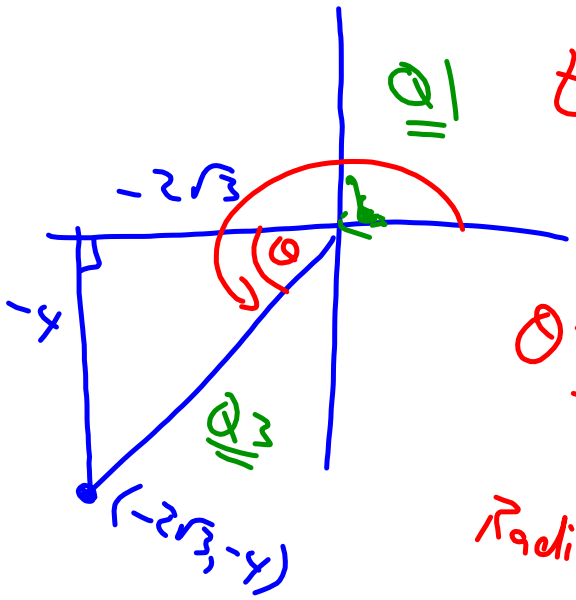
Example

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$



$\tan\theta = \frac{4}{2\sqrt{3}}$ (+)

$\theta = 49°$ (Ref. ∠)

$\theta = 180° + 49°$

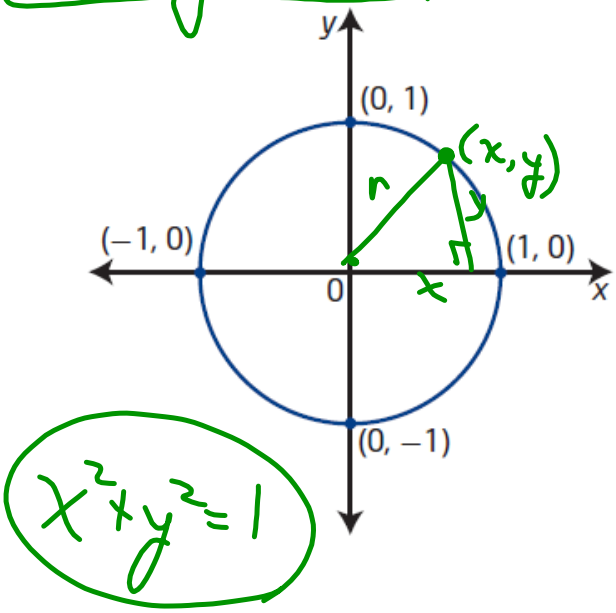
$= 229°$

Radians $\Rightarrow \frac{229\pi}{180}$

229π/180
3.996803987

Unit Circle

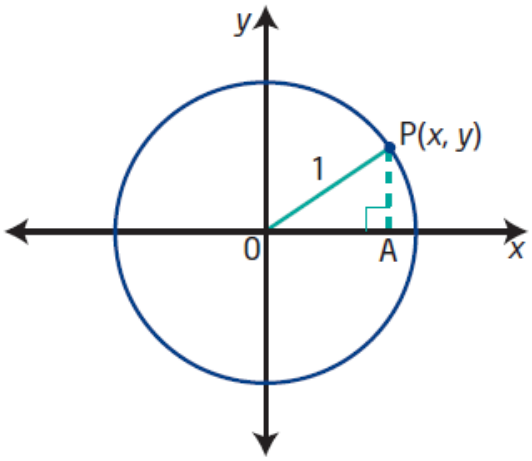
$x^2 + y^2 = r^2$



$x^2 + y^2 = 1$

unit circle

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

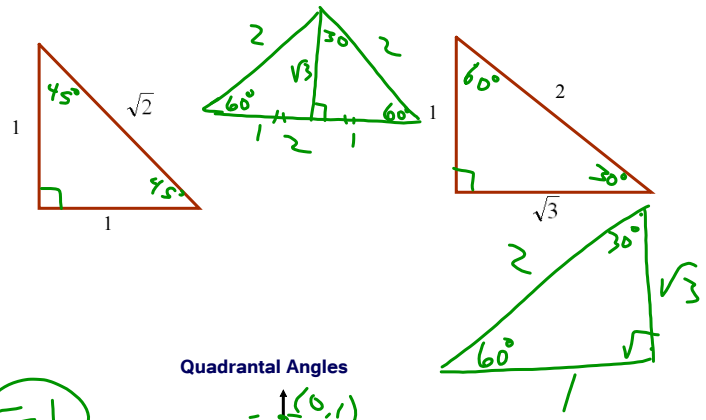


The equation of the unit circle is $x^2 + y^2 = 1$.

