1) $y = \csc \underbrace{7x^3}_{u}$

$y' = -\csc 7x^3 \cot 7x^3 (21x^2)$

2) $y = \cot^3 5x^7 \Rightarrow (\cot 5x^7)^3$ $(\underline{BLOB})^{\text{exponent}}$

$y' = 3(\cot 5x^7)^2(-\csc^2 5x^7 (35x^6))$

3) $y = \sin\sqrt{x} \cos^2 3x \sec x^7$

$y' = \underline{\cos\sqrt{x}\left(\tfrac{1}{2}x^{-1/2}\right)} \cos^2 3x \sec x^7 + \underline{2(\cos 3x)^1(-\sin 3x(3))} \sin\sqrt{x} \sec x^7$

$+ \underline{\sec x^7 \tan x^7 (7x^6)} \sin\sqrt{x} \cos^2 3x$

4) $y = \sec \underbrace{(\cos x^4)}_{u}$

$y' = \sec(\cos x^4) \tan(\cos x^4)(-\sin x^4 (4x^3))$

4. $f(x)=\tan\left[\cos\left(8x^{-3}\right)\right]$

$f'(x)=\sec^2(\cos(8x^{-3}))(-\sin(8x^{-3})(-24x^{-4}))$

5. $f(x)=\sin\left\{\underbrace{\cos\left[\tan^3(7x)\right]}_{u}\right\}$

$f'(x)=\cos(\cos(\tan^3(7x)))(-\sin(\tan^3 7x)(3\tan^2 7x\sec^2 7x(7)))$

6. $y=\dfrac{6x\sqrt{5\cot\sqrt{x}+\cos^3 3x}}{\tan\left(\sin\sqrt[3]{x}\right)+8\cot x^7-\csc\left(x^4-1\right)^5}$

Jacob was here

$\frac{dy}{dx}$

$$y'=\dfrac{\left\{18x^2(5\cot\sqrt{x}+\cos^3 3x)^{\frac{1}{2}}+6x^3\left[\frac{1}{2}(5\cot\sqrt{x}+\cos^3 3x)^{-\frac{1}{2}}\cdot(-5\csc^2\sqrt{x}(\tfrac{1}{2}x^{-\frac{1}{2}})+3(\cos 3x)^2(-\sin 3x)(3))\right]\right\}\left[\tan\left(\sin\sqrt[3]{x}\right)+8\cot x^7-\csc\left(x^4-1\right)^5\right]-6x\sqrt{5\cot\sqrt{x}+\cos^3 3x}\left[\sec^2(\sin x^{\frac{1}{3}})(\cos x^{\frac{1}{3}}(\tfrac{1}{3}x^{-\frac{2}{3}}))+8(-\csc^2 x^7(7x^6))+\csc(x^4-1)^5\cot(x^4-1)^5(5(x^4-1)^4(4x^3))\right]}{\left[\tan\left(\sin\sqrt[3]{x}\right)+8\cot x^7-\csc\left(x^4-1\right)^5\right]^2}$$

Rubric Based | Holistic Marking

① [4]

② [5]

③ [6]

→ Transfer/Careless

$y = 3x^{-2}$

$y' = -6x^{-1}$

$y = 3(x^2 - 5x)^4$

$y' = 12(2x-5)^3(2)$ Conceptual Error

$(x^2+3x)(x+2) + (x+2)(2x+3)$

$-5x \qquad -5x^{8/3}$

$+5(x^3)^{1/8}$

$+\frac{5}{8}(x^3)^{-7/8}(3x^2)$

Implicit Differentiation

- Sometimes an equation only <u>implicitly</u> defines y as a function (or functions) of x .
- Examples
  - $x^2 + y^2 = 25$
  - $x^3 + y^3 = 6xy$

