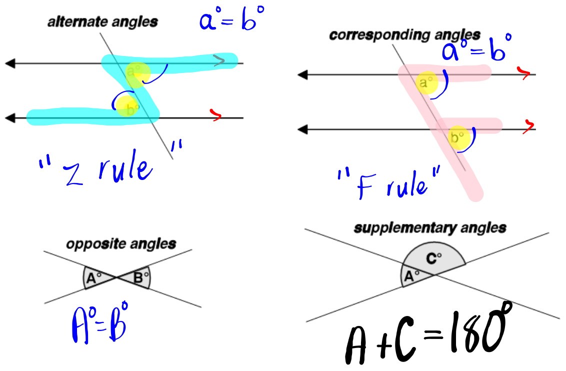
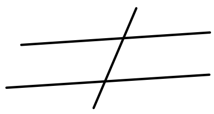
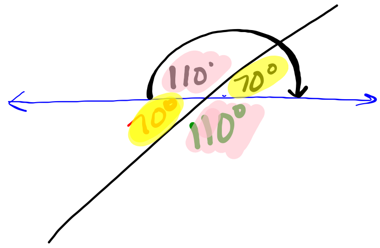
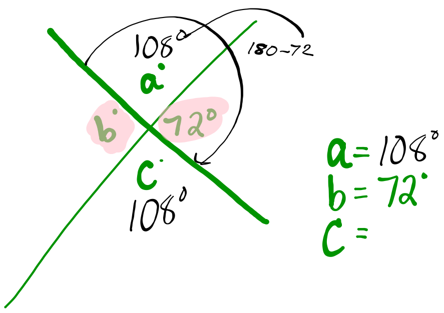
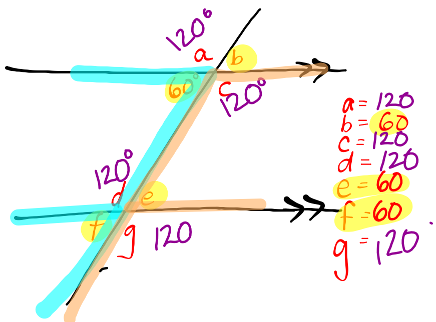
Chapter 7 - Angles and Parallel Lines

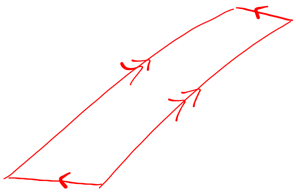


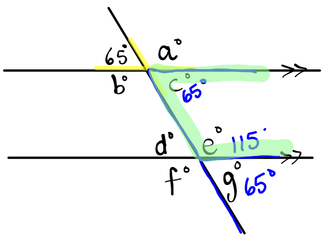




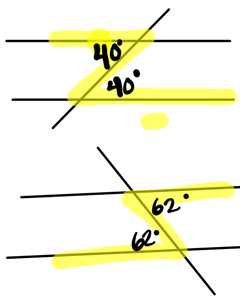


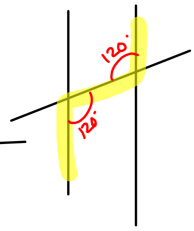


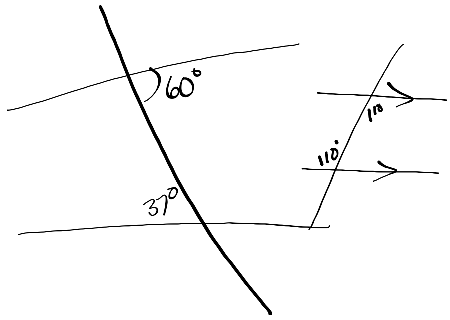




$a^\circ = 115^\circ$
$b^\circ = 115^\circ$
$c^\circ = 65^\circ$
$d^\circ = 65^\circ$
$e^\circ = 115^\circ$
$f^\circ = 115^\circ$
$g^\circ = 65^\circ$







Let's talk angles...

Take a moment to look at the structures in your classroom that contain **angles**. Consider who would have been involved in creating the structures that have those angles, for example, architects, designers, surveyors, and carpenters. Angles are also useful to people who do not make structures. Aircraft pilots and boat pilots use angles for navigation. Astronomers use angles to locate objects in the sky.

vertex

So, what exactly is an angle? An angle is formed when two rays meet at a common endpoint called a vertex. Angles are measured with tools, such as a protractor, that are marked in degrees.

N W E S

Visualize an angle that is used to express direction in navigation and mapping, such as east. In this case, the angle is measured relative to true north, which is 0° and may be expressed as a bearing. A **true bearing** describes the number of degrees, measured clockwise, between an imaginary line pointing towards true north (geographic north) and another imaginary line pointing towards an intended direction or along a pathway. East is represented in land navigation and mapping at a 90° angle from true north.

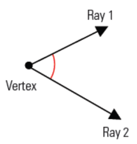
Angle measures can be estimated by using **referents**, which are common measurements like 90°, 45°, 30°, 60°, and 22.5°.

How can we draw angles? The tools used to measure angles can also be used to draw or replicate angles having specific measures. Tools have been designed to measure and create angles having only one or two specific measures, such as a set square used in technical drawings to draw right angles.

