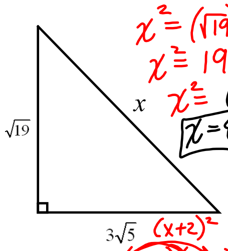
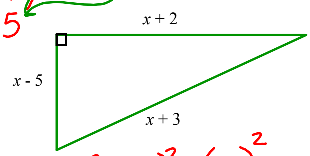
Warm Up

Determine the measure of the variable in each of the following diagrams:





$\sqrt{9}\sqrt{5}$
$\sqrt{45}$

$x^2 = (\sqrt{19})^2 + (3\sqrt{5})^2$ $9(5)$

$x^2 = 19 + 45$

$x^2 = 64$

$\boxed{x = 8}$

$(x+2)^2$

$(x+2)(x+2)$

$x^2 + 4x + 4$

$(x+2)^2 + (x-5)^2 = (x+3)^2$

$\cancel{x^2} + 4x + 4 + \cancel{x^2} - 10x + 25 = \cancel{x^2} + 6x + 9$

$x^2 - 12x + 20 = 0$

$(x-2)(x-10) = 0$

$x = 2 \quad x = 10$

$(x-10)^2$

$x^2 - 20x + 10$

$(x-5)^2$

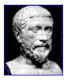
$(x-5)(x-5)$

$x^2 - 5x - 5x + 25$

$x^2 - 10x + 25$

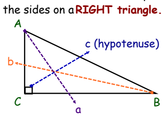
$(x+7)^2$

$x^2 + 14x + 49$



Pythagorean Theorem

- is a fundamental relationship amongst the sides on a **RIGHT triangle.**



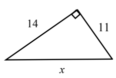
$$\boxed{c^2 = a^2 + b^2}$$

OPTIONS...

#1. Finding the unknown hypotenuse:

$$\boxed{c^2 = a^2 + b^2}$$

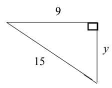
ex:



#2. Finding an unknown side

$$\boxed{a^2 = c^2 - b^2}$$

ex:



Pythagorean Triples

Verifying a Pythagorean Triple...

LS	RS

LS	RS

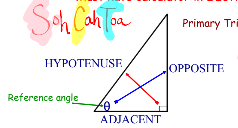
Trigonometric Table:

Degrees	Radian Measure	Sin	Cos	Tan	Degrees	Radian Measure	Sin	Cos	Tan
0	0.00000	0.00000	1.00000	0.00000	46	0.80285	0.71934	0.69466	1.03553
1	0.01745	0.01745	0.99985	0.01746	47	0.82030	0.73135	0.68200	1.07237
2	0.03491	0.03490	0.99939	0.03492	48	0.83776	0.74314	0.66913	1.11061
3	0.05236	0.05234	0.99863	0.05241	49	0.85521	0.75471	0.65606	1.15037
4	0.06981	0.06976	0.99756	0.06993	50	0.87266	0.76604	0.64279	1.19175
5	0.08727	0.08716	0.99619	0.08749	51	0.89012	0.77715	0.62932	1.23490
6	0.10472	0.10453	0.99452	0.10510	52	0.90757	0.78801	0.61566	1.27994
7	0.12217	0.12187	0.99255	0.12278	53	0.92502	0.79864	0.60182	1.32704
8	0.13963	0.13917	0.99027	0.14054	54	0.94248	0.80902	0.58779	1.37638
9	0.15708	0.15643	0.98769	0.15838	55	0.95993	0.81915	0.57358	1.42815
10	0.17453	0.17365	0.98481	0.17633	56	0.97738	0.82904	0.55919	1.48256
11	0.19199	0.19081	0.98163	0.19438	57	0.99484	0.83867	0.54464	1.53986
12	0.20944	0.20791	0.97815	0.21256	58	1.01229	0.84805	0.52992	1.60033
13	0.22689	0.22495	0.97437	0.23087	59	1.02974	0.85717	0.51504	1.66428
14	0.24435	0.24192	0.97030	0.24933	60	1.04720	0.86603	0.50000	1.73205
15	0.26180	0.25882	0.96593	0.26795	61	1.06465	0.87462	0.48481	1.80405
16	0.27925	0.27564	0.96126	0.28675	62	1.08210	0.88295	0.46947	1.88073
17	0.29671	0.29237	0.95630	0.30573	63	1.09956	0.89101	0.45399	1.96261
18	0.31416	0.30902	0.95106	0.32492	64	1.11701	0.89879	0.43837	2.05030
19	0.33161	0.32557	0.94552	0.34433	65	1.13446	0.90631	0.42262	2.14451
20	0.34907	0.34202	0.93969	0.36397	66	1.15192	0.91355	0.40674	2.24604
21	0.36652	0.35837	0.93358	0.38386	67	1.16937	0.92050	0.39073	2.35585
22	0.38397	0.37461	0.92718	0.40403	68	1.18682	0.92718	0.37461	2.47509
23	0.40143	0.39073	0.92050	0.42447	69	1.20428	0.93358	0.35837	2.60509
24	0.41888	0.40674	0.91355	0.44523	70	1.22173	0.93969	0.34202	2.74748
25	0.43633	0.42262	0.90631	0.46631	71	1.23918	0.94552	0.32557	2.90421
26	0.45379	0.43837	0.89879	0.48773	72	1.25664	0.95106	0.30902	3.07768
27	0.47124	0.45399	0.89101	0.50953	73	1.27409	0.95630	0.29237	3.27085
28	0.48869	0.46947	0.88295	0.53171	74	1.29154	0.96126	0.27564	3.48741
29	0.50615	0.48481	0.87462	0.55431	75	1.30900	0.96593	0.25882	3.73205
30	0.52360	0.50000	0.86603	0.57735	76	1.32645	0.97030	0.24192	4.01078
31	0.54105	0.51504	0.85717	0.60086	77	1.34390	0.97437	0.22495	4.33148
32	0.55851	0.52992	0.84805	0.62487	78	1.36136	0.97815	0.20791	4.70463
33	0.57596	0.54464	0.83867	0.64941	79	1.37881	0.98163	0.19081	5.14455
34	0.59341	0.55919	0.82904	0.67451	80	1.39626	0.98481	0.17365	5.67128
35	0.61087	0.57358	0.81915	0.70021	81	1.41372	0.98769	0.15643	6.31375
36	0.62832	0.58779	0.80902	0.72654	82	1.43117	0.99027	0.13917	7.11537
37	0.64577	0.60182	0.79864	0.75355	83	1.44862	0.99255	0.12187	8.14435
38	0.66323	0.61566	0.78801	0.78129	84	1.46608	0.99452	0.10453	9.51436
39	0.68068	0.62932	0.77715	0.80978	85	1.48353	0.99619	0.08716	11.43005
40	0.69813	0.64279	0.76604	0.83910	86	1.50098	0.99756	0.06976	14.30067
41	0.71558	0.65606	0.75471	0.86929	87	1.51844	0.99863	0.05234	19.08114
42	0.73304	0.66913	0.74314	0.90040	88	1.53589	0.99939	0.03490	28.63625
43	0.75049	0.68200	0.73135	0.93252	89	1.55334	0.99985	0.01745	57.28996
44	0.76794	0.69466	0.71934	0.96569	90	1.57080	1.00000	0.00000	
45	0.78540	0.70711	0.70711	1.00000					

