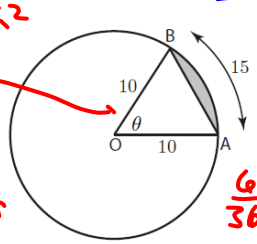
Example

Refer to Figure 8. Suppose we have a circle of radius 10cm and an arc of length 15cm. Suppose we want to find (a) the angle θ, (b) the area of the sector OAB, (c) the area of the minor segment (shaded).

$A_{circle} = \pi r^2$

$\theta = \frac{a}{r}$

$\theta = \frac{15}{10} = 1.5$ Radians



$A_{sector} = \left(\frac{\theta}{360}\right)\pi r^2$ Degrees

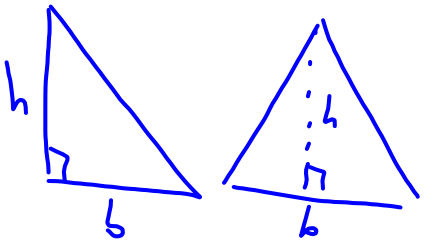
$\frac{60}{360} = \frac{1}{6}$

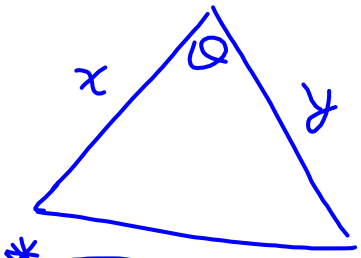
Radians $= \left(\frac{\theta}{2\pi}\right)\pi r^2$

Figure 8. The shaded area is called the minor segment.

Area of triangle 2 sides & contained Angle

$A = \frac{1}{2}bh$





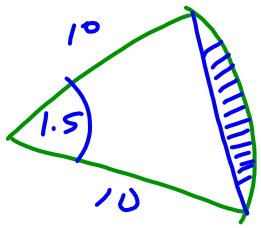
$A = \frac{1}{2}xy\sin\theta$

(b) $\theta = 1.5$ Rad

DRG Δ

$A_{sector} = \left(\frac{1.5}{2\pi}\right)\pi(10)^2$ Mode

$= 75u^2$



(c) $A_\Delta = \frac{1}{2}(10)(10)\sin 1.5$ Radians

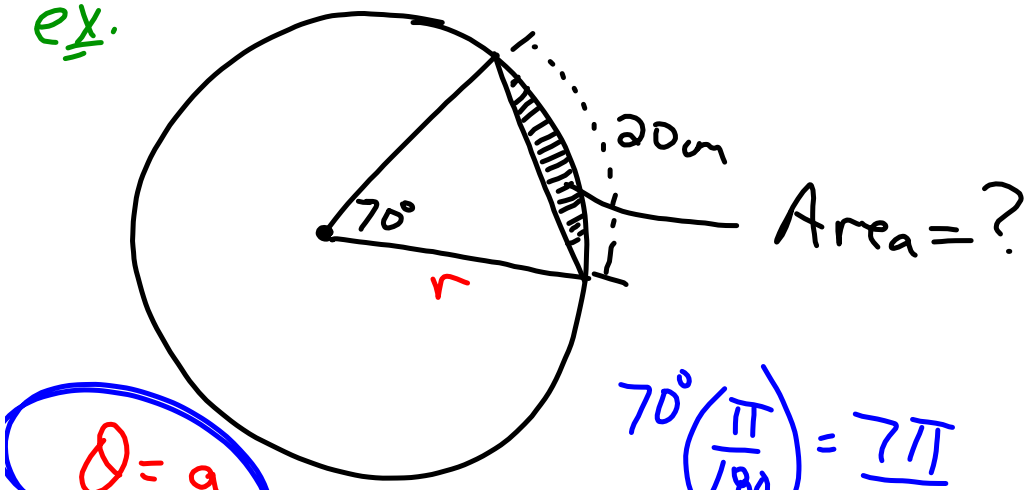
$= 49.87u^2$

$A_{segment} = A_{sector} - A_{Triangle}$

$= 75u^2 - 49.87u^2$

$= 25.13u^2$

ex.



