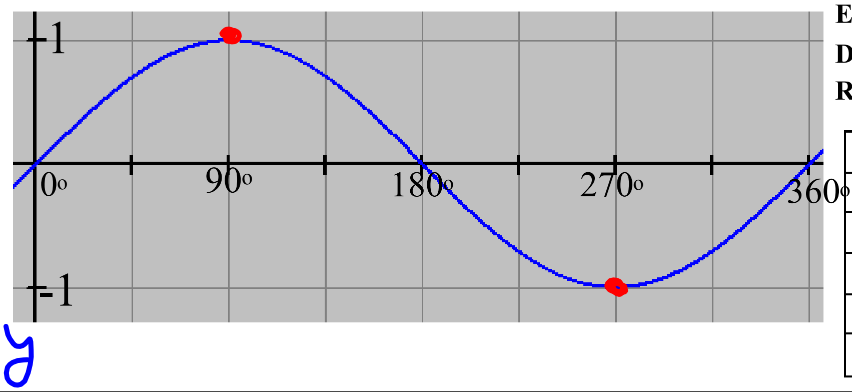
Basic Trig Graphs

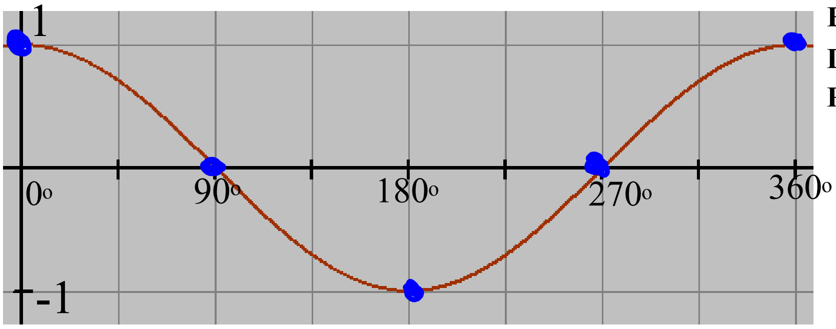
y = sin θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R }
Range: { -1 ≤ y ≤ 1 }

θ	y
0°	0
90°	1
180°	0
270°	−1
360°	0

y = cos θ



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0°	1
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360°	1

Transformations of the Sinusoidal Function

Recall...

$$y = -2(x-3)^2 + 4$$

reflection in the x-axis

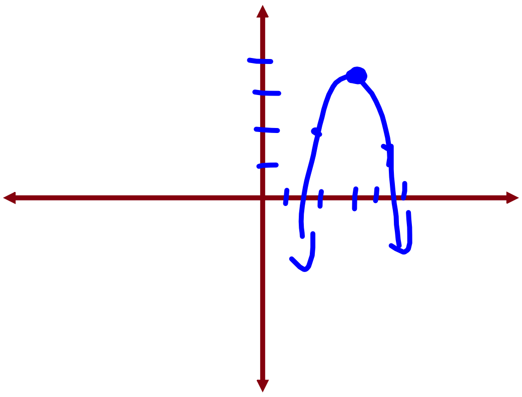
vertical stretch factor

horizontal translation

vertical translation

Vertex $\Rightarrow$ (3, 4)

Sketch $\Rightarrow$



Now, let's look at a sinusoidal function...

$$y = -2\sin[3(\theta - 60^\circ)] - 1$$

Reflection

Amplitude (v. stretch)

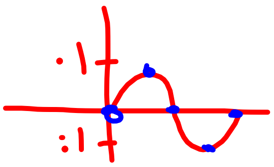
Horizontal Stretch (Period)

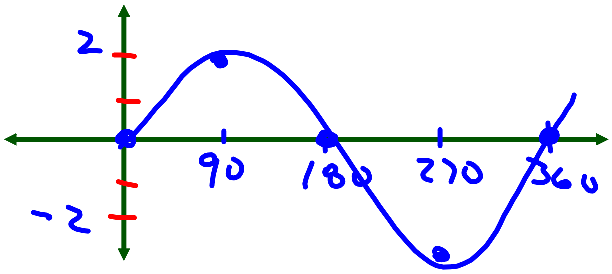
Phase Shift (h. translation)