Trigonometric Ratios

*** Must have calculator in DEGREE mode ***

SohCahToa



Primary Trigonometric Ratios

$$\boxed{\sin\theta = \frac{opp}{hyp}}$$

$$\boxed{\cos\theta = \frac{adj}{hyp}}$$

$$\boxed{\tan\theta = \frac{opp}{adj}}$$

Memory Aid: "SOH CAH TOA"

Reciprocal Trigonometric Ratios

$$\text{cosecant}\,\theta = \frac{hypotenuse}{opposite}$$

$$\text{secant}\,\theta = \frac{hypotenuse}{adjacent}$$

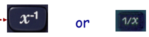
$$\text{cotangent}\,\theta = \frac{adjacent}{opposite}$$

Notice that these ratios are each the reciprocal of one of the primary trig ratios

Summary

Primary Ratios	Reciprocal Ratios
$\sin\theta = \frac{opp}{hyp}$	$\csc\theta = \frac{hyp}{opp}$
$\cos\theta = \frac{adj}{hyp}$	$\sec\theta = \frac{hyp}{adj}$
$\tan\theta = \frac{opp}{adj}$	$\cot\theta = \frac{adj}{opp}$

Reciprocal ratios are not found on a calculator....we must learn how to use the reciprocal function on our calculator.

Reciprocal Functions  or

Inverse Trigonometric Functions (Arc Trig Functions)

Trigonometric Functions

Evaluate each of the following:

$\sin 78° =$ ________

[redacted]

$\cos\theta = 0.6469$

$\theta =$ ______

[redacted]

$\cot 118° =$ ________

[redacted]

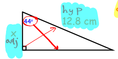
$\sec\theta = 3.2361$

$\theta =$ ______

[redacted]

EXAMPLE - Finding an unknown side

SohCahToa

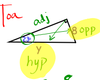


opp

$\cos 44° = \frac{x}{12.8}$

$12.8 \cos 44° = x$

$9.2 = x$

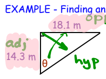


$\sin 23° = \frac{8}{y}$

$y = \frac{8}{\sin 23°}$

$y = 20.5$

EXAMPLE - Finding an unknown angle



SohCahToa

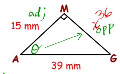
$\tan\theta = \frac{18.1}{14.3}$

$\tan\theta = 1.265...$

$\theta = \tan^{-1} 1.265...$

$\theta = 51.7°$

EXAMPLE - Solve the triangle (find ALL sides and angles)

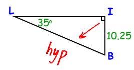


tanθ=

hyp

$x^2 = 39^2 - 15^2$

$x = 36$



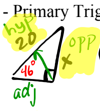
hyp

SohCahToa

HOMEWORK...

Worksheet - Primary Trig Ratios.doc

2 a)



$\sin 46° = \frac{x}{20}$

$20 \sin 46° = x$

$14.4 = x$

c)

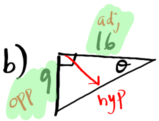
160.3

$\sin 86° = \frac{140}{a}$

$a = \frac{140}{\sin 86°}$

$a = 140.3$

3 b)



$\tan\theta = \frac{9}{16}$

$\theta = \tan^{-1}\left(\frac{9}{16}\right)$

$\theta = 29.4°$

$\theta = 29°$

Warm Up

1. Evaluate each of the following:

(a) $\csc A = 1.1924$
$A =$ _____

(b) $\sec 168° =$ __________

2. Solve the following triangles:

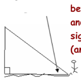
(a) ΔRST, given that S = 90°, $r = 12$ cm and $t = 25$ cm.

(b) ΔMVH, given that M = 90°, H = 14° and $m = 44$ cm.

Applications of Right Angle Trigonometry

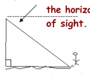
ANGLE OF ELEVATION/DEPRESSION

Angle of elevation - is the angle between the ground and the line of sight. (angle of inclination)



Always from the GROUND up

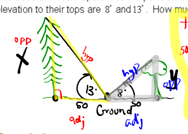
Angle of Depression - is the angle between the horizon and the line of sight.



Always outside the triangle

Example 1:

Two trees are 100m apart. From a point on midway between them, the angles of elevation to their tops are 8° and 13°. How much taller is one tree than the other?



$\tan 13° = \frac{x}{50}$

$50 \tan 13° = x$

$11.5 = x$

$\tan 8° = \frac{y}{50}$

$50 \tan 8° = y$

$7.0 = y$

$11.5 - 7.0 = 4.5$ m

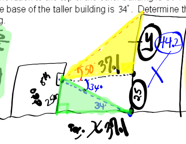
Example 2:

The 8th floor of an apartment building is 25m above the ground. From the 8th floor, the angle of elevation to the top of the other building is 50°. The angle of depression to the base of the taller building is 34°. Determine the height of the taller building.

$\tan 34° = \frac{25}{x}$

$x = \frac{25}{\tan 34}$

$x = 37.1$



$\tan 50 = \frac{y}{37.1}$

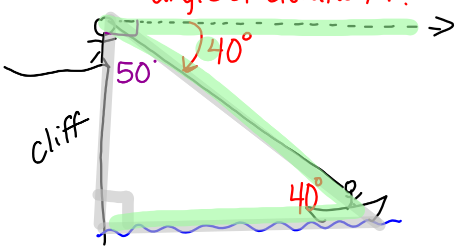
$37.1 \tan 50 = y$

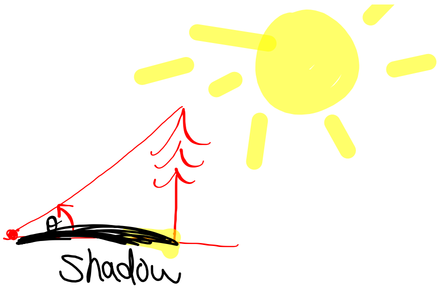
$44.2 = y$

$+ 25$

69.2 m

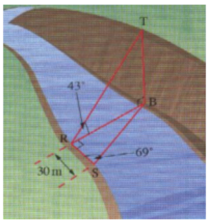
angle of depression equals angle of elevation !!





