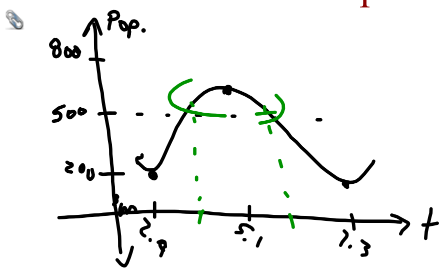
$y = 40.16\text{cm}$

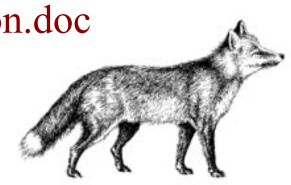
Biology!!

Naturalists find that the populations of some animals varies periodically with time. Records started being taken at t = 0 years. A minimum number, 200 foxes, occurred when t = 2.9 years. The next maximum, 800 foxes, occurred at t = 5.1 years.

Give two different times at which the fox population is 625.

Bonus Soln - Fox Population.doc





$4.4 = \frac{360}{K}$

$K = \frac{360}{4.4}$

$y = -300\cos\left[\frac{360}{4.4}(t-2.9)\right]+500$

$625 = -300\cos\left(\frac{660}{4.4}(t-2.9)\right)\cancel{+500}$ -500

$\frac{625-500}{-300} = \frac{\cancel{-300}\cos\left[\frac{660}{4.4}(t-2.9)\right]}{\cancel{-300}}$

$\frac{125}{-300} = \cos\left[\frac{660}{4.4}(t-2.9)\right]$

$0.75 = \cos x$

$\cos^{-1}\left(\frac{-125}{300}\right) = \frac{660}{4.4}(t-2.9)$

$\cos^{-1}(0.75) = \cancel{\cos^{-1}}(\cancel{\cos}\, x)$

$= x$

Ref ∢ 65.4°
Q 2, 3

114.6° OR 245.4°

①

$114.6 = \frac{\cancel{360}}{\cancel{4.4}}(t-2.9)\ \frac{\cancel{4.4}}{\cancel{360}}$

$\frac{114.6(4.4)}{360} = t-2.9$

$\frac{114.6(4.4)}{360}+2.9 = t$

$t = 4.3$ years

②

$245.4 = \frac{360}{4.4}(t-2.9)$

$245.4\left(\frac{4.4}{360}\right)+2.9 = t$

$t = 5.9$ years

Attachments

Bonus Soln - Fox Population.doc