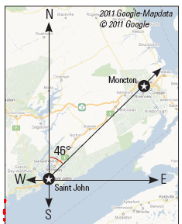
You have used a protractor and ruler to draw angles. You can also draw certain angles with a ruler and compass, and you can replicate any angle with these tools.

Key Terms...

angle: two rays that meet at a point called the vertex

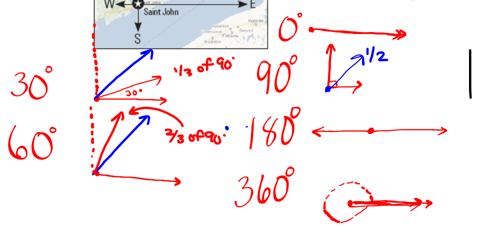


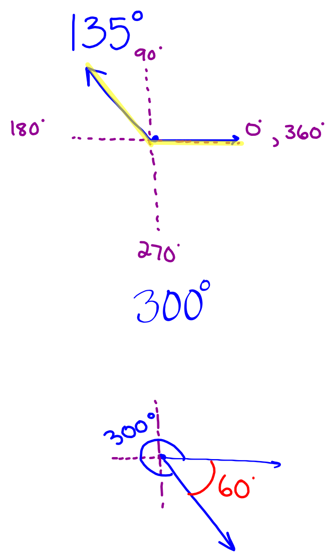
true bearing: the angle measured clockwise between true north and an intended path or direction, expressed in degrees



angle measure: a number representing the spread of the two rays of an angle, expressed in degrees

angle referent: a common standard of angle measure, for example, 0°, 45°, 90°, 180°, and 360°; they are used to estimate angles

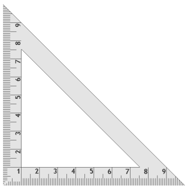




Geometry Set... Bring tomorrow!

Protractor

Right Triangle



Compass

Ruler

Some More Key Terms...

Acute angle - measure is between 0° and 90°

Right angle - measure is 90°; the two rays are perpendicular to

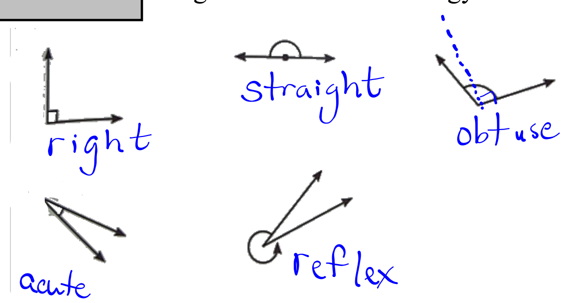
Obtuse angle - measure is between 90° and 180°

Straight angle - measure is 180°

Reflex angle - measure is between 180° and 360°

FIVE TYPES OF ANGLES	
Definition of angle	*Kind of angle*
greater than 0° but less than 90°	acute
90°	right
greater than 90° but less than 180°	obtuse
180° (two rays share a vertex and point in opposite directions)	straight
greater than 180° but less than 360°	reflex

EXERCISE: Identify each of the following angles using the correct terminology...



More Key Terms...

complementary angles: two angles that have measures that add up to 90°

supplementary angles: two angles that have measures that add up to 180°

Sort the following angles into pairs of **complementary** and **supplementary** angles.

∠A = 42°	∠E = 121°
∠B = 107°	∠F = 31°
∠C = 59°	∠G = 19°
∠D = 48°	∠H = 73°

complementary

$\angle A + \angle D = 90°$

$\angle C + \angle F = 90°$

supplementary

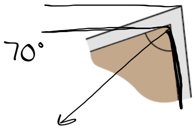
$\angle C + \angle E = 180°$

$\angle B + \angle H = 180°$

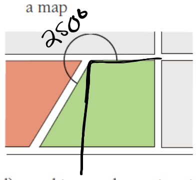
Estimating Angles using Referent Angles

Estimations are made in many trades that use angles. Imagine that you are working as a tradesperson in the situations below and make the following estimations (aim to be within 5°).

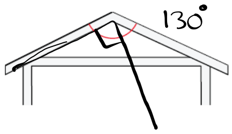
a) a landscaper estimating the angle of the corner of a garden bed



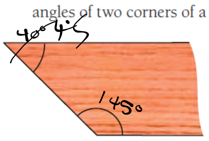
b) a surveyor estimating the angle of a property boundary line on a map



c) a roofer estimating the angle of the peak of a roof



d) a cabinet-maker estimating the angles of two corners of a shelf



SOLUTION

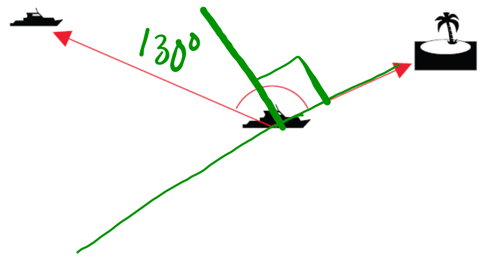
a) Students will be able to recognize a 90° angle without measuring, so they should be able to look at this angle and estimate it to be slightly less than 90°. They might also compare it to a real-life example of 90°, such as a corner. They will probably rotate the page on which the diagram appears, to position the angle so that one of the rays is horizontal, which will show clearly that the angle is an acute angle, less than 90°. A close estimate would be within 76° to 80°.