Vertical Translation

EXAMPLES: Sketch each of the following...

a) $y = 2\sin\theta$

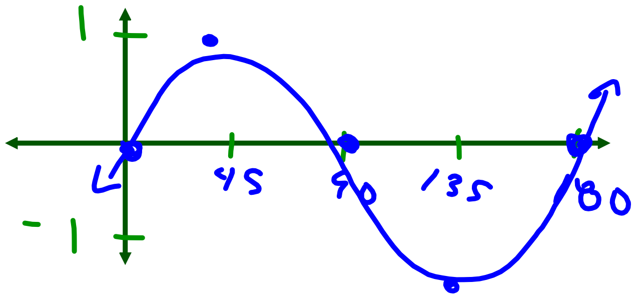




b) $y = \sin 2\theta$

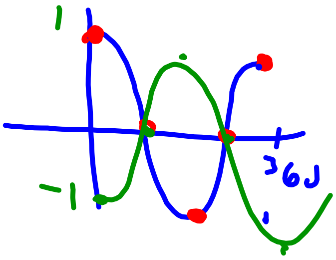
Period $\Rightarrow \frac{360°}{2}$

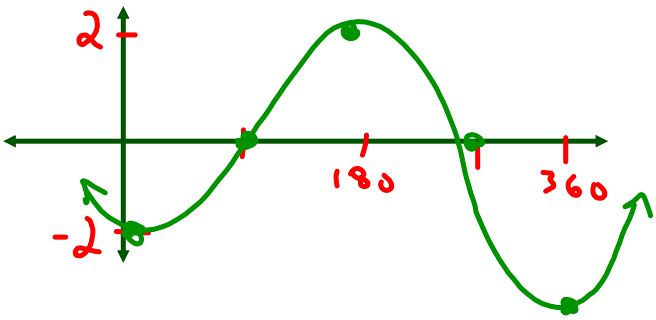
$= 180°$



RST

c) $y = -2\cos\theta$





Sketching Sinusoidal Functions using Transformations

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $f(\theta) = a \sin k(\theta - c) + d$

1. Reflection: If a < 0 the graph will be reflected in the *x*-axis.

2. Amplitude: The amplitude of the graph will be equal to | a | .

3. Period: The period of the graph will be equal to $\frac{360°}{k}$

4. Horizontal Phase Shift: The graph will shift "c" units to the right. (Think Opposite)

5. Vertical Translation: The graph will shift "d" units up.

Mapping Notation: $(x, y) \rightarrow \left(\frac{1}{k}\theta + c, ay + d\right)$

Transformations of Sinusoidal Functions

Example: $f(\theta) = -2 \sin 3(\theta + 30°) - 2$

Domain	$\theta \in R$
Range	$-4 \leq y \leq 0$
Reflection	yes, in x-axis
Amplitude	2
Horizontal Phase Shift	Left 30°
Vertical Translation	Down 2
Period	120°