$(x-3)^2 + 7$

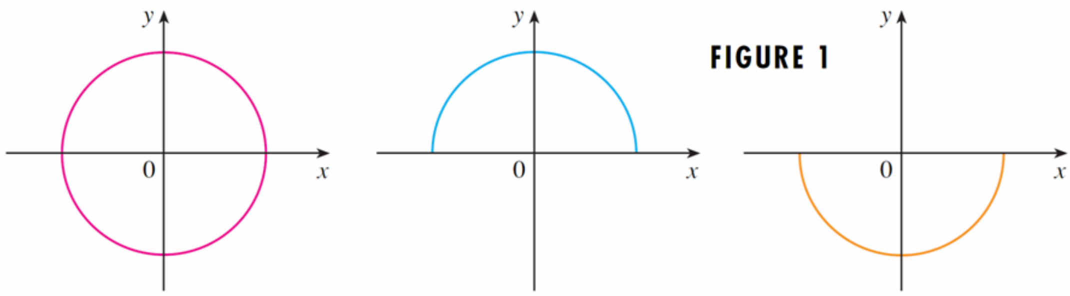
$(x+3)^2 + (y-4)^2 = 9$ $x^2 + y^2 = r^2$ Radius

- The first equation could easily be rearranged for $y =$...

$(-3, 4)$
$r = 3$

$$y = \pm\sqrt{25 - x^2}$$ Actually gives two functions

FIGURE 1



(a) $x^2 + y^2 = 25$ (b) $f(x) = \sqrt{25 - x^2}$ (c) $g(x) = -\sqrt{25 - x^2}$

- The other sample equation

$$x^3 + y^3 = 6xy$$

  - can be solved for y but
  - the results are very complicated.

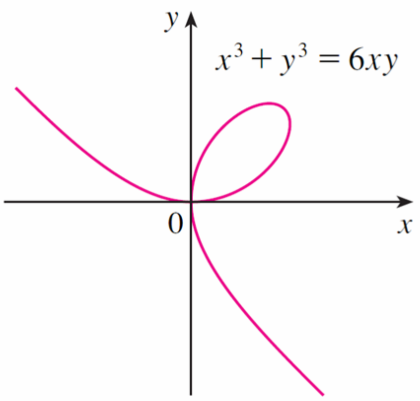


FIGURE 2 The folium of Descartes

Implicit Differentiation

- There is a way called *implicit differentiation* to find dy/dx without solving for y :
  - First differentiate both sides of the equation with respect to x ;
  - Then solve the resulting equation for y' .
- We will always assume that the given equation does indeed define y as a differentiable function of x .

Example

- For the circle $x^2 + y^2 = 25$, find
  a) dy/dx
  b) an equation of the tangent at the point (3, 4) .

Solution:

Start by differentiating both sides of the equation:

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

Remembering that y is a function of x and using the Chain Rule, we have

$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2)\frac{dy}{dx} = 2y\frac{dy}{dx}$$

Thus... $2x + 2y\dfrac{dy}{dx} = 0$

Solving for $\frac{dy}{dx}$ ... $\dfrac{dy}{dx} = -\dfrac{x}{y}$

Therefore at the point (3,4) the equation of the tangent would be...

$$y - 4 = -\tfrac{3}{4}(x - 3) \qquad \text{or} \qquad 3x + 4y = 25$$

ex. $(x)^2 + y^2 = 25$

$\cancel{\frac{d}{dx}}\quad 2(x)^1(1) + 2(y)^1 y' = 0$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{-\cancel{2}x}{\cancel{2}y}$

$\frac{dy}{dx} = -\frac{x}{y}$

$\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$

$\frac{dx}{dy}$

Same Example Revisited

- Since it is easy to solve this equation for y, we
  - do so, and then
  - find the equation of the tangent line by earlier methods, and then
  - compare the result with our preceding answer:

Solution

- Solving the equation gives $y = \pm\sqrt{25 - x^2}$ as before.
- The point (3, 4) lies on the upper semicircle $y = \sqrt{25 - x^2}$ and so we consider the function $f(x) = \sqrt{25 - x^2}$

Differentiate *f*:

$$f'(x) = \tfrac{1}{2}(25 - x^2)^{-1/2} \frac{d}{dx}(25 - x^2)$$

$$= \tfrac{1}{2}(25 - x^2)^{-1/2}(-2x) = -\frac{x}{\sqrt{25 - x^2}}$$

$$f'(3) = -\frac{3}{\sqrt{25 - 3^2}} = -\frac{3}{4}$$

Solution (cont'd)

- So $f'(3) = -\dfrac{3}{\sqrt{25-3^2}} = -\dfrac{3}{4}$,

  leading to the same equation

  $$3x + 4y = 25$$

  for the tangent that we obtained earlier.
- Note that although this problem <u>could</u> be done both ways, implicit differentiation was easier!