c) Students will see that this is greater than 90° but not quite wide enough to add another 45°. The will be able to suggest a close estimate of around 130°.

b) Students will be able to see instantly that this angle is more than 180°, but not quite big enough to add another 45°. So they may describe it as "a straight angle (180°) plus about a bit less than one-half of a right angle (45°)", and come up with a close estimate of 219° to 223°.

d) The acute angle is the top left corner can be seen to be a 45° angle by using a referent. The obtuse angle in the lower left corner can be seen to be a right angle plus a little more than a 45° angle so a close estimate will be about 145°.

Estimate the measure of this angle without using a measuring device.



7.1 - Measuring, Drawing and Estimating Angles

MATH *ON THE JOB*

Sue Rendell's job takes her from the digital world of graphic design and social media to granite cliffs inhabited by caribou, mink, and red fox. Sue and her partner Bob Hicks own and operate Gros Morne Adventures. Their guiding company takes customers on kayaking, hiking, snowshoeing, and ski touring trips through Gros Morne National Park. When guiding guests "we work with maps and a compass on some of our outings, which involves angles, bearings, and declination," says Sue.

Sue was born in Gander, NL, but grew up in Goose Bay, Labrador. Her ancestors arrived in Newfoundland from England and Ireland in the late 1700s and mid 1800s.

Susan went to high school at St. Paul's High School in Gander, NL.

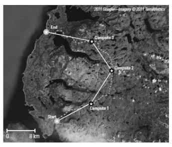
When not in the park, Sue markets her business through social media, photography, presentations, and print ads that she designs. This involves calculating dimension when she scales photos to include in advertisements. Her business also includes a café, so Sue must calculate food and labour costs and menu pricing. She also estimates staffing costs.

Sue guides hikes through Gros Morne National Park, NL, which is a United Nations Educational, Scientific and Cultural Organization (UNESCO) World Heritage Site. While they hike, her guests enjoy the park's mountains, fjords, waterfalls, and beaches.

Sue is planning to take guests on a challenging four-day hike along the Long Range Traverse route. Over the course of the hike, her guests will spend three nights at rough campsites along the route.

1. As a safety precaution, all hikers going into the backcountry need to know how to use a compass and map. Compasses are divided into 360°, as shown. Using the map provided, give the direction in degrees the hikers will need to take:
   a) From the start point to the first campsite.
   b) From the first campsite to the second campsite.
   c) From the second campsite to the third campsite.
   d) From the third campsite to the end point.
2. Using the scale provided, estimate the approximate length of the hike if hikers followed the route shown here.

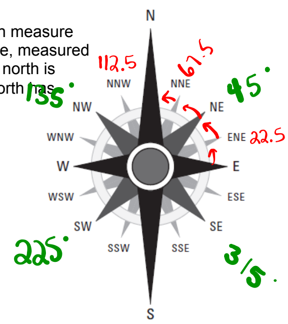
SOLUTION



1. Using Blackline Master 7.1 (p. 458), have students draw line segments to show what the directions the hikers will roughly follow between destinations and then use a protractor to calculate the number of degrees between north and the line the hikers will be travelling along. Suggest to students that they can more easily measure the degrees by extending the lines beyond the destination points to that they line up with the markings on their protractor.
   a) 65°
   b) 33°
   c) 325°
   d) 282°
2. As students are being asked to estimate the length of the hike, the answers here will vary. Students should measure the route with a ruler then convert the measured value using the scale provided. Answers should range between 40 and 50 km.

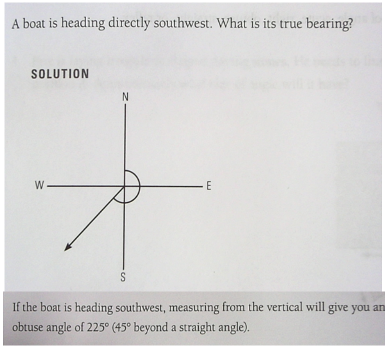
Working with True Bearing

In navigation and map-making, people often measure angles from the vertical, or north. The angle, measured in a clockwise direction from a line pointing north is referred to as the ***true bearing.*** Straight north has a bearing of 0°



NAVIGATIONAL BEARING	
Direction	*Bearing*
N	0°
NNE	22.5°
NE	45°
ENE	67.5°
E	90°
ESE	112.5°
SE	135°
SSE	157.50°
S	180°
SSW	202.5°
SW	225°
WSW	247.5°
W	270°
WNW	292.5°
NW	315°
NNW	337.5°

EXAMPLE...


A boat is heading directly southwest. What is its true bearing?

SOLUTION

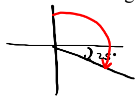
If the boat is heading southwest, measuring from the vertical will give you an obtuse angle of 225° (45° beyond a straight angle).

EXERCISE:

1) If a boat is travelling 25° south of straight east, what is its true bearing?

