Warm Up

Differentiate the following...

$$\left(x^3 + y^5\right)^6 + 3xy = 2x^4y^5$$

$6(x^3+y^5)^5\left(3x^2+5y^4\frac{dy}{dx}\right)+3y+3x\frac{dy}{dx}=8x^3y^5+2x^4(5y^4)\frac{dy}{dx}$

$18x^2(x^3+y^5)^5+30y^4(x^3+y^5)^5\frac{dy}{dx}+3y+3x\frac{dy}{dx}=8x^3y^5+10x^4y^4\frac{dy}{dx}$

$\frac{dy}{dx}\left(30y^4(x^3+y^5)^5+3x-10x^4y^4\right)=8x^3y^5-18x^2(x^3+y^5)^5-3y$

$$\frac{dy}{dx}=\frac{8x^3y^5-18x^2(x^3+y^5)^5-3y}{30y^4(x^3+y^5)^5+3x-10x^4y^4}$$

ex. $\sec x^2 - 3x^2y = \cos(x^2+y^3)$

$\sec x^2 \tan x^2 (2x) - \left(6xy + 3x^2\frac{dy}{dx}\right) = -\sin(x^2+y^3)\left(2x + 3y^2\frac{dy}{dx}\right)$

$2x\sec x^2 \tan x^2 - 6xy - 3x^2\frac{dy}{dx} = -2x\sin(x^2+y^3) - 3y^2\sin(x^2+y^3)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-2x\sin(x^2+y^3) - 2x\sec x^2\tan x^2 + 6xy}{-3x^2 + 3y^2\sin(x^2+y^3)}$$

Higher Order Derivatives

We can continue to find the derivatives of a derivative. We find the

- second derivative by taking the derivative of the first,
- third derivative by taking the derivative of the second ... etc

Examples:

1. Determine the higher order derivatives for $f(x)$...

$$f(x) = x^4 - 2x^3 + 3x - 5$$

$f'(x) = 4x^3 - 6x^2 + 3$ $\quad f^{(4)} =$

$f''(x) = 12x^2 - 12x$ $\quad f^{(5)} =$

$f'''(x) = 24x - 12$

$f''''(x) = 24$

$f'''''(x) = 0$

2. Determine $f'''(x)$ given that $f(x) = \dfrac{5}{\sqrt{2-3x}}$

$f(x) = 5(2-3x)^{-\frac{1}{2}}$

$f'(x) = -\frac{5}{2}(2-3x)^{-\frac{3}{2}}(-3)$

$f'(x) = \frac{15}{2}(2-3x)^{-\frac{3}{2}}$

$f''(x) = -\frac{45}{4}(2-3x)^{-\frac{5}{2}}(-3)$

$f''(x) = \frac{135}{4}(2-3x)^{-\frac{5}{2}}$

$f'''(x) = -\frac{675}{8}(2-3x)^{-\frac{7}{2}}(-3)$

$f'''(x) = \frac{2025}{8}(2-3x)^{-\frac{7}{2}}$

$$f(x) = 5(1-3x)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{5}{2}(1-3x)^{-\frac{3}{2}}(-3)$$

$$f''(x) = -\frac{5}{2}\left[-\frac{3}{2}(1-3x)^{-\frac{5}{2}}(-3)\right](-3)$$

$$f'''(x) = -\frac{5}{2}\left[-\frac{3}{2}\left[-\frac{5}{2}(1-3x)^{-\frac{7}{2}}(-3)\right](-3)\right](-3)$$

Ex. $f(x)=\dfrac{6}{(5x-2)^4}$ $f'''(x)=?$

$f(x)=6(5x-2)^{-4}$

$f'(x)=-24(5x-2)^{-5}(5)$

$f'(x)=-120(5x-2)^{-5}$

$f''(x)=600(5x-2)^{-6}(5)$

$f''(x)=3000(5x-2)^{-6}$

$f'''(x)=-18000(5x-2)^{-7}(5)$

$f'''(x)=-90000(5x-2)^{-7}$

$f''''(x)=630000(5x-2)^{-8}(5)$

3. Find the second derivative of the implicit function $xy + y^2 = 4$.

$$xy + y^2 = 4$$

$$y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$(x + 2y)\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{x+2y}$$

y' $\frac{dy}{dx}$

y'' $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = -\frac{\frac{dy}{dx}(x+2y) + y\left(1 + 2\frac{dy}{dx}\right)}{(x+2y)^2} \qquad -y(x+2y)^{-1}$$

$$\frac{d^2y}{dx^2} = \left(\frac{y}{x+2y}\right)(x+2y) + y\left(1 + 2\left(\frac{-y}{x+2y}\right)\right)$$

$$= y + y - \frac{2y^2}{(x+2y)}$$

$$\frac{(x+2y)^2}{(x+2y)^2} = \frac{2y - \frac{2y^2}{x+2y}}{(x+2y)^2}$$

4. *Determine the fourth derivative of* $y = \cos(5x)$

HOMEWORK

Page 111
(Higher Order Derivatives)

#2, 3, 4, 5, 7 (a)

Attachments

Bonus Soln - Fox Population.doc