Example 3

A climbing club plans to scale a cliff overlooking a river. To prepare for the climb, a surveyor visited the site and took some measurements to calculate the height of the cliff. From point R on the shore directly across the river, the angle of elevation to the top of the cliff is $\angle TRB = 43^{\circ}$. From a point S, 30m down the river, $\angle BSR = 69^{\circ}$. How high is the cliff?



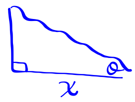
Example 4

An antenna is on the top of the CN Tower in Toronto. From a point 2400 m away, the angles of elevation to the top and bottom of the antenna are 12.1° and 9.9° respectively. How tall is the antenna?

10.8 Exercise #1-6

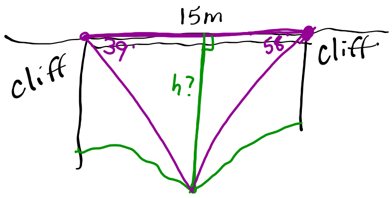
note

#2. Slope $= \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$



Warm Up

2. A new bridge is to be built across a gorge which is known to be 15 m wide. A support pier is to be built at the deepest point of the gorge. If the angles of depression to that point are 39° and 58° from the two ends of the bridge, what must the height of this support pier be?



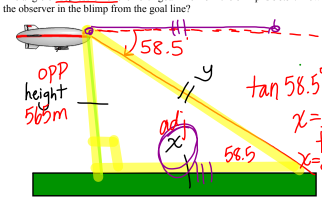
1. A surveyor who wishes to know the width of a river sights a tree on the opposite bank as bearing N 72° E. He then walks 46 m due east along the bank of the river until he is directly across the river from the tree. How wide is the river?

REVIEW of ...

Applications of Trig Ratios

Examples...

#1. The Goodyear Blimp is 565 m above the ground during a Super Bowl game. The angle of depression of the north goal line from the blimp is 58.5°. How far is the observer in the blimp from the goal line?

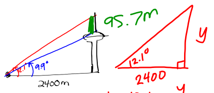


$\tan 58.5^\circ = \frac{565}{x}$

$x = \frac{565}{\tan 58.5}$

$x = 346.2$

#2. An antenna is on the top of the CN Tower in Toronto. From a point 2400 m away, the angles of elevation to the top and bottom of the antenna are 12.1° and 9.9° respectively. How tall is the antenna?



$\tan 12.1 = \frac{y}{2400}$

$2400 \tan 12.1 = y$

$514.52 = y$

$\tan 9.9 = \frac{x}{2400}$

$2400 \tan 9.9 = x$

$416.87 = x$

95.65

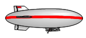
95.7m antenna

REVIEW of ...

Applications of Trig Ratios

Examples...

#1. The Goodyear Blimp is 565 m above the ground during a Super Bowl game. The angle of depression of the north goal line from the blimp is 58.5°. How far is the observer in the blimp from the goal line?

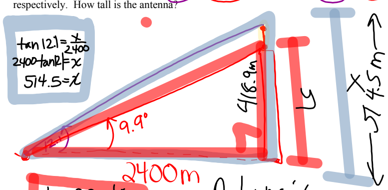


#2. An antenna is on the top of the CN Tower in Toronto. From a point 2400 m away, the angles of elevation to the top and bottom of the antenna are 12.1° and 9.9° respectively. How tall is the antenna?

$\tan 12.1 = \frac{x}{2400}$

$2400 \tan 12.1 = x$

$514.5 = x$



$\tan 9.9 = \frac{y}{2400}$

$418.9 = y$

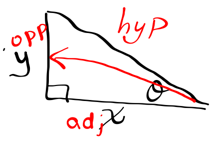
Antenna is $514.5 - 418.9\text{m} = 95.6\text{m}$

HOMEWORK...

(Solving Right Triangles / Word Problems)

Worksheet - Applications of Primary Trig.doc

2.

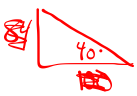


slope = 0.84

$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = 0.84 = \frac{\text{opp}}{\text{adj}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\tan\theta = 0.84$



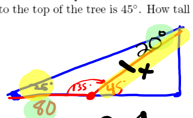
$\theta = \tan^{-1}(0.84)$

$\theta = 40^\circ$

Warm Up

Law of Sines

In order to measure the height of a tree, two measurements are made. At one spot, the angle from the horizontal to the top of the tree is 25°. 80 feet closer to the tree, the angle from the horizontal to the top of the tree is 45°. How tall is the tree (to the nearest foot)?



$\frac{x}{\sin 25} = \frac{80}{\sin 20}$

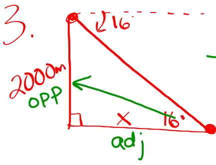
$x = \frac{80}{\sin 20} \cdot \sin 25$

$x = 98.9$

$\sin 45 = \frac{y}{98.9}$

To find the length AB of a small lake, a surveyor at point C measures angle ACB to be 115°, length AC to be 500 feet, and length BC to be 325 feet. What is the length of the lake (to the nearest foot)? Circle your answer.

3.

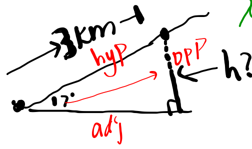


$\tan 16^\circ = \frac{2000}{x}$

$x = \frac{2000}{\tan 16}$

$x = 6974.83$

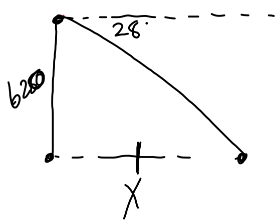
5.



$\sin 17^\circ = \frac{h}{3}$

$0.87 = h$

6.

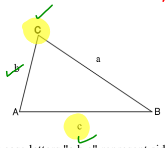


Law of Sines

** Used when the triangle does not contain 90° angle (Oblique Triangle)

** In order to use you must be given 1) an angle and an opposite side AND 2) any other side or angle

"PAIR"



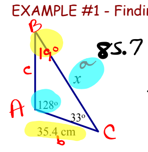
Lower case letters "a,b,c" represent side lengths
Upper case letters "A,B,C" represent angle measures

Law of Sines

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \qquad \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

"when looking for a side" "when looking for an angle"

EXAMPLE #1 - Finding a side.