(Solution - 115°)

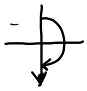
???



???

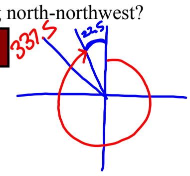
2) What is the true bearing of a boat travelling south?

(Solution - 180°)



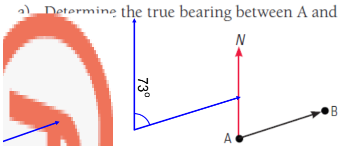
3) What is the true bearing of a boat travelling north-northwest?

???

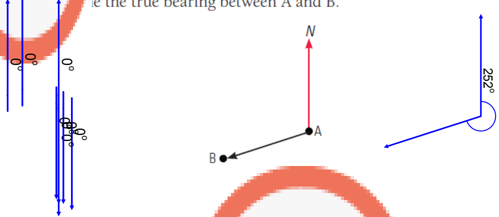


Examples...

a) Determine the true bearing between A and B.



e the true bearing between A and B.

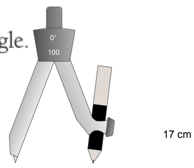


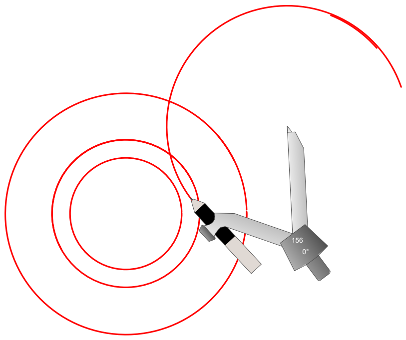
Example 1

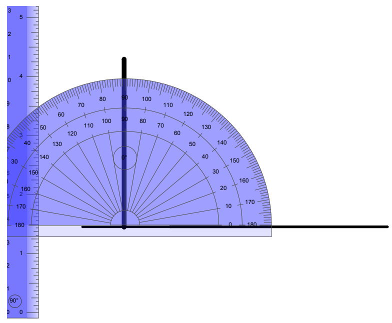
Use a ruler and compass to create the following angles.

a) Draw a 90° angle.

b) Replicate any existing angle.



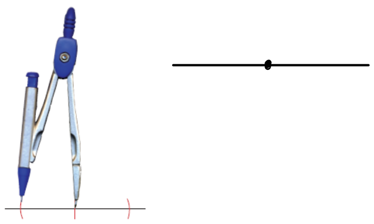


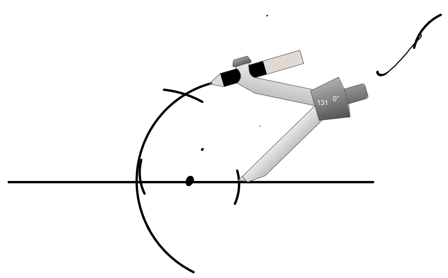


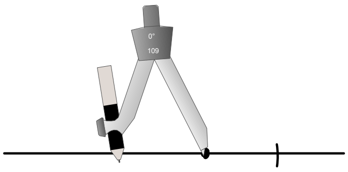
Angle Constructions... 1) Perpendicular Bisector

STEP 1: $\perp$

Put the compass point at the mark you made. Open the compass slightly and make two more marks on each side of the first mark. Ensure they are the same distance from the first mark.

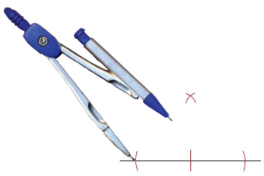






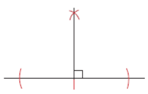
STEP 2:

Widen the compass a bit more, and place the compass point at one of the new marks. Make a small arc, then do the same thing after placing the compass point at the other new mark. Ensure the two arcs intersect each other.



STEP 3:

Draw a line segment that goes between or through the point where the arcs intersect and the first mark you made. The two line segments are perpendicular to each other, and therefore form a 90° (right) angle.

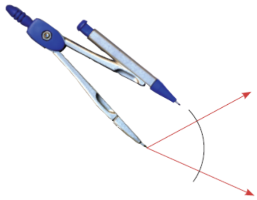


Angle Constructions... 2) Replicate an Angle

STEP 1:

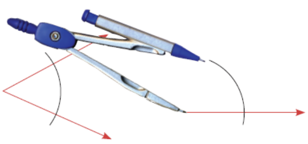
b) To replicate any existing angle, follow these steps.

Use a compass to lightly draw an arc centred at the vertex of the original angle.



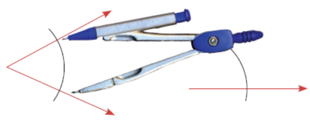
STEP 2:

Use a ruler to draw one side of the new angle, and draw an arc of the same radius and arc length as the one you just drew on the original angle.



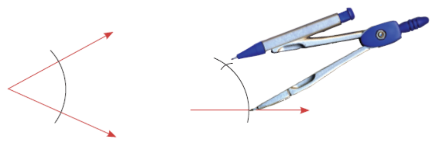
STEP 3:

Bring the compass up to the original angle, and set it so that its point and the tip of the pencil touch the points where the original arc intersects the sides of the angle.