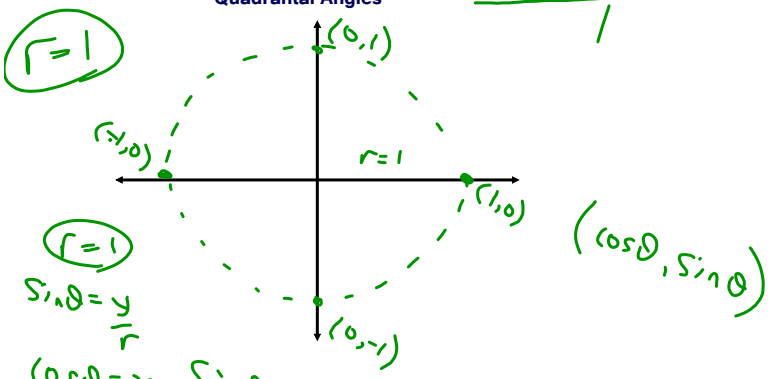
Determine the equation of a circle with centre at the origin and radius 6.

$x^2 + y^2 = 36$

Special Angles (in radians)



Quadrantal Angles

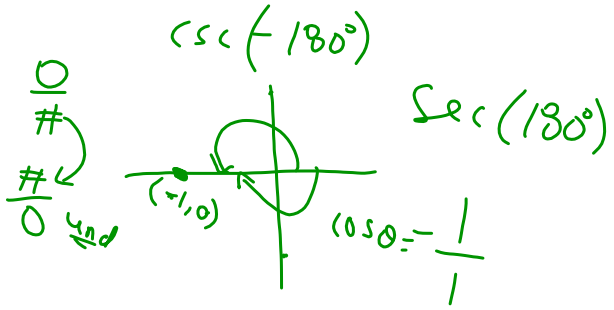


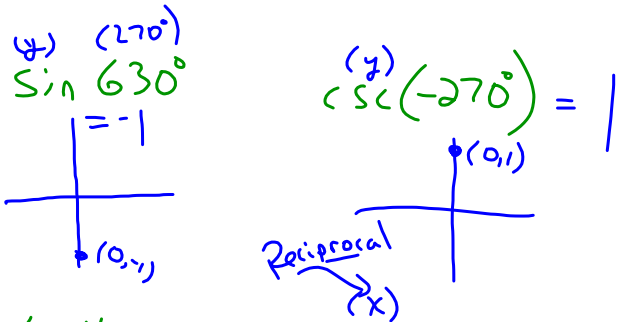
$(\cos\theta, \sin\theta)$

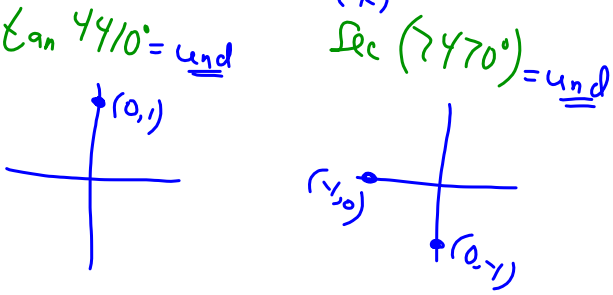
$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$ $\quad \sin\alpha = y$