$B+128^\circ+33=180^\circ$

$B=19^\circ$

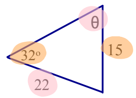
$$\frac{a}{\sin A}=\frac{b}{\sin B}$$

$$\frac{x}{\sin 128}=\frac{35.4}{\sin 19}$$

$$x=\frac{35.4}{\sin 19}\times \sin 128$$

$\boxed{x=85.7\text{cm}}$

EXAMPLE #2 - Finding an angle.



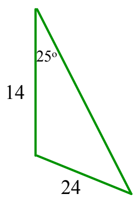
$$\frac{\sin\theta}{22}=\frac{\sin 32}{15}$$

$$\sin\theta=\frac{\sin 32}{15}\times 22$$

$$\theta=\sin^{-1}\left(\frac{\sin 32}{15}\times 22\right)$$

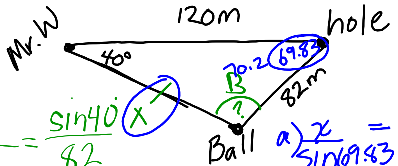
$\theta=51^\circ$

EXAMPLE #3 - Solve the triangle.



EXAMPLE #4 - Application

Suppose that Mr. Watters was playing a straight par-3 golf hole that was 120 m long. He hits one of his regular old slices that ends up 40° off line and is still 82 m from the hole.
(a) How far did his tee shot travel?
(b) If he somehow miraculously hits his next shot onto the green, what was the angle between the path of his first shot and the path followed by the second shot?



$$\frac{\sin B}{120}=\frac{\sin 40}{82}$$

$$\sin B=\frac{\sin 40}{82}\times 120$$

$\sin B=0.94$

b) $B=\sin^{-1}(0.94)$ $=70.2^\circ$ 70

a) $\frac{x}{\sin 69.83}=\frac{82}{\sin 40}$

$x=119.75$

$\boxed{x=119.8}$

Homework...

Worksheet - Law of Sines.doc

Left Side... #1 - 6

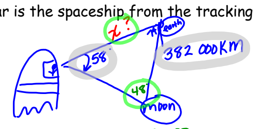
Right Side... #1 - 4

WARM-UP...

Ask yourself...
1. What am I given?
2. What am I trying to find?

EXAMPLE...

On a space flight, astronaut Neil Armstrong reports that the angle formed by his lines of sight to the earth and to the moon was 58°. At the same time, the observer on the earth reports that the angle formed by her lines of sight to the spaceship and to the moon is 74°. If the moon is 382 000 km from the earth, how far is the spaceship from the tracking station?

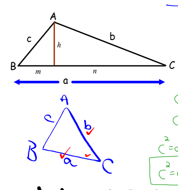


$$\frac{x}{\sin 48}=\frac{382\,000}{\sin 58}$$

$x=334746.7\text{ km}$

Law of Cosines

Derivation of the law of cosines...



Don't Copy

$c^2=h^2+m^2$ $m=a-n$

$c^2=h^2+(a-n)^2$

$c^2=h^2+a^2-2an+n^2$

$c^2=\boxed{h^2+n^2}+a^2-2an$ $h^2+n^2=b^2$

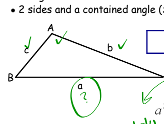
$c^2=b^2+a^2-2an$ $\cos C=\frac{n}{b}$, $n=b\cos C$

$c^2=a^2+b^2-2a(b\cos C)$

$\boxed{c^2=a^2+b^2-2ab\cos C}$

LAW OF COSINES

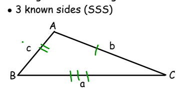
Finding an unknown side...
• 2 sides and a contained angle (SAS)



$$a^2=b^2+c^2-2bc\cos A$$

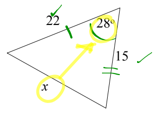
$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

Finding an unknown angle...
• 3 known sides (SSS)



$$\boxed{\cos A=\frac{b^2+c^2-a^2}{2bc}}$$

EXAMPLE: Finding an unknown side.



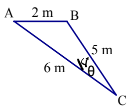
$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

$x^2 = 22^2 + 15^2 - 2(22)(15)\cos 28°$

$x^2 = 126.25$

$x = 11.2$

EXAMPLE: Finding an unknown angle.



$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\theta = \frac{5^2 + 6^2 - 2^2}{2(5)(6)}$

$\cos\theta = 0.95$

$\theta = \cos^{-1}(0.95)$

$\theta = 18.2$

$\boxed{18°}$

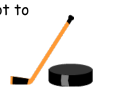
Application Questions - Law of Cosines

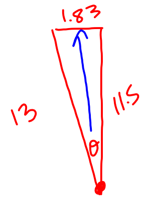
Ask yourself...

1. What am I given?
2. What am I trying to find?

EXAMPLE...

A hockey net is 1.83m wide. A player shoots from a point where the puck is 13m from one goal post and 11.5m from the other. Within what angle must he make his shot to score?





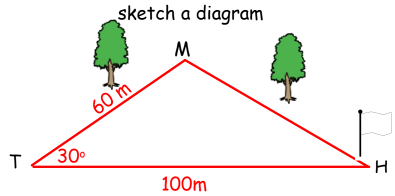
$\cos\theta = \frac{13^2 + 11.5^2 - 1.83^2}{2(13)(11.5)}$

$\theta = 5°$

Example #2:

From T, a golfer aims a ball towards the hole at H which is 100m away. But the ball actually sliced in a direction 30° off course and lands at M, 60m away. If the next shot is hit 50 m towards the hole, will the ball go in the hole?

sketch a diagram



Homework...

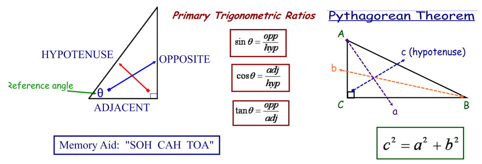
Worksheet - Law of Cosines.doc QUESTIONS???

10.11 # 1a, 2a, 7abc

10.12 # 3

REVIEW - What formula do I use? Ask yourself...

- Is it a right triangle? If Yes, then...



- If you are finding a side, do you have **SAS**? If Yes, then...

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

- If you are finding an angle, do you have **SSS**? If Yes, then...