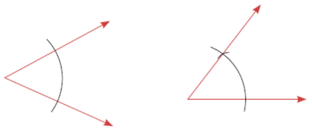
STEP 4:

Place the compass point over to the point of intersection of the side of the new angle and the new arc. Draw a short arc through the new arc.

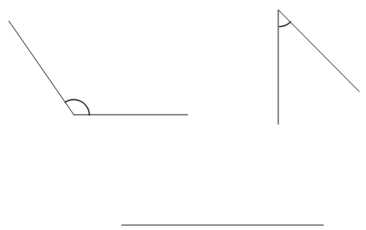


STEP 5:

Use the ruler to draw the other side of the angle, from the left end of the first side (the vertex) through the point of intersection of the two arcs. The result is a new angle with the same measure as the original.



Let's try a few...



HOMEWORK...

p. 284 #1 - 7 (omit #6)

7.1 - Build Your Skills Detailed Solutions.pdf

7.2 - Angle Bisectors and Parallel Lines

Students at the Hungry Heart Café learn how to work in a restaurant kitchen from chef Kathy Jaeger.

MATH *ON THE JOB*

The Hungry Heart Café is a unique restaurant in St. John's, Newfoundland. Founded by Stelle Burry Community Services, the Hungry Heart Café is both a restaurant and a job-training program for people who have experienced significant personal troubles including abuse, addictions, violence, and incarceration.

Chef Kathy Jaeger was instrumental in creating the Hungry Heart Café program. As a teacher, she gives her students classroom work, on the job training, and life skills development. "My main duties include instructing students in all aspects of introductory restaurant cooking and catering in hopes that it will improve their prospects for employment. I am also responsible for overall food production of the Café and staff supervision."

Originally from Ontario, but with family ties to the Maritimes, Kathy received her culinary training at George Brown College in Toronto, has her national certification in Food Safety Training from the Canadian Restaurant and Food Service Association, and her Red Seal certification. She uses math to calculate food, beverage, and labour costs; to measure ingredients using volume and weight measures; to order supplies and calculate menu pricing; to cut and slice foods proportionately; to calculate and track inventory; and to calculate conversions between imperial and SI units.

The Hungry Heart Café has been asked to cater a fundraising dinner, featuring apple pie for dessert.

1. If there are 60 guests, and Kathy bakes 10 pies, how many slices of pie will Kathy need to cut out of each pie to make sure there is a piece of pie for each guest?
2. What will be the approximate size of the central angles of the pieces of pie?
3. One of the guests wants only half a piece of pie. What would be the approximate size of the central angle of the half-piece of pie?

Solutions...

1. To calculate how many pieces of pie there will be in each pie, students will need to divide the number of guests by the number of pies.

   $60 \div 10 = 6$ pieces of pie

2. One pie is equal to 360°. Divide by the number of pieces.

   $360° \div 6 = 60°$

   The central angles of the pieces of pie will each be 60°.

3. Divide the size of the angle of a full slice by 2.

   $60° \div 2 = 30°$

   The central angles of the children's pie pieces will be 30°.

How Can We Bisect An Angle???

angle bisector: a segment, ray, or line that separates two halves of a bisected angle

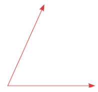
Method #1 - Paper Folding

Method #2 - Protractor and straight edge

Method #3 - Compass and straight edge

Example

Accurately bisect an angle like the one shown here.



SOLUTION

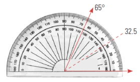
Measure the angle using a protractor. Divide that measure by 2.

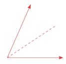
The angle measure is 65°.

$65 \div 2 = 32.5$

Use a protractor to measure and mark off a 32.5° angle.

Draw a line segment from the vertex to the mark you made.





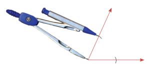
The angle has been successfully divided into two equal parts.

ALTERNATIVE SOLUTION

Trace the angle on above onto a sheet of paper. Place one side of the angle over the other side, creating a fold that goes through the vertex of the angle. The angle has been successfully divided into two equal parts.

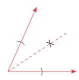
ALTERNATIVE SOLUTION 2

Replicate the angle drawn in the previous solution. Set a compass so that the gap between the pivot point and pencil is a few centimetres. Put the pivot point on the vertex. Mark each side of the angle with the pencil.

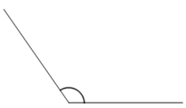


Adjust the compass so that the gap between the pivot point and pencil is over half the distance between the two marks on the sides of the angle. Put the pivot point on one mark and mark off a short arc inside the angle. Put the pivot point on the other mark and mark off another short arc inside the angle, to intersect with the first arc.

Draw a line segment from the vertex to the point of intersection.



Let's TRY one more...



THE ROOTS OF MATH

GEOMETRIC PERSPECTIVE IN ART

The School of Athens, completed by Raphael in 1511, is famous for its use of perspective.

Panels painted with olive trees and placed behind stage actors were some of the first artistic attempts to make closer objects appear larger and distant objects appear smaller, or to produce perspective. This occurred in fifth-century Greece. Medieval and Byzantine art also incorporated perspective in paintings.