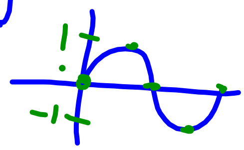
$\frac{360°}{k}$

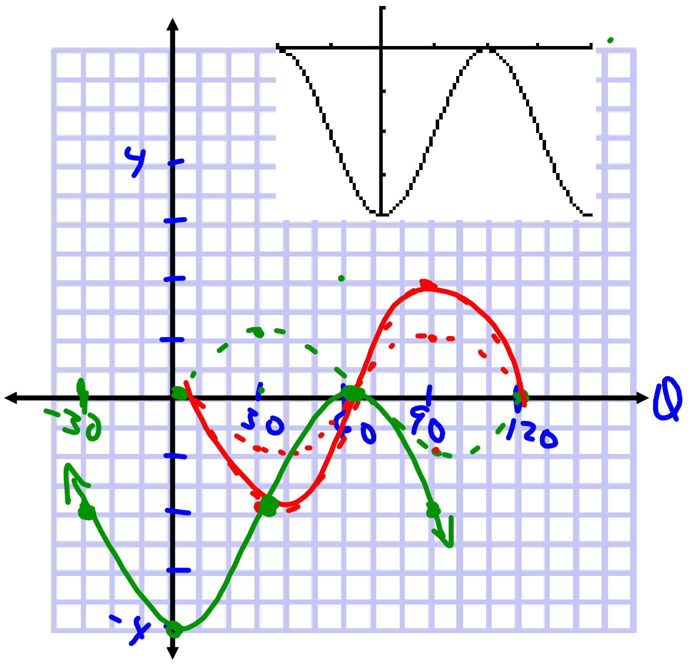
EXAMPLE #1

Now let's sketch a graph of $f(\theta) = -2\sin 3(\theta + 30°) - 2$

" THINK: RST "

Sketching using transformations:
- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*





DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

Check our graph using a graphing calculator

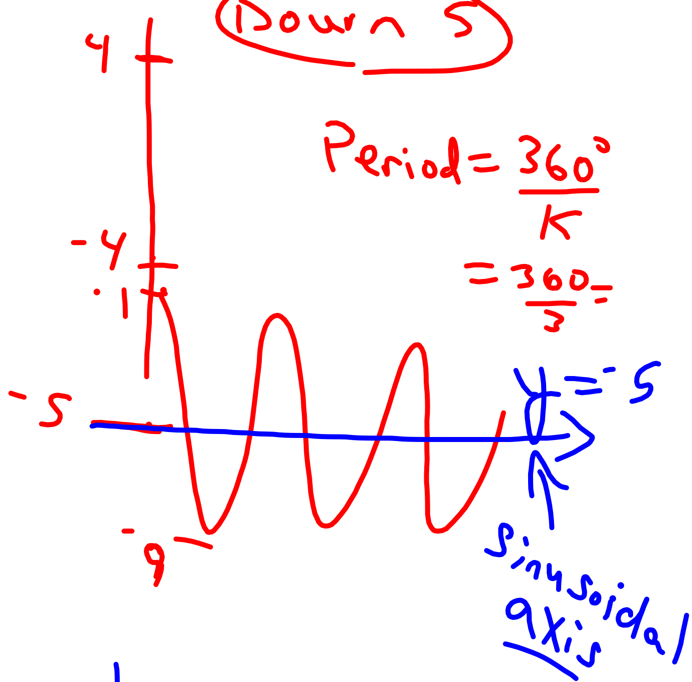
$k=3 \quad 3(\theta+30^\circ)$

① $y=-4\sin(3\theta+90^\circ)-5$

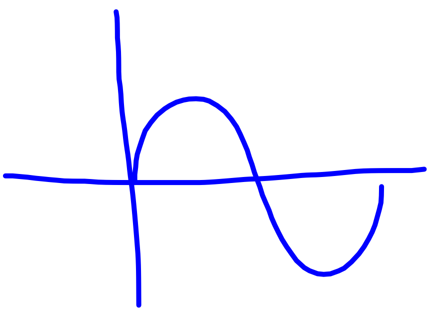
Down 5

Period $= \frac{360^\circ}{k}$

$= \frac{360}{3}$



DOMAIN	$\theta \in R$
RANGE	$-9 \leq y \leq -1$
AMPLITUDE	4
PERIOD	120°
PHASE SHIFT	30° Left
VERTICAL TRANSLATION	Down 5
EQUATION OF SINUSOIDAL AXIS	$y=-5$



Mapping:

$(x,y) \rightarrow \left(\frac{1}{3}x-30^\circ, -4y-5\right)$

$y = 7\cos(5w - 40^\circ) + 10$

$y = 7\cos(5(w - 8^\circ)) + 10$

7 ↑ up 10

7 ↑ up

$\frac{360^\circ}{5} = 72^\circ$

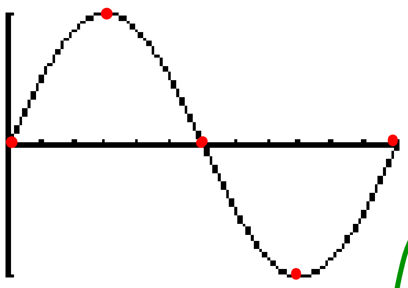
DOMAIN	$\theta \in R$
RANGE	$3 \leq y \leq 17$
AMPLITUDE	7
PERIOD	72°
PHASE SHIFT	Rt. 8
VERTICAL TRANSLATION	up 10
EQUATION OF SINUSOIDAL AXIS	$y = 10$