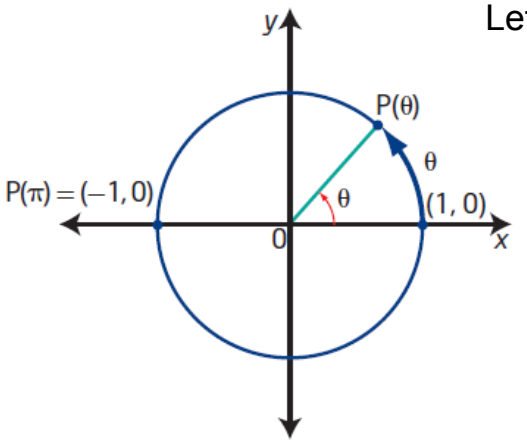
$\tan\theta = \frac{y}{x}$ $\quad \cos\alpha = x$





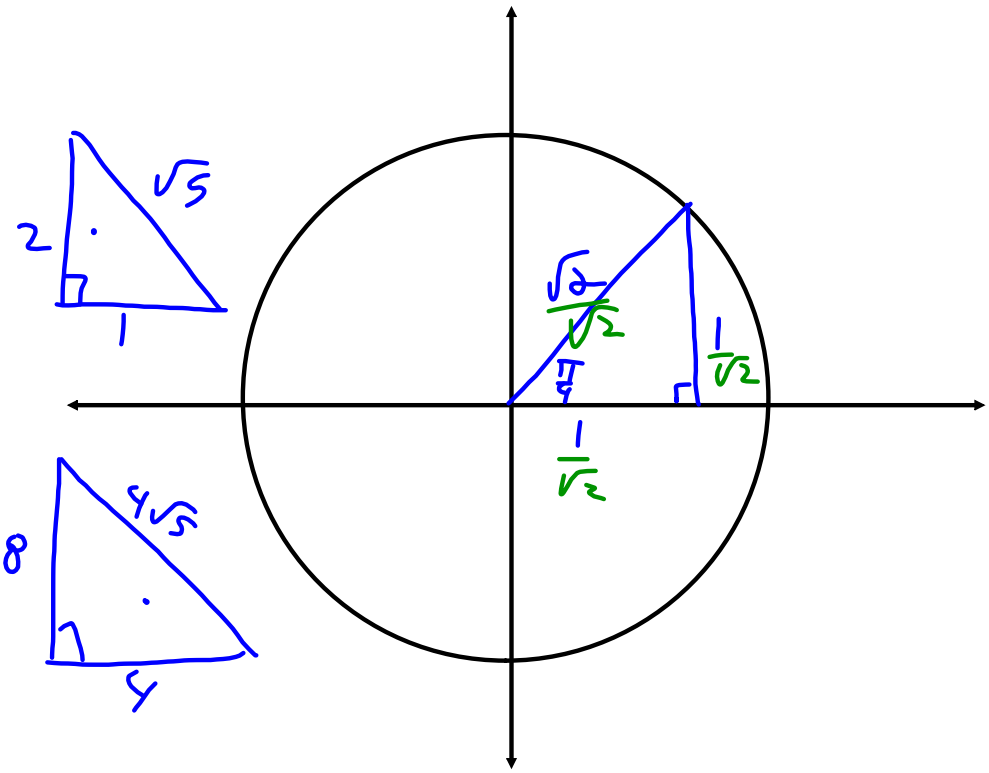


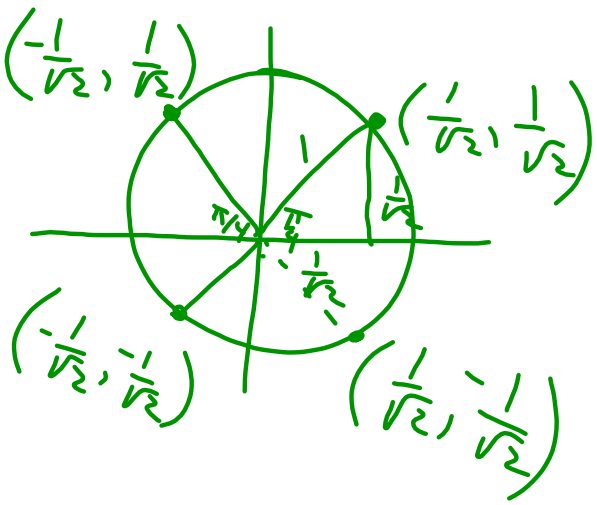
Special Angles on the Unit Circle:

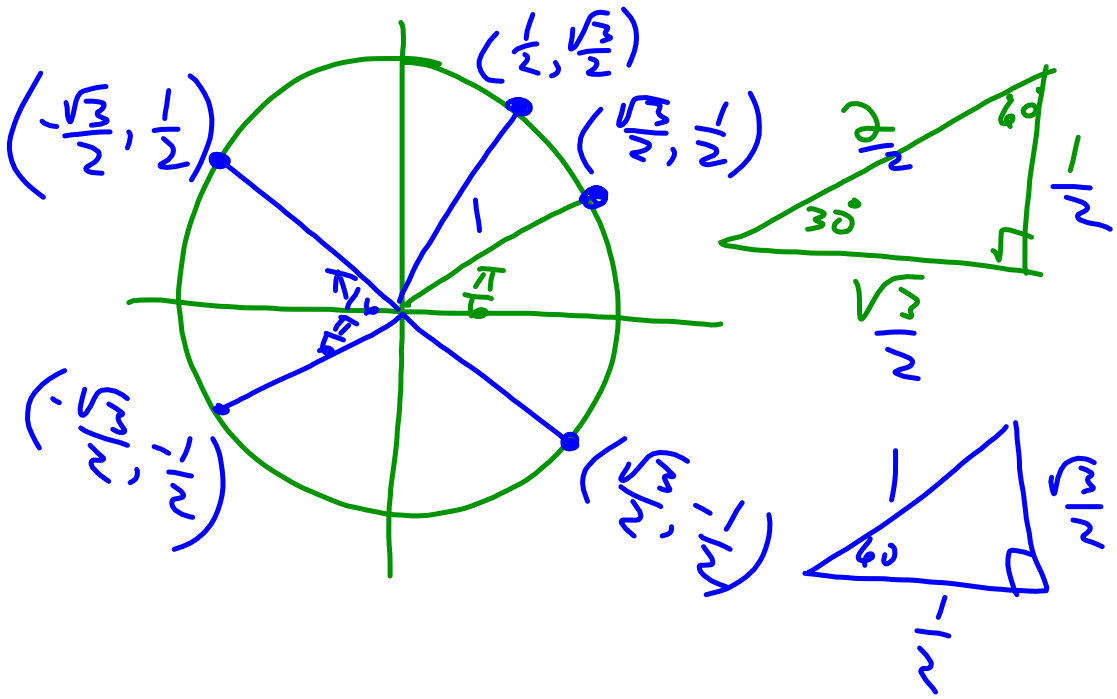


Let's use $\frac{\pi}{4}$ as our reference angle

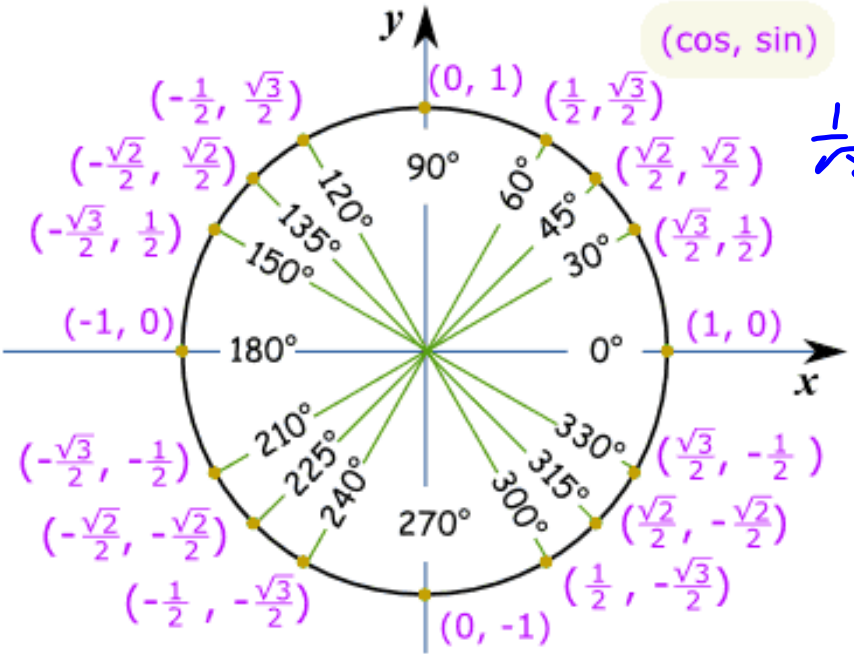
Construct reference triangles for all multiples of $\pi/4$ between 0 and 2π