Renaissance artists such as Michelangelo are most frequently celebrated for their use of perspective, since they were the first to use the same principles of perspective and scale that artists use today. Between the fourteenth and seventeenth centuries, Renaissance artists used geometric perspective to create the appearance of three-dimensional space within two-dimensional paintings.

Geometric perspective uses four elements to create a three-dimensional effect, the first being a horizon line. It is often found at the viewer's eye level, and represents the horizon. The second is a vanishing point, a spot on the horizon line where the parallel lines in the painting converge and seem to disappear. Perspective lines—those drawn from the edges of objects and leading back into the distance—and angular lines are also used. Geometric scale also allows artists to create perspective by accurately representing the size of one object in relation to another.

1. Think about a photograph you took, a painting you like, or a poster you own. In what ways does it represent geometrical perspective? Identify parallel lines, a vanishing point, or a horizon line that it contains.
2. How could a picture have more than one vanishing point? Is it possible for a picture not to have a vanishing point? Explain your reasoning.
3. How would you define the term "parallel lines"? Describe a method you could use to prove whether or not two lines are parallel.

STUDENT BOOK, p. 297

1. Many artworks and photographs use geometrical perspective as the basis for their composition. Ask students to describe their suggestions, bring in images of them, or look them up on the internet. Then ask them to describe where the parallel lines and horizon line are (these may be imaginary, that is, the artists may have used them to compose the work only) and where the vanishing point is in the work they've chosen.
2. Pictures can contain more than one vanishing point or no vanishing point.
3. From previous learning, students should be able to define parallel lines as lines on the same plane that never intersect and are always the same distance apart at any given point. They may suggest that to prove they are parallel, a line perpendicular to the parallel lines will meet at a right angle to both lines.

HOMEWORK...

- Do Activity 7.4 on p. 290
- Build Your Skills #1 - 7 on p. 292

7.2 - Build Your Skills Detailed Solutions.pdf

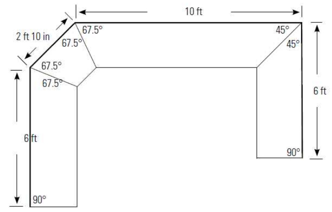
#1, 2, 6

ACTIVITY 7.4
DRAW A KITCHEN COUNTERTOP PLAN

STUDENT BOOK, p. 290

In this activity, students develop their visualization skills, to imagine or sketch the countertop, as well as their reasoning skills, to deduce that they will need four differently shaped countertop pieces. They will also deduce that, in order to be systematic, they can work from start to finish, but that this is not strictly necessary.

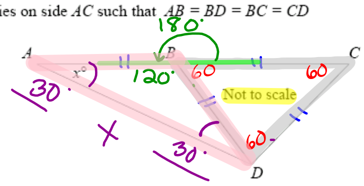
SOLUTION



Warm Up...

ACD is a triangle and point B lies on side AC such that $AB = BD = BC = CD$

Find angle BAD ($x°$):



7.4 - Parallel Lines and Transversals

MATH *ON THE JOB*

Hal is a carpenter from Bathurst, NB. After graduating from high school he enrolled in the carpentry program at BayTech Institute of Trades and Technology, in Moncton. Shortly after graduating, he began working for his uncle's construction company, building and renovating houses primarily in northern New Brunswick.

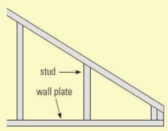
Hal often works on wood-framed houses and buildings. The frames are made of studs (parallel, vertical pieces), and wall plates (pieces that are attached along the top and bottom of the studs). Frames are usually constructed on the ground or floor and then erected. The wall plates hold the studs in position.

Part of Hal's job is to make sure that the studs are exactly perpendicular to the bottom wall plate and parallel to each other. To do this, he uses a measuring device such as a carpenters' square, which is used to measure and mark off 90° angles.

Hal is constructing a partial wall for the side of a staircase. The top of the wall follows the slope of the staircase. A partial diagram of the framing for the staircase is shown here.

- Decide upon a reasonable angle for the staircase. Staircase angles range between 33° and 42°.
- To make the studs parallel, what angle measure will Hal need to make between the studs and the bottom wall plate?
- To make the ends of the studs align with the top wall plate, what angle will Hal need to make between the studs and the top wall plate?

Carpenters use a variety of tools to do different jobs.



SOLUTION

The average rise of a staircase is between 33° and 42°. The studs must be perpendicular (90°) to the bottom wall plate. If the student has drawn a top wall plate at a 33° angle to the horizontal, the left side of each stud must make a 122° angle with the top wall plate. The right side of each stud must make a 58° angle with the top wall plate.

Notes - Geometry Theorems.doc

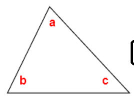
*** Now that the notes are taken care of...

let's do some examples to <u>UNDERSTAND</u> these BIG ideas!!!

Geometry Theorems...

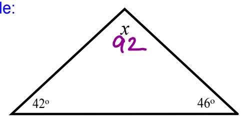
Triangle Angle Sum Theorem:

The sum of the interior angles of any triangle is 180°.