Law of Cosines (rearranged)

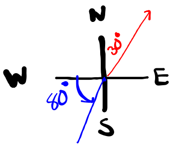
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

- Anything else...use your Law of Sines! Pair

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ "when looking for a side"

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ "when looking for an angle"

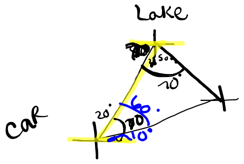
Bearings



N 30°E
=
E 60°N

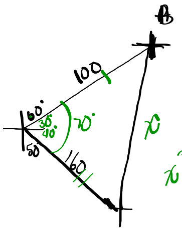
W 80°S = S 10°W

due east →



10.12 → #8-12
10.13 → #1-5

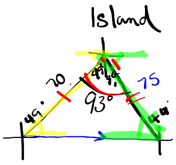
10.12 8.

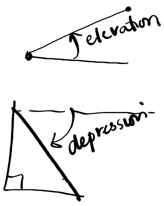


$x^2 = 100^2 + 160^2 - 2(100)(160)\cos 70°$

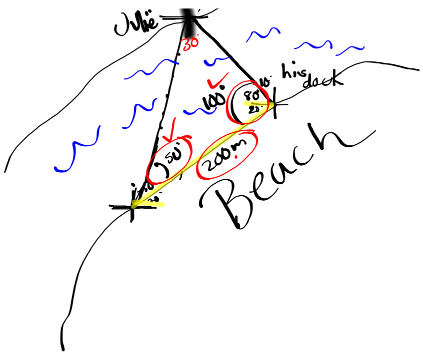
$x^2 =$
$x =$

12.

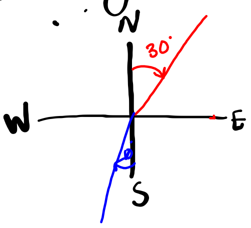




#1

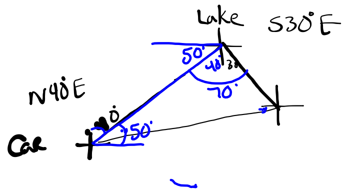


Bearings

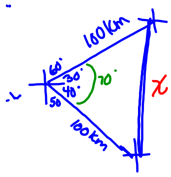


N30°E
E60°N

S10°W
W80°S



#8.



SAS.
Law of Cosines

$x^2 = 100^2 + 100^2 - 2(100)(100)\cos 70°$

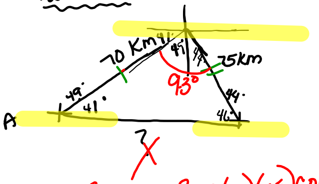
$x^2 = 13159.59$

$\boxed{x = 114.7\text{ Km}}$

10.12 # 8-12

10.13 harder!! #1-5

12.



$x^2 = 70^2 + 75^2 - 2(70)(75)\cos 93°$

$x =$

EXTRA PRACTICE TIME...Finish for HW!!!

1) Review - Primary Trig Ratios_Law of Sines_Cosines.pdf

Corrections to solutions: #1c) 19.1 #1d) 20.63 scratch (e) & (f)
#4. 877.3 km

2) Puzzle Review - Primary Trig, Law of Sines_Cosines.pdf

$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$

$c^2 = a^2 + b^2$

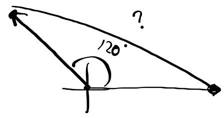
$a^2 = b^2 + c^2 - 2bc\cos A$

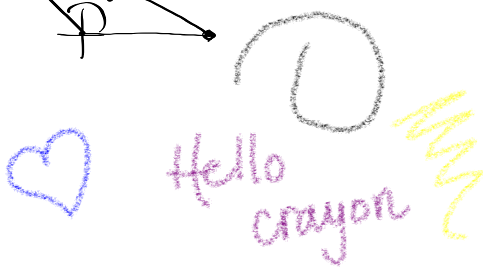
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

9.





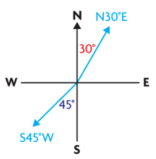
Hello crayon

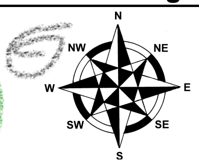
MORE APPLICATIONS... Bearings

NOTE:

Communication | Tip

Directions are often stated in terms of north and south on a compass. For example, N30°E means travelling in a direction 30° east of north. S45°W means travelling in a direction 45° west of south.





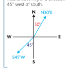
crayon

Applications: Bearings

NOTE:

Communication Tip

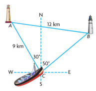
Directions are often stated in terms of north and south on a compass. For example, N30°E means travelling in a direction 30° east of north. S45°W means travelling in a direction 45° west of south.

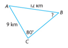


Compass Rose Animation

Ex #1:(p. 122) Using reasoning to determine the measure of an angle

The captain of a small boat is delivering supplies to two lighthouses, as shown. His compass indicates that the lighthouse to his left is located at N30°W and the lighthouse to his right is located at N50°E. Determine the compass direction he must follow when he leaves lighthouse *B* for lighthouse *A*.





I drew a diagram. I labelled the sides of the triangle I knew and the angle I wanted to determine.

$$\frac{\sin B}{AC} = \frac{\sin C}{AB}$$

I knew *AC*, *AB*, and ∠*C*, and I wanted to determine ∠*B*. So I used the sine law that includes these four quantities.

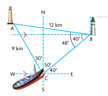
I used the proportion with sin *B* and sin *C* in the numerators so the unknown would be in the numerator.

$$\frac{\sin B}{9} = \frac{\sin 80^\circ}{12}$$
$$9\left(\frac{\sin B}{9}\right) = 9\left(\frac{\sin 80^\circ}{12}\right)$$
$$\sin B = 9\left(\frac{\sin 80^\circ}{12}\right)$$
$$\sin B = 0.7386...$$
$$\angle B = \sin^{-1}(0.7386...)$$
$$\angle B = 47.612...^\circ$$

I substituted the given information and then solved for sin *B*.

The measure of ∠*B* is 48°.

The answer seems reasonable. ∠*B* must be less than 80°, because 9 km is less than 12 km.



I drew a diagram and marked the angles I knew. I knew east-west lines are all parallel, so the alternate interior angle at *B* must be 40°.

The captain must head N82°W from lighthouse *B*.

The line segment from lighthouse *B* to lighthouse *A* makes an 8° angle with west-east. I subtracted this from 90° to determine the direction west of north.

EX #2: Solving an application question...

(p. 166)

Colleen and Juan observed a tethered balloon advertising the opening of a new fitness centre. They were 250 m apart, joined by a line that passed directly below the balloon, and were on the same side of the balloon. Juan observed the balloon at an angle of elevation of 7° while Colleen observed the balloon at an angle of elevation of 82°. Determine the height of the balloon to the nearest metre.

HOMEWORK...

