

10. c) $\frac{3}{m} - \frac{3}{2m+3}$

$\frac{3}{m^2} + \frac{1}{2m+3}$

$\frac{3(2m+3) - 3m}{m(2m+3)}$

$\frac{6m+9 - 3m}{m^2(2m+3)}$

$\frac{3m+9}{m(2m+3)}$

$\frac{m^2+6m+9}{m^2(2m+3)}$

$\frac{3(m+3)}{m(2m+3)} \cdot \frac{m^2(2m+3)}{(m+3)}$
 $\frac{3m}{m+3}$

10a) $\frac{2 - \frac{6}{x}}{1 - \frac{9}{x^2}}$

$\frac{2x-6}{x}$

$\frac{2x-6}{x^2-9}$

$\frac{2(x-3)}{(x+3)(x-3)} = \frac{2x}{x+3}$

Rational Equations

Focus on...

- identifying non-permissible values in a rational equation
- determining the solution to a rational equation algebraically
- solving problems using a rational equation

Time to ACTIVATE prior knowledge on this concept...

Solve the following equation for x

$\frac{1}{2} \cdot \frac{2}{3} = \frac{3}{4}$

$\frac{6x}{12} - \frac{8}{12} = \frac{9}{12}$

$6x - 8 = 9$
 $6x = 17$
 $x = \frac{17}{6}$

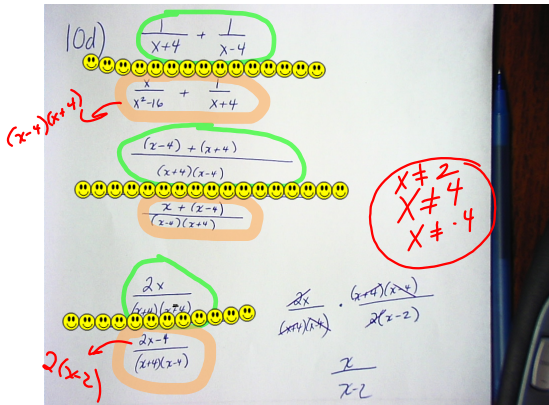
$6x - 8 = 9$
 $6x = 17$
 $x = \frac{17}{6}$

solve $\frac{3(x-4)}{12} + \frac{4x}{12} = \frac{72}{12}$

$\frac{3(x-4)}{12} + \frac{4x}{12} = \frac{72}{12}$

$3x - 12 + 4x = 72$
 $7x = 84$
 $x = 12$

$3(x-4) + 4x = 72$
 $x = 12$



STRATEGY!! Or demonstrating effective number sense.

Clear the Fractions from a Rational Equation. If your equation has rational expressions, multiply both sides of the equation by the least common denominator to clear the equation of rational expressions.

Example: solve $\rightarrow \frac{3}{2x} - \frac{2x}{x+1} = -2$

$$3(x+1) - 2x(2x) = -2(2x)(x+1)$$

$$3x+3 - 4x^2 = -4x^2 - 4x$$

$$3x+3 = -4x$$

$$7x = -3$$

$$x = -3/7$$

le: Solve $\rightarrow \frac{6}{x} - \frac{9}{x-1} = \frac{1}{4}$

$$6(4)(x-1) - 9x(4) = 1x(x-1)$$

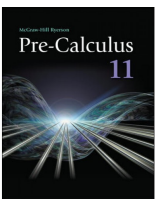
$$24(x-1) - 36x = x(x-1)$$

$$24x - 24 - 36x = x^2 - x$$

$$0 = x^2 + 11x + 24$$

$$0 = (x+8)(x+3)$$

$$x = -8 \quad x = -3$$



p348 #2,3

Extra help
12:10
and I.S. ↵
(sign up)

Example: solve $\rightarrow x - \frac{2}{x-3} = \frac{x-1}{3-x}$

Always FACTOR first!!

$$\frac{k^2 - 8k + 12}{4} = \frac{k-2}{k} + \frac{k-6}{k-6}$$

$$\frac{(k-2)(k-6)}{4} = \frac{k(k-6)}{k(k-6)} + \frac{1(k-2)(k-6)}{(k-2)(k-6)}$$

$$4 = k(k-6) + 1(k-2)$$

$$4 = k^2 - 6k + k - 2$$

$$0 = k^2 - 5k - 6$$

$$(k+1)(k-6)$$

$k = -1$ ~~$k = 6$~~ restriction

| | |
|------------------------|---------------------------------|
| LS | RS |
| $\frac{4}{(-3)(-7)}$ | $\frac{-1}{-3} + \frac{1}{-7}$ |
| $\frac{4}{21}$ | $\frac{1}{3} - \frac{1}{7}$ |
| | $\frac{7}{21} - \frac{3}{21}$ |
| | $\frac{4}{21}$ |
| LS | RS |
| $\frac{4}{(6-6)(6-2)}$ | $\frac{6}{6-2} + \frac{1}{6-6}$ |
| | D.N.E |
| | D.N.E |