$\boxed{a + b + c = 180^\circ}$

Example:

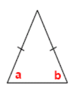


180−42−46=

Isosceles Triangle Theorem:

In an isosceles triangle, the base angles are equal.

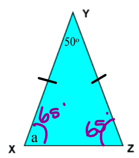
The two angles that are opposite to the equal sides.



$\boxed{a = b}$

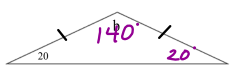
EXAMPLES...

1)



130°

2)

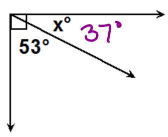


- **Complementary Angles:**
  Two or more angles that have a sum of 90°.

Examples:

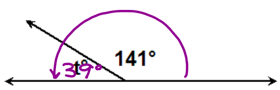
(1) What is the complement of a 50° angle? 40°+50°=90

(2) Determine the measure of the missing angle.



- **Supplementary Angles**:
  Two or more angles that have a sum of 180°.

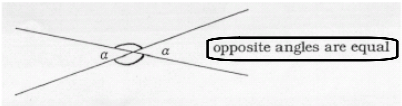
Examples:



180−141=39°

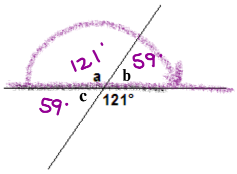
Opposite Angle Theorem...

When 2 straight lines cross, 2 pairs of opposite angles are formed. Opposite angles are equal in size



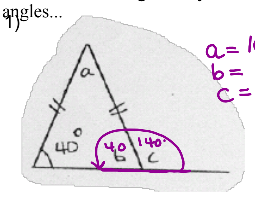
In geometry, angles or lines marked with the same symbol are the same size.

Example:



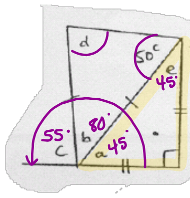
EXERCISE: Use geometry theroems to determine the measure of missing angles...

1)



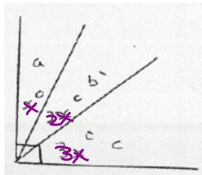
$a = 100°$
$b = 40°$
$c = 140°$

2)

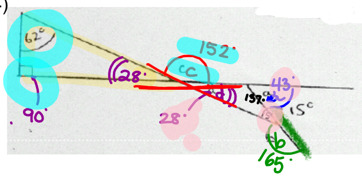


$a = 45°$
$b = 80°$
$c = 55°$
$d = 50°$
$e = 45°$

3)



4)



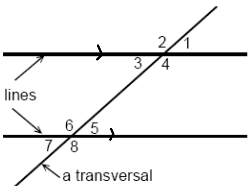
$a = 15°$
$b = 30°$
$c = 45°$

$x + 2x + 3x = 90$
$6x = 90$
$x = 15$

$a =$
$b =$
$c =$
$d =$

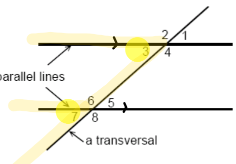
Parallel Line Theorems

A transversal is a third line that crosses two or more lines, as shown in the illustration to the right.

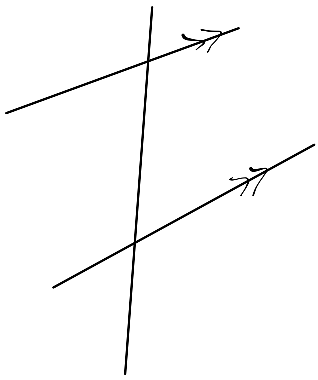


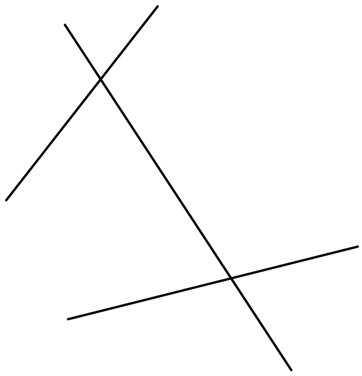
Corresponding Angles:

Pairs of angles on the same side of a transversal and the same side of the parallel lines



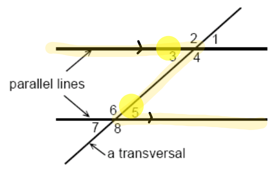
CORRESPONDING ANGLES ARE EQUAL





Alternate Interior Angles:

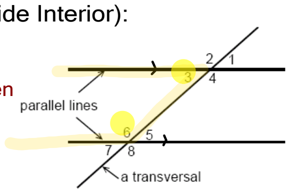
Pairs of angles on the opposite sides of a transversal and between the parallel lines



ALTERNATE INTERIOR ANGLES ARE EQUAL

Co-Interior Angles (Same-side Interior):

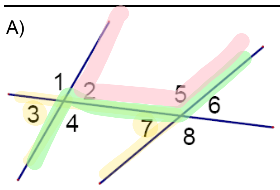
Pairs of angles on the same side of a transversal and between the parallel lines



CO-INTERIOR ANGLES ARE SUPPLEMENTARY

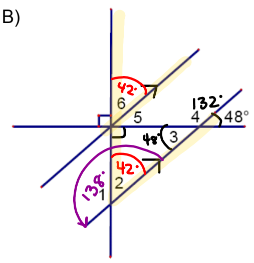
EXERCISE: Practice...