*** Quiz on Monday

- Primary Trig Ratios & Pythagorean Theorem
- Law of Sines/Cosines
- Finding angles/sides/solving/word problems

MORE PRACTICE!!!

Puzzle Review - Primary Trig, Law of Sines_Cosines.pdf

Solutions to the puzzle... Puzzle Review Solutions.pdf

Logic - Figure Out The Digits.doc

DUE FIRST OF CLASS MONDAY

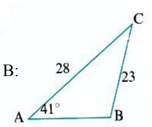
When your finished the quiz...

HOMEWORK: More Applications/Word Problems

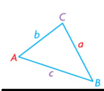
Page 154 #5, 6, 9, 10, 11 (bearings - see example from Friday)
Page 172 #9, 10, 12, 13, 14

Warm Up

Determine the measure of the obtuse angle B:



Trigonometry Summary AND 'The AMBIGUOUS Case'...



sine law
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine law
$$a^2 = b^2 + c^2 - 2bc \cos A$$

oblique triangle
A triangle that does not contain a 90° angle.

Need to Know

- The sine law and cosine law are used with obtuse triangles in the same way that they are used with acute triangles.

Use the sine law when you know ...	Use the cosine law when you know ...
- the lengths of two sides and the measure of the angle that is opposite a known side	- the lengths of two sides and the measure of the contained angle
- the measures of two angles and the length of any side	- the lengths of all three sides

Ambiguous Case

- Be careful when using the sine law to determine the measure of an angle. The inverse sine of a ratio always gives an acute angle, but the supplementary angle has the same ratio. You must decide whether the acute angle, θ, or the obtuse angle, $180^\circ - \theta$, is the correct angle for your triangle.

4.3 The Ambiguous Case of the Sine Law

GOAL

Analyze the ambiguous case of the sine law, and solve problems that involve the ambiguous case.

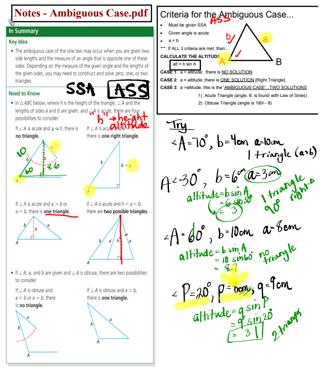
EXPLORE...

- Two sides in an obtuse triangle are 3 m and 4 m in length. The angle that is opposite the 3 m side measures 40°. Determine the measure of the angle that is opposite the 4 m side.

SAMPLE ANSWER

ambiguous case of the sine law

A situation in which two triangles can be drawn, given the available information; the ambiguous case may occur when the given measurements are the lengths of two sides and the measure of an angle that is not contained by the two sides (*SSA*).



Try

$\angle A = 70°$, $b = 4$cm $a = 10$cm

1 triangle ($a > b$)

$A < 30°$, $b = 6$cm $a = 3$cm

altitude $= b \sin A$
$= 6 \sin 30°$
$= 3$

1 triangle $90°$ right △

$\angle A = 60°$, $b = 10$cm $a = 8$cm

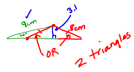
altitude $= b \sin A$
$= 10 \sin 60°$
$= 8.7$

no triangle

$\angle P = 20°$, $p = ~~6cm~~$, $q = 9$cm

altitude $= q \sin P$
$= 9 \sin 20°$
$\boxed{= 3.1}$

2 triangles



2 triangles

ex. $m = 4$cm $n = 3$cm $\angle M = 40°$

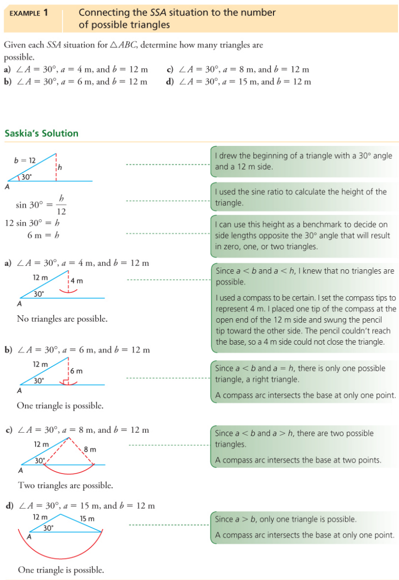
$m > n$
OR
$a > b$

1 triangle

ex. $b = 10$cm $c = 12$cm $\angle B = 30°$

altitude $= c \sin B$
$= 12 \sin 30°$
$= 12(0.5)$
$= 6$

2 triangles



EXAMPLE 1 Connecting the *SSA* situation to the number of possible triangles

Given each *SSA* situation for $\triangle ABC$, determine how many triangles are possible.

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m
b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m
c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m
d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

Saskia's Solution

$b = 12$, h, $30°$, A

I drew the beginning of a triangle with a 30° angle and a 12 m side.

$$\sin 30° = \frac{h}{12}$$
$$12 \sin 30° = h$$
$$6 \text{ m} = h$$

I used the sine ratio to calculate the height of the triangle.

I can use this height as a benchmark to decide on side lengths opposite the 30° angle that will result in zero, one, or two triangles.

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

12 m, 4 m, 30°, A

No triangles are possible.

Since $a < b$ and $a < h$, I knew that no triangles are possible.

I used a compass to be certain. I set the compass tips to represent 4 m. I placed one tip of the compass at the open end of the 12 m side and swung the pencil tip toward the other side. The pencil couldn't reach the base, so a 4 m side could not close the triangle.

b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

12 m, 6 m, 30°, A

One triangle is possible.

Since $a < b$ and $a = h$, there is only one possible triangle, a right triangle.

A compass arc intersects the base at only one point.

c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

12 m, 8 m, 30°, A

Two triangles are possible.

Since $a < b$ and $a > h$, there are two possible triangles.

A compass arc intersects the base at two points.

d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

12 m, 15 m, 30°, A

One triangle is possible.

Since $a > b$, only one triangle is possible.

A compass arc intersects the base at only one point.

MUST MEMORIZE THESE NOTES IN ORDER TO KNOW AMBIGUOUS CASE

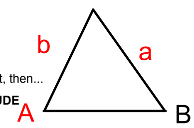
Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...

CALCULATE THE ALTITUDE

alt = b sin A



CASE 1: a < altitude; there is NO SOLUTION

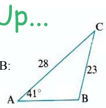
CASE 2: a = altitude; there is ONE SOLUTION [Right Triangle]

CASE 3: a >altitude; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is 180° - θ)

Back to the Warm-Up...