$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
1
$\frac{\pi}{6}$
$\frac{\pi}{6}$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
2
60
30
1
2
$\frac{\sqrt{3}}{2}$
1
60
$\frac{\sqrt{3}}{2}$
$\frac{1}{2}$

Unit Circle of Special Angles in Degrees



$\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

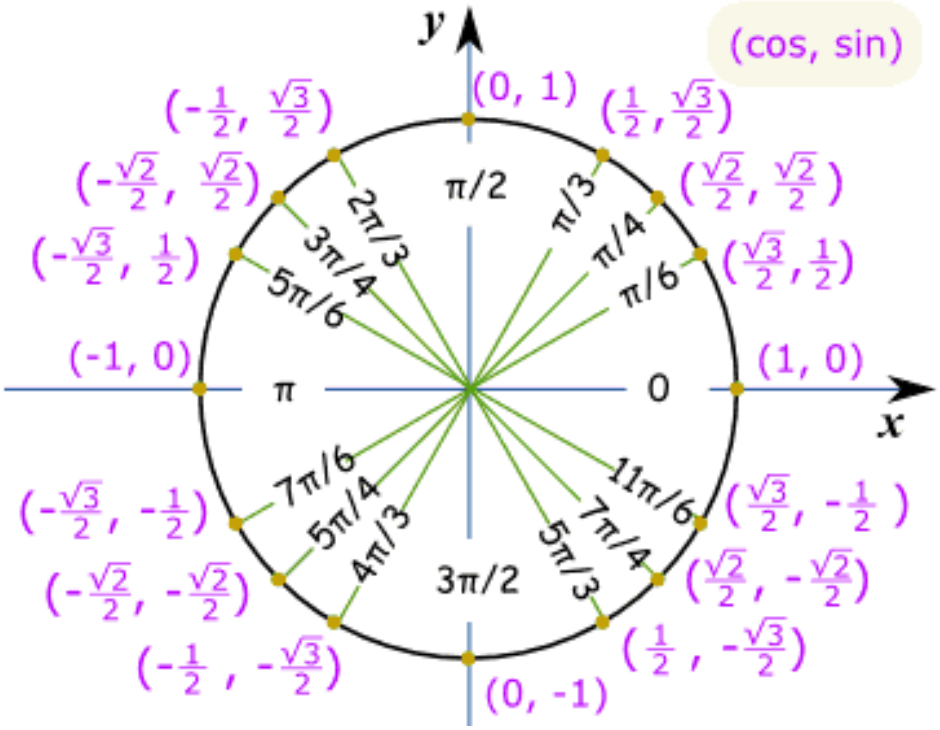
This is lovely...so what is it used for????

$30° \rightarrow \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$60° \rightarrow \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$45° \rightarrow \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