$(x, y) \to \left(\frac{1}{5}x + 8, 7y + 10\right)$

This time we will graph the same function using a mapping:

$$f(\theta) = -2\sin 3(\theta + 30^\circ) - 2$$

$y = \sin\theta$



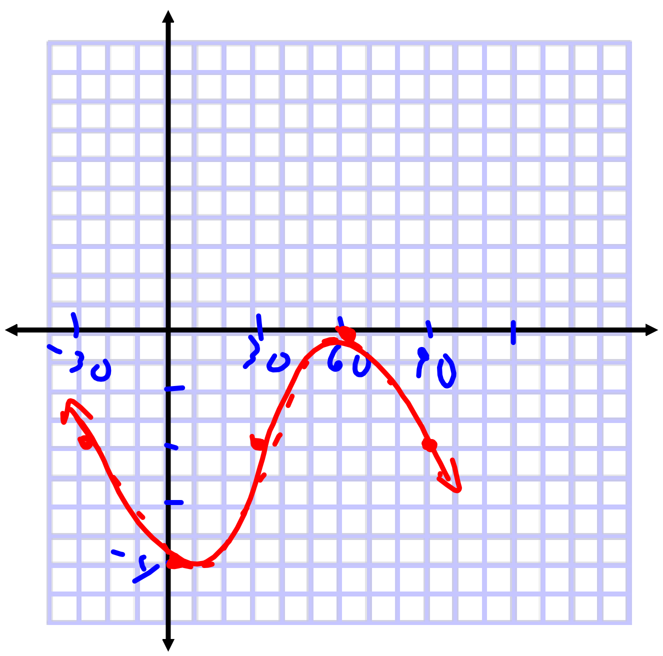
Mapping:

$(\theta, y) \rightarrow \left(\frac{1}{3}x - 30^\circ, -2y - 2\right)$

θ	y
0	0
90	1
180	0
270	−1
360	0

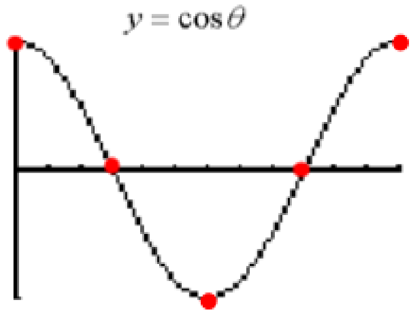
New points after mapping

θ	y
−30	−2
0	−4
30	−2
60	0
90	−2



Graph:

$$y = 3\cos[2(\theta - 135°)] + 2$$



Mapping:

$$(x, y) \rightarrow \left(\tfrac{1}{2}\theta + 135°,\ 3y + 2\right)$$

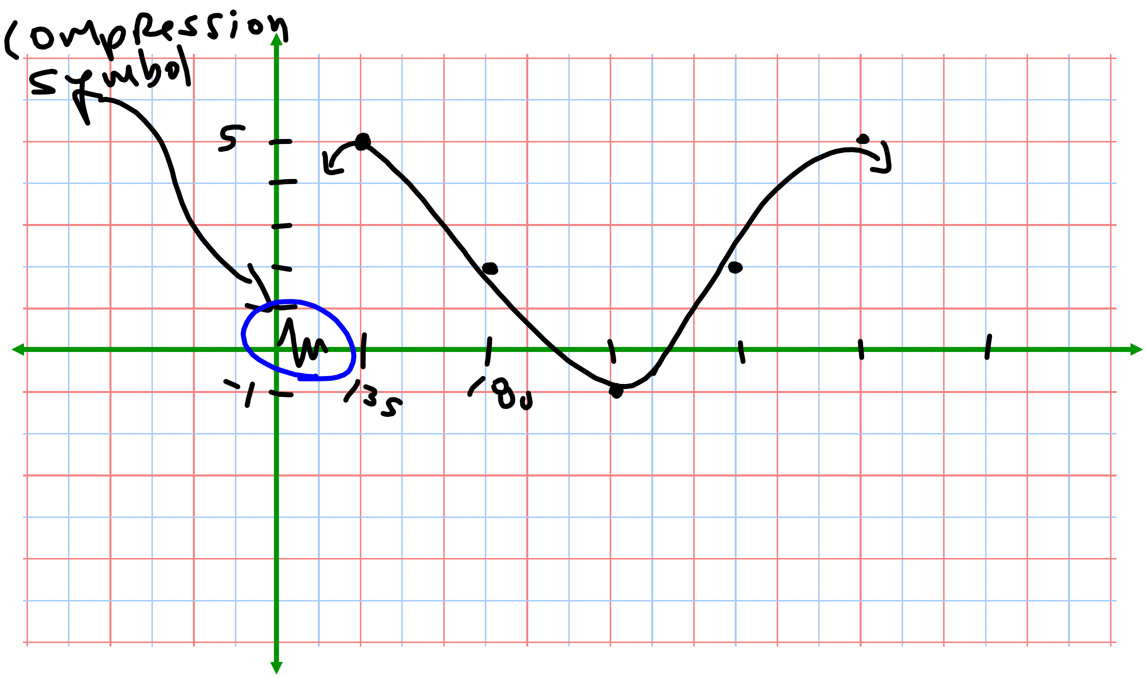
θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
135°	5
180°	2
225°	-1
270°	2
315°	5

+45°

+45°



$$y = a\cos(k(\theta+c))+d$$

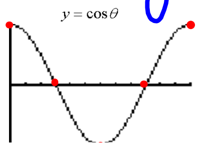
$$\frac{1}{2}(y+1)=3\cos\left(\frac{1}{2}\theta-90^\circ\right)+2$$

Remember...Put in standard form first!!

$$y+1 = 6\cos\left(\frac{1}{2}(\theta-180^\circ)\right)+4$$

$$y = 6\cos\left(\frac{1}{2}(\theta-180^\circ)\right)+3$$

Remember what the graph of cosine looks like ??

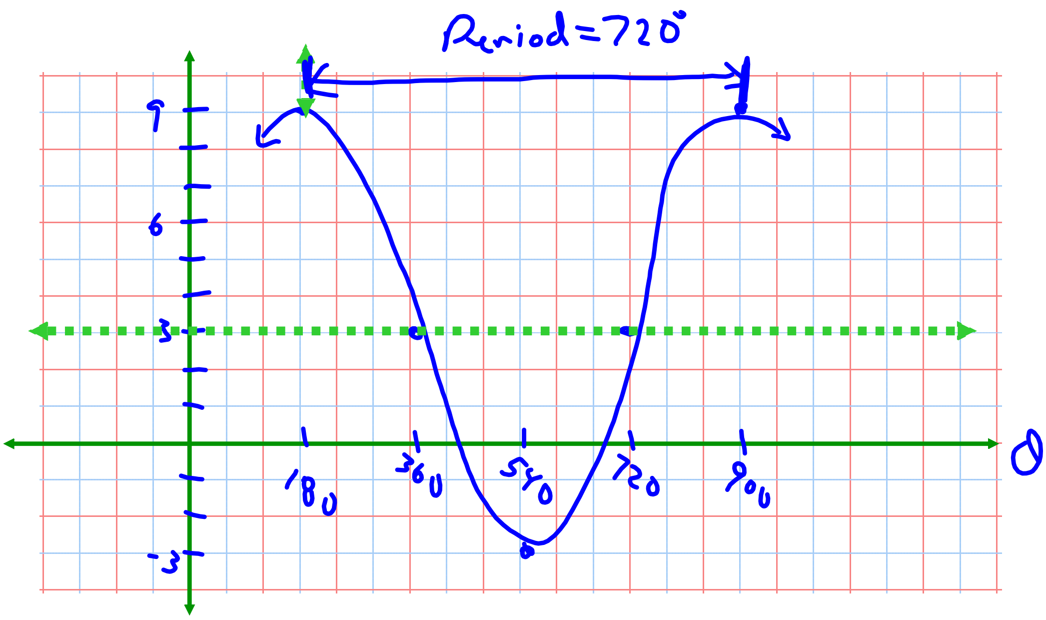


Mapping:
$(x, y) \rightarrow (2x + 180^\circ, 6y + 3)$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