Determine the measure of the obtuse angle B:

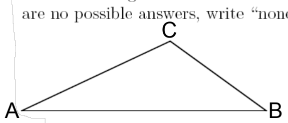


Example 2:

Solve the triangle ABC if $a = 10$, $b = 12$ and angle A = 72°.

Example 3:

Given that $A = 25°$, $a = 15$, and $b = 33$, find the measure of angle B to the nearest degree. If there are two answers, give both of them. If there are no possible answers, write "none".



HOMEWORK...

Worksheet - Ambiguous Case.pdf

Do questions #1, 2 & 4

MEMORIZE!!!

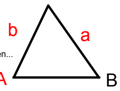
Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...

CALCULATE THE ALTITUDE

$\boxed{\text{alt} = b \sin A}$



CASE 1: a < altitude; there is NO SOLUTION

CASE 2: a = altitude; there is ONE SOLUTION [Right Triangle]

CASE 3: a >altitude; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is 180° - θ)

The Ambiguous Case of the Law of Sines

Ambiguous Case Slide Show.ppt

am·big·u·ous [am-**big**-yoo-*uh* s] Show IPA

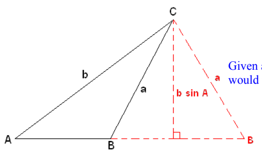
adjective

1. open to or having several possible meanings or interpretations; equivocal: *an ambiguous answer.*
2. *Linguistics* . (of an expression) exhibiting constructional homonymity; having two or more structural descriptions, as the sequence *Flying planes can be dangerous.*
3. of doubtful or uncertain nature; difficult to comprehend, distinguish, or classify: *a rock of ambiguous character.*
4. lacking clearness or definiteness; obscure; indistinct: *an ambiguous shape; an ambiguous future.*

REVIEW: Solving Oblique Triangles... The Ambiguous Case

The ambiguous case

When using the law of sines to solve triangles, under special conditions there exists an ambiguous ca
where two separate triangles can be constructed (i.e., there are two different possible solutions to th
triangle).



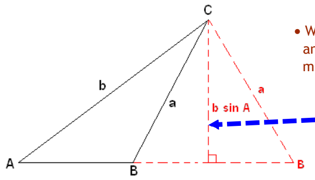
Given a general triangle *ABC*, the following conditions would need to be fulfilled for the case to be ambiguous:

- The only information known about the triangle is the angle A and the sides a and b, where the angle A is not the included angle of the two sides.
- The angle A is acute (i.e., A < 90°).
- The side a is shorter than the side b (i.e., $a < b$).
  (a is the altitude of a right triangle with angle A)
- The angle B is not a right angle (i.e., $a > b \sin A$).

Given all of the above premises are true, the angle B may be acute or obtuse; meaning, one of the following is true:

$$B = \sin^{-1}\left[\frac{b \sin A}{a}\right] \quad \text{OR} \quad B = 180^\circ - \sin^{-1}\left[\frac{b \sin A}{a}\right]$$

Summary: Ambiguous Case



- When given two sides and a non-contained angle that is acute the ambiguous case must be examined

The altitude of the triangle will dictate how many solutions are possible.

$a < b \bullet \sin A$	No Solutions
$b \bullet \sin A < a < b$	Two Solutions
$b < a$	One Solution

Example 4: Solving a problem using the sine law

Martina and Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36.0° with the ground. Carl's rope is 5.9 m long. Assuming that Martina and Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina and Carl, to the nearest tenth of a metre?

AMBIGUOUS???

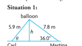
Example 4: Solving a problem using the sine law

Martina and Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36.0° with the ground. Carl's rope is 5.9 m long. Assuming that Martina and Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina and Carl, to the nearest tenth of a metre?

Sandra's Solution: Using the sine law and then the cosine law

Let h represent the height of the weather balloon.
Let θ represent the angle for Carl's rope.

Situation 1:

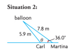


$$\sin 36.0 = \frac{h}{7.8}$$
$$7.8(\sin 36.0) = 7.8\left(\frac{h}{7.8}\right)$$
$$4.5847... = h$$

I drew the triangle.

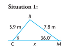
I noticed that this is a *SSA* situation. I had to determine the height of the triangle to determine if this is an ambiguous case.

Situation 2:



Carl's rope is longer than the height and shorter than Martina's rope, so there are two possible triangles. I drew the second triangle.

Situation 1:



$$\frac{\sin\theta}{7.8} = \frac{\sin 36^\circ}{5.9}$$
$$\sin\theta = \frac{7.8 \sin 36^\circ}{5.9}$$
$$\sin\theta = 0.7770...$$
$$\theta = \sin^{-1}(0.7770...)$$
$$\theta = 50.9932...^\circ$$

I substituted the side lengths and angles (including θ) into the formula for the sine law and isolated θ.

$$\angle B = 180^\circ - 36.0^\circ - 50.9932...^\circ$$
$$\angle B = 93.0067...^\circ$$

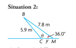
The measures of the angles in a triangle sum to 180°.

$$x^2 = 5.9^2 + 7.8^2 - 2(5.9)(7.8)\cos 93.0067...^\circ$$
$$x^2 = 100.4777...$$
$$x = 10.0238...$$

I used the cosine law to determine the distance, x, between Martina and Carl. I substituted the known measurements into the cosine law.

In Situation 1, Martina and Carl are 10.0 m apart.

Situation 2:



I also considered the situation in which Carl is closer to Martina.

$$\frac{\sin\theta}{7.8} = \frac{\sin 36^\circ}{5.9}$$
$$\sin\theta = \frac{7.8 \sin 36^\circ}{5.9}$$
$$\sin\theta = 0.7770...$$
$$\theta = \sin^{-1}(0.7770...)$$
$$\theta = 50.9932...^\circ$$
$$\theta = 180^\circ - 50.9932...^\circ$$
$$\theta = 129.0067...^\circ$$

I used the sine law to determine θ.

I determined the measure of the supplementary angle, which is suitable for this situation.

$$\angle B = 180^\circ - 36.0^\circ - 129.0067...^\circ$$
$$\angle B = 14.9932...^\circ$$

The measures of the angles in a triangle sum to 180°.

$$y^2 = 5.9^2 + 7.8^2 - 2(5.9)(7.8)\cos 14.9932...^\circ$$
$$y^2 = 6.7433...$$
$$y = 2.5968...$$

I can use $\angle B$ in the cosine law to determine the distance, y, between Martina and Carl.

I substituted the measure of $\angle B$ and the given side lengths into the cosine law.