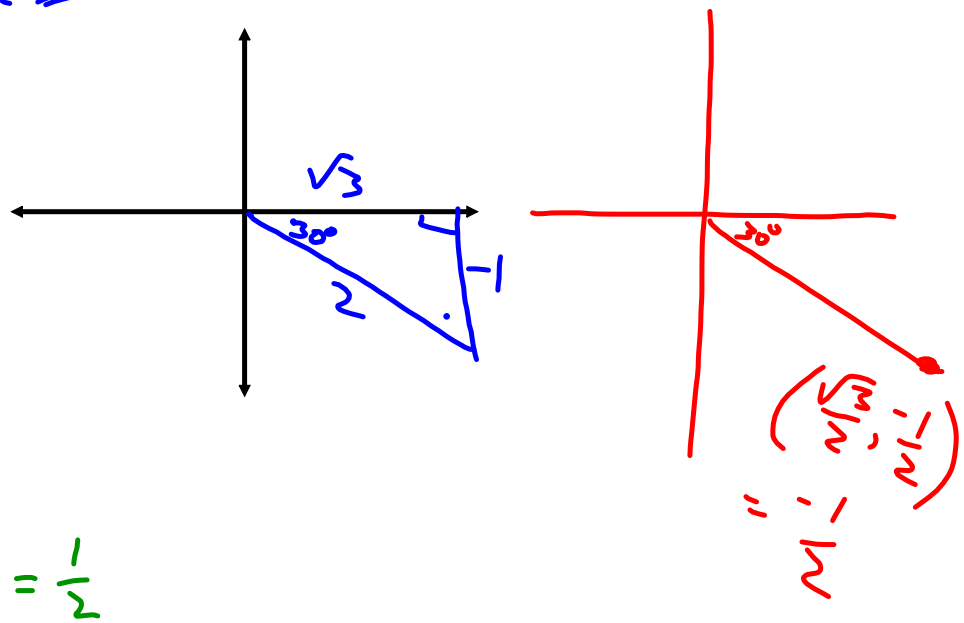
Unit Circle of Special Angles in Radians



Sketching Angles in Radians

ex. $\sin 690°$ (330°)