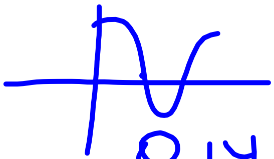
θ	y
180	9
360	3
540	-3
720	3
900	9



Ex. $y = \cos\left(2x - \frac{\pi}{3}\right) - 1$ $\frac{2\pi}{k}$

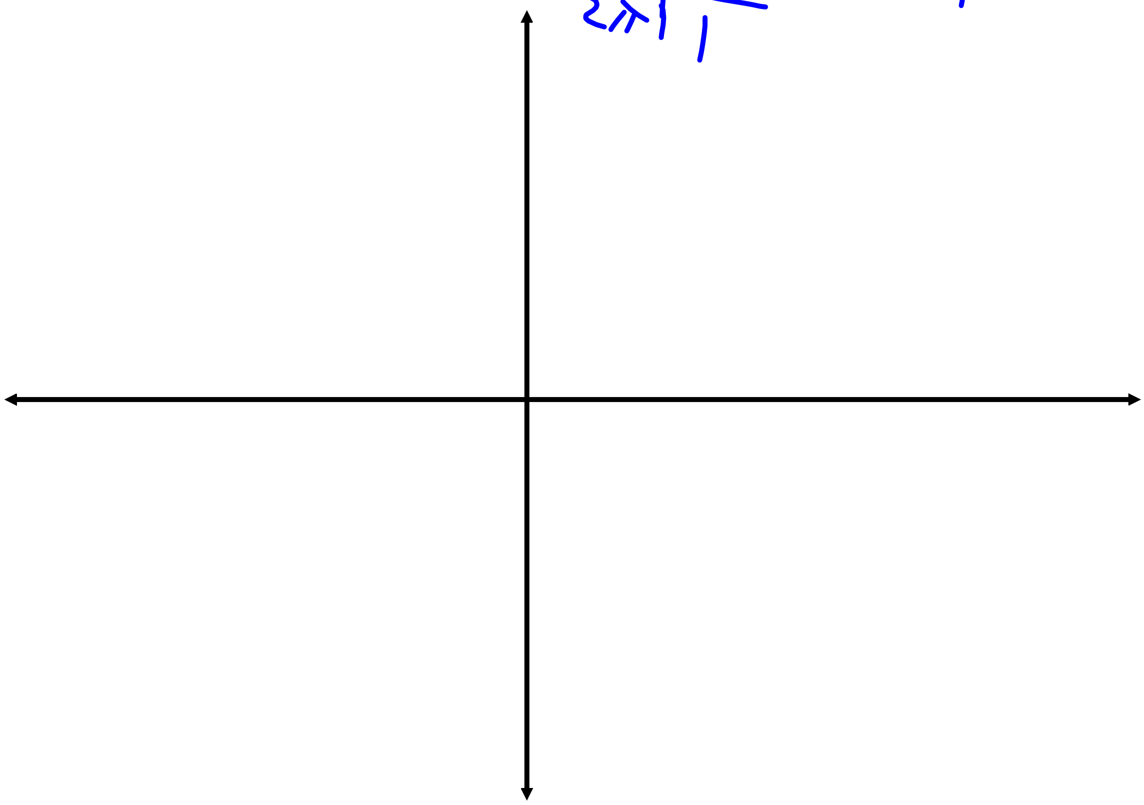
$2\left(x - \frac{\pi}{6}\right)$

AMPLITUDE	1
PERIOD	π
PHASE SHIFT	Rt. $\pi/6$
VERTICAL TRANSLATION	Down 1
EQUATION OF SINUSOIDAL AXIS	$y = -1$



θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

x	y



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#6, 9, 10, 14, 15

Attachments

Worksheet - Sketching Angles in Radians.doc

Warm-Up - Intro to Limits.docx

Review - Factoring.pdf

Worksheet - Factoring Review.doc

Worksheet - Function Notation.pdf