1a) $\frac{x^2(x-1)}{3} - \frac{3(x-3)}{4} = \frac{9(5)}{12} + \frac{15(x)}{6}$

$$4(x-1) - 3(3x-9) = 5 + 2x$$

$$4x - 4 - 9x + 27 = 5 + 2x$$

$$-5x + 23 = 5 + 2x$$

$$-7x = -18$$

$$x = \frac{18}{7}$$

2a) $\frac{f+3}{3} - \frac{4f-2}{3} = \frac{6}{3}$

$$f+3 - 4f+2 = 6$$

$$-3f+5 = 6$$

$$-3f = 1$$

$$f = -\frac{1}{3}$$

2c) $\frac{9}{w-3} - \frac{4}{w-6} = \frac{18}{(w-3)(w-6)}$

$$9(w-6) - 4(w-3) = 18$$

$$9w - 54 - 4w + 12 = 18$$

$$5w - 42 = 18$$

$$5w = 60$$

$$w = 12$$

3b) $\frac{6}{c-3} = \frac{c+3}{(c+3)(c-3)} - 5$

$$6 = 1 - 5(c-3)$$

$$6 = 1 - 5c + 15$$

$$-10 = -5c$$

$$2 = c$$

3d) $\frac{x^2+x+2}{x+1} - x = \frac{x^2-5}{(x+1)(x-1)}$

$$(x^2+x+2)(x-1) - x(x+1)(x-1) = x^2-5$$

$$x^3 - x^2 + x^2 - x + 2x - 2 - x^3 + x = x^2 - 5$$

$$2x - 2 = x^2 - 5$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$x = 3$ $x = -1$

| | |
|---|-------------------------------|
| LS | RS |
| $\frac{3^2+3+2}{3+1} - 3$ | $\frac{3^2-5}{(3+1)(3-1)}$ |
| $\frac{14}{4} - 3$ | $\frac{4}{(4)(2)}$ |
| $\frac{14}{4} - \frac{12}{4} = \frac{2}{4} = \frac{1}{2}$ | $\frac{1}{2}$ |
| LS | RS |
| $\frac{(-1)^2+(-1)+2}{(-1)+1}$ | $\frac{(-1)^2-5}{(-1)(-1)-5}$ |
| $\frac{0}{0}$ | |

not a solution


Correct answer for these --> Professional Solver!!

$$\frac{3}{x^2+2x-15} + \frac{4}{x^2-9} = \frac{8}{x^2+8x+15}$$

$$\frac{3(x+3)(x-3)(x+3)}{(x+5)(x-3)} + \frac{4(x+3)(x-3)(x+3)}{(x+3)(x-3)} = \frac{8(x+3)(x-3)(x+3)}{(x+5)(x-3)}$$

$$3(x+3) + 4(x+5) = 8(x-3)$$

$$3x+9+4x+20 = 8x-24$$

$$x = 53$$


Warm-Up...

Simplify the following expression: $\frac{\frac{5}{x+4}}{\frac{1}{x-4} - \frac{2}{x^2-16}}$

Solve each of the following:

$$\frac{2x}{x+3} - \frac{x}{x+7} = \frac{x^2-1}{x^2+10x+21}$$

$$\frac{10}{x+3} - \frac{3}{5} = \frac{10x+1}{3x+9}$$

Poor Tyler could use your help...

Tyler has begun to solve a rational equation. His work is shown below.

$$\begin{aligned} \frac{2}{x-1} - 3 &= \frac{5x}{x+1} \\ 2(x+1) - 3(x+1)(x-1) &= 5x(x-1) \\ 2x+2 - 3x^2+3 &= 5x^2-5x \\ 0 &= 8x^2-7x-5 \end{aligned}$$

Check his work... make corrections... and then finish the problem

$$\begin{aligned} -x &= -40 \\ -x &= -7 \end{aligned}$$

Time to start applying some of our number sense!!

8. The sum of two numbers is 25. The sum of their reciprocals is $\frac{1}{4}$. Determine the two numbers.

Erin and Andrea set off at the same time on a 30-km walk for charity. Erin, who has trained all year for this event, walks 1 km/h faster than Andrea. Erin finishes the walk 1 h ahead of Andrea. How fast was each sister walking, and how long did it take for each sister to finish the walk?