$= -\frac{1}{2}$

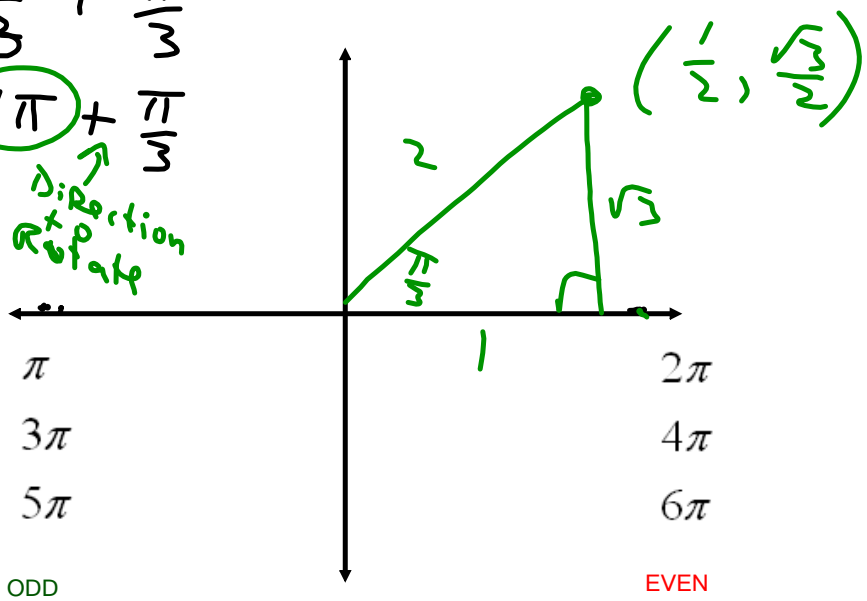


Ex. $\cos\frac{13\pi}{3}$ $= \frac{1}{2}$

$\frac{12\pi}{3} + \frac{\pi}{3}$

$4\pi + \frac{\pi}{3}$

Direction of Rotation



π 2π

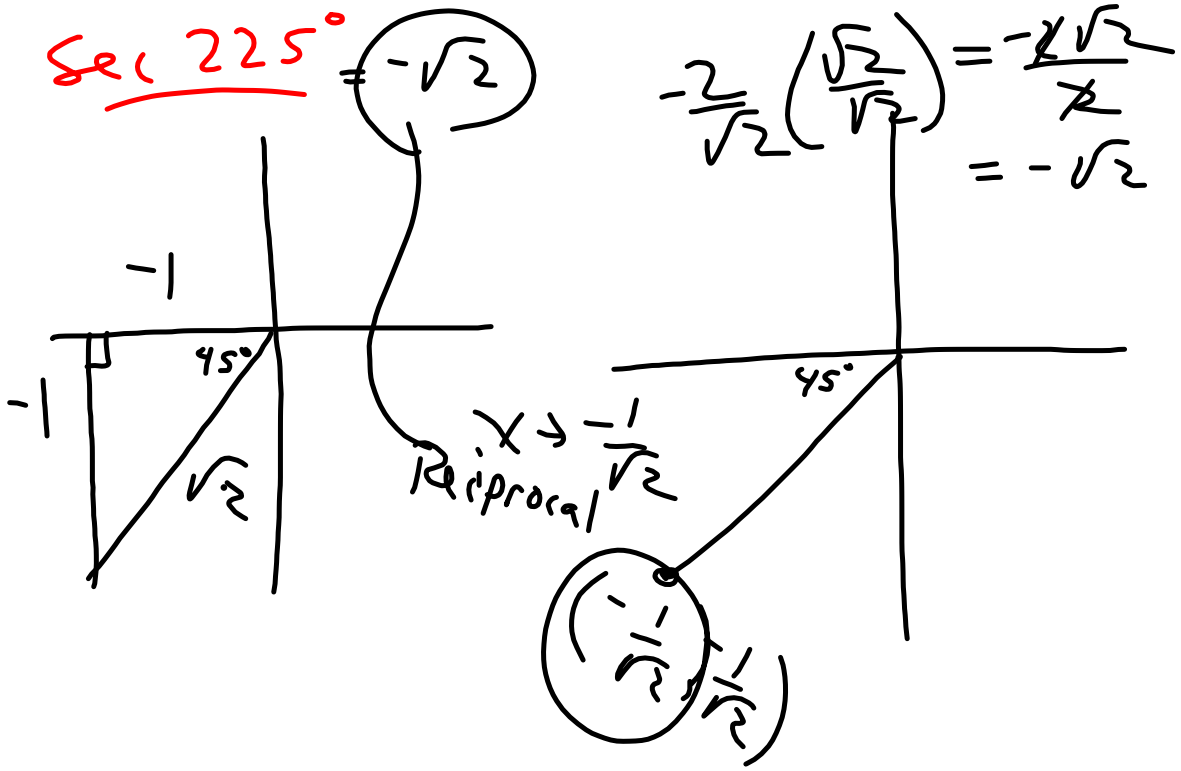
3π 4π

5π 6π

ODD EVEN

Break it apart

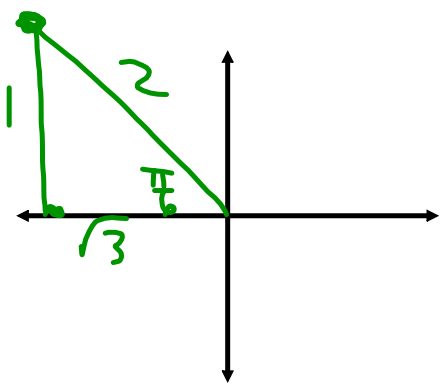
$\cos\frac{13\pi}{3}$


Sec 225° = -√2
-1
-1
45°
√2
Reciprocal
-2/√2 (√2/√2) = -2√2/2
= -√2
45°
-1/√2
-1/√2

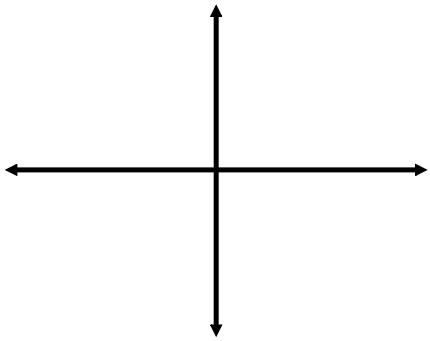
Ex. $\tan\frac{17\pi}{6}$ $= \frac{1}{\sqrt{3}}$

$\frac{18\pi}{6} - \frac{\pi}{6}$

$3\pi - \frac{\pi}{6}$



Ex. $\sin\frac{15\pi}{4}$



Ex. $\cos\left(-\frac{21\pi}{4}\right)$