Example

- For the folium of Descartes $x^3 + y^3 = 6xy$,
  - Find y'
  - Find the tangent to the curve at the point (3, 3)
  - At what points on the curve is the tangent line horizontal?

$$x^3+y^3=6xy$$

$$3x^2+3y^2\frac{dy}{dx}=6\left[1y+x\frac{dy}{dx}\right]$$

$$y \Rightarrow \frac{dy}{dx}$$

$$x \Rightarrow \frac{dx}{dx} \Rightarrow 1$$

$$3y^2\frac{dy}{dx}-6x\frac{dy}{dx}=6y-3x^2$$

$$\frac{\frac{dy}{dx}\cancel{(3y^2-6x)}}{\cancel{3y^2-6x}}=\frac{6y-3x^2}{3y^2-6x}$$

$$\frac{dy}{dx}=\frac{2y-x^2}{y^2-2x}$$

Example:

$(y)^2 \quad 2(y)^1\left(\frac{dy}{dx}\right)$

Given $x^2 - \underbrace{3x^3}\underbrace{y^2} + y^2 = 5xy^3 - 4$

Find $\frac{dy}{dx}$

$$2x - \left(9x^2y^2 + 3x^3\left(2y\frac{dy}{dx}\right)\right) + 2y\frac{dy}{dx} = 5y^3 + 5x\left(3y^2\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left(-6x^3y + 2y - 15xy^2\right) = 5y^3 - 2x + 9x^2y^2$$

$$\frac{dy}{dx} = \frac{5y^3 - 2x + 9x^2y^2}{-6x^3y + 2y - 15xy^2}$$

$$7x^3y^4 - y^7 + \underbrace{xy} = 7x^2 - \underbrace{3x^4y^8}$$

$$21x^2y^4 + 7x^3\left(4y^3\frac{dy}{dx}\right) - 7y^6\frac{dy}{dx} + y + x\frac{dy}{dx} =$$

$$14x - \left(12x^3y^8 + 3x^4\left(8y^7\frac{dy}{dx}\right)\right)$$

$$\frac{dy}{dx}\left(28x^3y^3 - 7y^6 + x + 24x^4y^7\right) = 14x - 12x^3y^8 - 21x^2y^4 - y$$

$$\frac{dy}{dx} = \frac{14x - 12x^3y^8 - 21x^2y^4 - y}{28x^3y^3 - 7y^6 + x + 24x^4y^7}$$

Example:

Find $\frac{dy}{dx}$, given the curve $x^2 - 3xy = (5x^2 - 8y)^5$

$$2x - \left(3y + 3x\frac{dy}{dx}\right) = 5(5x^2 - 8y)^4\left(10x - 8\frac{dy}{dx}\right)$$

$$2x - 3y - 3x\frac{dy}{dx} = 50x(5x^2 - 8y)^4 - 40(5x^2 - 8y)^4\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(-3x + 40(5x^2 - 8y)^4\right) = 50x(5x^2 - 8y)^4 - 2x + 3y$$

$$\frac{dy}{dx} = \frac{50x(5x^2 - 8y)^4 - 2x + 3y}{40(5x^2 - 8y)^4 - 3x}$$

$$\sqrt{x^2-3y} = 8x^3y^5$$

$$\frac{1}{2}(x^2-3y)^{-\frac{1}{2}}\left(2x-3\frac{dy}{dx}\right) = 24x^2y^5 + 8x^3(5y^4)\frac{dy}{dx}$$

$$1x(x^2-3y)^{-\frac{1}{2}} - \frac{3}{2}(x^2-3y)^{-\frac{1}{2}}\frac{dy}{dx} = 24x^2y^5 + 40x^3y^4\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{24x^2y^5 - x(x^2-3y)^{-\frac{1}{2}}}{-\frac{3}{2}(x^2-3y)^{-\frac{1}{2}} - 40x^3y^4}$$

Homework

Page 107

1 d, f, h

2 c, d

3 c, d

5 a

6 a, b, c

Attachments

Bonus Soln - Fox Population.doc