A)



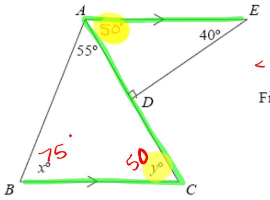
1. <3 and <7 are corresponding angles.
2. <4 and <5 are alternate interior angles.
3. <5 and <2 are same-side interior angles.

B)



1. m<1 = 138°
2. m<2 = 42
3. m<3 = 48°
4. m<4 = 132°
5. m<5 = 48°
6. m<6 = 42

C)

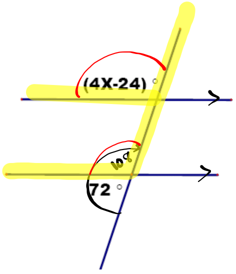


~~<A = 50~~

<EAD = 50°

Find x° and y°.

D)



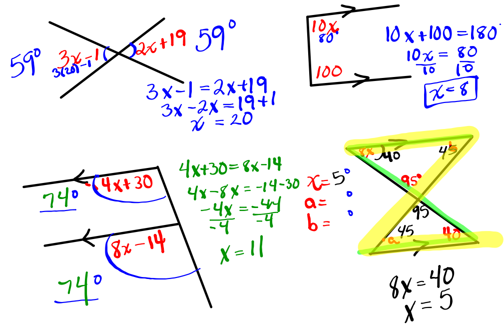
$4x - 24 = 108$

$\frac{4x}{4} = \frac{132}{4}$

$x = 33$

x = 33

NOT DRAWN TO SCALE



$3x - 1 = 2x + 19$

$3x - 2x = 19 + 1$

$x = 20$

$10x + 100 = 180$

$\frac{10x}{10} = \frac{80}{10}$

$\boxed{x = 8}$

$4x + 30 = 8x - 14$

$4x - 8x = -14 - 30$

$\frac{-4x}{-4} = \frac{-44}{-4}$

$x = 11$

$x = 5°$

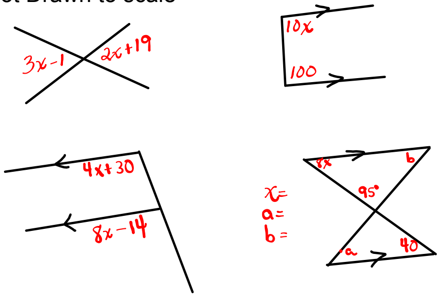
$a =$ °

$b =$ °

$8x = 40$

$x = 5$

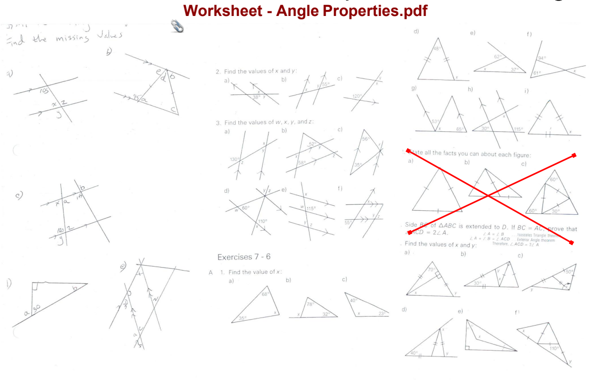
Not Drawn to scale



QUIZ TIME... Use a ruler and compass!

When finished... HW: Complete the following...

Worksheet - Angle Properties.pdf



Solutions from the HOMEWORK...Questions???

Worksheet Solutions - Angle Properties.pdf

PRACTICE TIME...

Worksheet - Parallel Lines and Transversals.pdf

Worksheet Solutions - Parallel Lines and Transversals.pdf

IN-CLASS ASSIGNMENT TIME!!!

In-Class Assignment - Parallel Lines and Transversals.pdf

Parallel Lines???

What you should have found by measuring the angles above is that when two lines are parallel and intersected by a transversal:

- The measures of corresponding angles, alternate interior angles, and alternate exterior angles will be equal. (If such angles do not have equal measures, then the lines are not parallel.)
- Interior and exterior angles on the same side of a transversal will be supplementary. (If they are not, then the lines are not parallel.)

HOMEWORK...

p. 314: #1 - 7

7.4 - Build Your Skills Detailed Solutions.pdf

Attachments

- 7.1 - Build Your Skills Detailed Solutions.pdf
- 7.2 - Build Your Skills Solutions.pdf
- Notes - Geometry Theorems.doc
- Worksheet - Parallel Lines and Transversals.pdf
- In-Class Assignment - Parallel Lines and Transversals.pdf
- 7.4 - Build Your Skills Detailed Solutions.pdf
- Worksheet - Angle Properties.docx
- Worksheet - Angle Properties.pdf
- Worksheet Solutions - Parallel Lines and Transversals.pdf
- Worksheet Solutions - Angle Properties.pdf