Would something like this chart help organize things?

| | Distance (km) | Rate (km/h) | Time (h) |
|--------|---------------|-------------|------------------|
| Erin | 30 | $x+1$ | $\frac{30}{x+1}$ |
| Andrea | 30 | x | $\frac{30}{x}$ |

$$\frac{30}{x+1} + 1 = \frac{30}{x}$$

$$30x + x(x+1) = 30(x+1)$$

$$30x + x^2 + x = 30x + 30$$

$$x^2 + x - 30 = 0$$

$$(x+6)(x-5) = 0$$

$$x = 6 \text{ or } x = 5$$

Erin's time $\frac{30}{6} = 5 \text{ hours}$
Andrea $\frac{30}{5} = 6 \text{ hours}$

The Olympic world record for the women's 600m speed skating is 42.76s. At the Olympic trials Jean knew that if she skated 3m/s faster she could cut 10s from her time

| | Distance (km) m | Rate (km/h) m/s | Time (h) seconds |
|------------|-----------------|-----------------|-------------------|
| Jean now | 600 | x | $\frac{600}{x}$ |
| Super Jean | 600 | x+3 | $\frac{600}{x+3}$ |

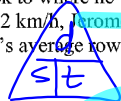
ignore

$$\frac{600}{x} - \frac{600}{x+3} = 10$$

$$600(x+3) - 600x = 10x(x+3)$$



Jerome rows his boat 24 km downstream and back to where he began. When the average speed of the current is 2 km/h, Jerome can complete the journey in 9 h. What is Jerome's average rowing speed in still water?



| | Distance (km) | Rate (km/h) | Time (h) |
|------------|---------------|-------------|------------------|
| Downstream | 24 | x+2 | $\frac{24}{x+2}$ |
| Upstream | 24 | x-2 | $\frac{24}{x-2}$ |

up + down = 9

$$\frac{24}{x+2} + \frac{24}{x-2} = 9$$

$$24(x-2) + 24(x+2) = 9(x-2)(x+2)$$

$$24x + 48 + 24x + 48 = 9(x^2 - 4)$$

$$-18x^2 = -36$$

$$\frac{-18x^2}{-18} = \frac{-36}{-18}$$

$$x^2 = 2$$

$$3x^2 - 18x + 2x - 12$$

$$3x(x-6) + 2(x-6)$$

$$(x-6)(3x+2)$$

$$48x = 9x^2 - 36$$

$$0 = 9x^2 - 48x - 36$$

$$0 = 3x^2 - 16x - 12$$

$$0 = (x-6)(3x+2)$$

$$x = 6$$

$3x = \frac{2}{x}$
 $x = -2/3$

How long would it take to fill the pool if only hose B were used?

14. Two kayakers paddle 18 km downstream with the current in the same time it takes them to go 8 km upstream against the current. The rate of the current is 3 km/h.

a) Complete a table like the following in your notebook. Use the formula distance = rate x time.

| | Distance (km) | Rate (km/h) | Time (h) |
|------------|---------------|-------------|------------------|
| Downstream | 18 | x+3 | $\frac{18}{x+3}$ |
| Upstream | 8 | x-3 | $\frac{8}{x-3}$ |

- b) What equation could you use to find the rate of the kayakers in still water?
- c) Solve your equation.
- d) Which values are non-permissible?

Did You Know?

When you are travelling with the current, add the speed of the current to your rate of speed. When you are travelling against the current, subtract the speed of the current.

#12
#13

$$\frac{18}{x+3} = \frac{8}{x-3}$$

$$18(x-3) = 8(x+3)$$

$$18x - 54 = 8x + 24$$

$$10x = 78$$

$$x = 7.8$$

12.

| | | | |
|------|---|-----|---------|
| cold | 2 | 1/2 | 1/2 |
| hot | 3 | 1/3 | 1/3 |
| both | x | 1/x | 1/x = 1 |

$$\frac{3x + 2x}{2 \cdot 3} = 1$$

$$3x + 2x = 6$$

$$5x = 6$$

$$x = 6/5 = 1.2$$

13.

| | | | |
|------|---|-----|-----|
| A | 3 | 1/3 | 1/3 |
| B | x | 1/x | 1 |
| Both | 2 | 1/2 | 1/2 |

$$\frac{2x + 6}{2 \cdot 3} = 1$$

$$2x + 6 = 3x$$

$$6 = x$$

Jerome rows his boat 24 km downstream and back to where he began. When the average speed of the current is 2 km/h, Jerome can complete the journey in 9 h. What is Jerome's average rowing speed in still water?



| | Distance (km) | Rate (km/h) | Time (h) |
|------------|---------------|-------------|------------------|
| Downstream | 24 | $x+2$ | $\frac{24}{x+2}$ |
| Upstream | 24 | $x-2$ | $\frac{24}{x-2}$ |

Handwritten solution:

$$\frac{24}{x+2} + \frac{24}{x-2} = 9$$

$$24(x-2) + 24(x+2) = 9(x^2-4)$$

$$24x - 48 + 24x + 48 = 9x^2 - 36$$

$$0 = 9x^2 - 48x - 36$$

$$0 = 3x^2 - 16x - 12$$

$$0 = (x-6)(3x+2)$$

$x = 6$ (boxed)
 ~~$x = -\frac{2}{3}$~~

A thought bubble contains:
 $3x^2 - 16x - 12$
 $3x^2 - 18x + 2x - 12$
 $3x(x-6) + 2(x-6)$
 $(x-6)(3x+2)$