$70^\circ\left(\frac{\pi}{180}\right) = \frac{7\pi}{18}$

$\theta = \frac{a}{r}$

$\frac{7\pi}{18} = \frac{20}{r}$

$7\pi r = 360$

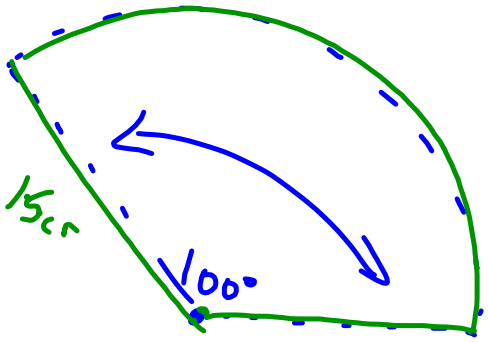
$r = \frac{360}{7\pi} \Rightarrow 360 \div (7\pi)$

$r = 16.4\text{ cm}$

$A_{segment} = A_{sector} - A_{Triangle}$

$= \left(\frac{70}{360}\right)\pi(16.4)^2 - \frac{1}{2}(16.4)^2(\sin 70^\circ)$

$= 37.9\text{ cm}^2$



Perimeter of watered area?

$A_{\text{sector}} = \frac{100}{360}\pi(15)^2$

$=$

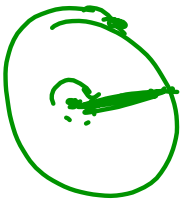
$\theta = \frac{a}{r}$

$(15)\frac{100\pi}{180} = \frac{a}{15}(15)$

$\frac{1500\pi}{180}\text{m} = \text{arc}$

Angular Velocity

Angular velocity - amount of rotation around a central point per unit of time



$$v_a = \frac{\theta}{t} \qquad \theta = \frac{a}{r}$$

θ = angle (radians)

v_a = angular velocity

t = time

a = arc length

r = radius

Ex. The roller on a computer printer makes 2200 rpm (revolution per minute). Find the roller's angular velocity.

$V_A = \frac{\theta}{t}$

$V_A = \frac{4400\pi \text{ Rad}}{1 \text{ minute}}$

$= 4400\pi \text{ Rad/min.}$

$4400\pi \frac{\text{Rad}}{\text{min.}} \times \frac{1 \text{ min}}{60 \text{ sec}}$

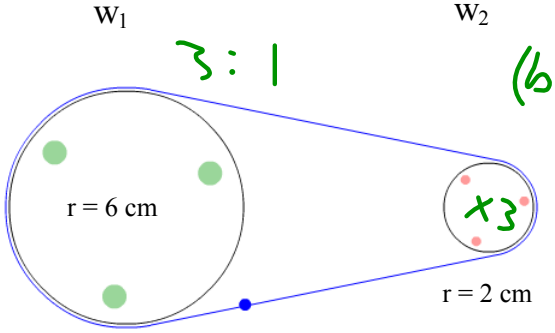
$1 \text{ Rev} = 360^\circ = 2\pi$

$2200 \text{ Rev} = 2200(2\pi) \text{ Rad}$

$\theta = 4400\pi \text{ Rad}$

Ex. (a) If wheel 1 rotates 40 radians, how far has the belt traveled? arc = ??

(b) Given the 40 rad rotation of wheel 1, what was the angle of rotation for wheel 2?



(a) $\theta = \frac{a}{r}$

$40 = \frac{a}{6}$

$a = 240 \text{ cm}$

(b) $40 \text{ Rad} \times 3 = 120 \text{ Rad}$

Ex. A small electrical motor turns at 2200 rpm.

(a) Express the angular velocity in rad/s.
(b) Find the distance a point 0.8cm from the center of rotation travels in 0.008 s.

Ex. A Ferris Wheel rotates 3 times each minute. The passengers sit in seats that are 5 m from the center of the wheel. What is the angular velocity of the wheel in radians per second? What distance do the passengers travel in 6.5 seconds?

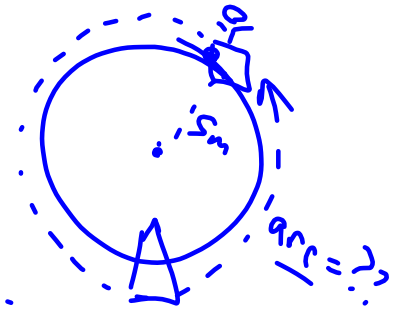
(a) $V_A = \frac{\theta}{t}$

$\theta = 3 \times 2\pi = 6\pi$ Rad

$= \frac{6\pi \text{ Rad}}{60 \text{ sec}}$

$= \frac{\pi}{10}$ Rad/sec

(b)



θ for 6.5 seconds

$\theta = 6.5 \text{ sec} \times \frac{\pi \text{ Rad}}{10 \text{ sec}}$

$= \frac{6.5\pi}{10}$ Rad

$\theta = \frac{a}{r}$

$\frac{6.5\pi}{10} = \frac{a}{5}$

$10a = 6.5\pi(5)$

$\frac{6.5\pi(5)}{10} = \frac{10a}{10}$

$a = 10.2$ m