In the second situation, Martina and Carl are 2.6 m apart.
Martina and Carl are either 10.0 m apart or 2.6 m apart.

HOMEWORK...

Worksheet - Ambiguous Case.pdf

#5, 6, & 7

Page 184: #7, 8, 11

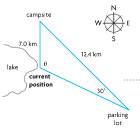
Warm Up

Given ΔRST has angle R = 58°, r = 48 and s = 25.
Solve the triangle, if there is more than one possible, solve both!!

EXAMPLE #5: Reasoning about ambiguity

(p. 180)
Leanne and Kerry are hiking in the mountains. They left Leanne's car in the parking lot and walked northwest for 12.4 km to a campsite. Then they turned due south and walked another 7.0 km to a glacier lake. The weather was taking a turn for the worse, so they decided to plot a course directly back to the parking lot. Kerry remembered, from the map in the parking lot, that the angle between the path to the campsite and the path to the glacier lake measures about 30°. What compass direction should they follow to return directly to the parking lot?

Austin's Solution



Since I am given specific directions, I know exactly how to draw a sketch of the situation. There is only one way to draw the sketch, so this is not ambiguous.

Leanne and Kerry left the parking lot and walked northwest and then south.

Because the campsite is due north of the lake, I knew that the angle at the lake vertex of the triangle, θ, would help me determine the compass direction that Leanne and Kerry need to travel.

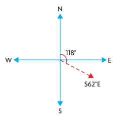
My diagram shows that Leanne and Kerry need to travel approximately southeast.

$$\frac{\sin\theta}{12.4} = \frac{\sin 30°}{7.0}$$
$$12.4\left(\frac{\sin\theta}{12.4}\right) = 12.4\left(\frac{\sin 30°}{7.0}\right)$$
$$\sin\theta = 0.8857\ldots$$
$$\theta = \sin^{-1}(0.8857)$$
$$\theta = 62.3395\ldots°$$

I noticed two side-angle pairs, so I substituted the values into the sine law and solved for θ.

The angle seemed too small, according to my diagram. To correct the angle measure, I needed the supplementary angle.

Correction: AMBIGUOUS!!!
$\theta = 180° - 62.3395\ldots°$
$\theta = 117.6604\ldots°$



$180° - 117.6604\ldots° = 62.3395\ldots°$
Leanne and Kerry would need to travel in the direction S62°E to reach the parking lot.

I subtracted the measure of the angle in my triangle from 180° to determine the direction of travel.

Warm up Nov 28 Name ______

Gail works as an aerial photographer. On one trip she takes off from the airport and flies for 52 km on a bearing of N50°E. Then she turns and flies southwest for 38 km, until she is due east of the airport.

a) Sketch a diagram of Gail's flight.

b) Explain how this situation could be ambiguous, and why it is not.

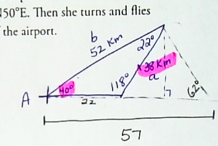
c) How far is Gail from the airport when she is due east of it?

 4.3 The Ambiguous Case of the Sine Law 97

Warm up Nov 28 Name: Answer

11. Gail works as an aerial photographer. On one trip she takes off from the airport and flies for 52 km on a bearing of N50°E. Then she turns and flies southwest for 38 km, until she is due east of the airport.

a) Sketch a diagram of Gail's flight.



b) Explain how this situation could be ambiguous, and why it is not.

$a < b$
$a >$ altitude
two possible situations
but told South West

altitude $= b\sin A$
$= 52\sin 40°$
$= 33.42$

c) How far is Gail from the airport when she is due east of it?

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{52} = \frac{\sin 40}{38}$$
$B = 61.6°$
$62°$

but need supplementary angle $180° - 62° = \boxed{118°}$

$$\frac{x}{\sin 22} = \frac{52}{\sin 118}$$
$\boxed{x = 22\text{ km From Airport}}$

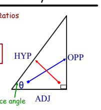
REVIEW - Trigonometry

- Pythagorean Theorem & Primary Trig Ratios

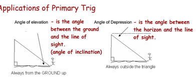
REMEMBER: "SOH CAH TOA"

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

$c^2 = a^2 + b^2$



- Applications of Primary Trig



Also, note that the angle of elevation = angle of depression

- Law of Sines & Its Applications

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ "when looking for a side"

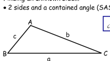
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ "when looking for an angle"

- Ambiguous Case??? (Law of Sines - finding an angle)
  - given a side (a), the angle opposite (A) and another side (b)...

CASE #1: a > b -> only one solution
CASE #2: a = b -> only one solution
CASE #3: a < b ... Determine the altitude length (bsinA)
(i) a < altitude -> no solution
(ii) a = altitude -> one solution (right triangle)
(iii) a > altitude -> two solutions... (Ambiguous Case)
(1) acute angle
(2) obtuse angle (180° - acute)

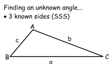
- Law of Cosines & Its Applications

Finding an unknown side...
• 2 sides and a contained angle (SAS)



$a^2 = b^2 + c^2 - 2bc\cos A$

Finding an unknown angle...
• 3 known sides (SSS)



$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

- Bearings and Multi-step Word Problems
- "Solving" - find ALL angles & sides

Review for Test - Lots of Practice from the Textbook!!!

Chapter Review...
(Frequently Asked Questions)

Thurs Test!!

- Page 128
- Page 153 } Chp 3
- Page 174
- Page 199 } Chp. 4

Practice Questions...

* Ambiguous Case → 4.3

Bearing #11, 12

- Page 129 #1 - 9
- Page 154 #1 - 12 } Chp 3
- Page 175 #1 - 9
- Page 200 #1 - 8 } Chp 4

Practice Tests...

Bearing #8

- Page 152 #1 - 8 Chp. 3
- Page 198 #1 - 7 Chp. 4

REVIEW TIME!!!

Review - Trigonometry.doc

Attachments

Worksheet - Primary Trig Ratios.doc

Worksheet - Applications of Primary Trig.doc

Worksheet - Law of Cosines.doc

Review - Primary Trig_Law of Sines_Cosines.doc

Worksheet - Area of a Triangle_Review Trig.doc

Review - Triggonometry.doc

Worksheet - Law of Sines.doc

Puzzle Review - Primary Trig, Law of Sines_Cosines.pdf

Review - Primary Trig Ratios_Law of Sines_Cosines.pdf

Logic - Figure Out The Digits.doc

Puzzle Review Solutions.pdf

Worksheet - Ambiguous Case.pdf

Notes - Ambiguous Case.pdf

Ambiguous Case Slide Show.ppt