Warm Up

Evaluate the following limits, if they exist:

$x+27$

$(\sqrt[3]{x}+3)((\sqrt[3]{x})^2-3\sqrt[3]{x}+9)$

12

1. $\lim_{x\to 8} \frac{x-8}{\sqrt[3]{x}-2}$

$\lim_{x\to 8} \frac{\cancel{(\sqrt[3]{x}-2)}((\sqrt[3]{x})^2+2\sqrt[3]{x}+4)}{\cancel{\sqrt[3]{x}-2}}$

$=(\sqrt[3]{8})^2+2\sqrt[3]{8}+4$

$=4+4+4$

$=12$

$-\frac{2}{27}$

2. $\lim_{x\to 3} \frac{x^{-2}-3^{-2}}{x-3}$

$\frac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$

$\lim_{x\to 3}\left(\frac{9-x^2}{9x^2}\right)\cdot\frac{1}{x-3}$

$\lim_{x\to 3} \frac{-1\cancel{(3-x)}(3+x)}{9x^2}\cdot\frac{1}{\cancel{x-3}}$

$=\frac{-1(3+3)}{9(3)^2}=\frac{-6}{81}=\frac{-2}{27}$

-1

3. $\lim_{x\to 1} \frac{x^3-1}{x^3-x^2-4x+4}$

x^3-x^2-4x+4

$x=1$

$1-1-4+4=0$

$(x-1)$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -4 & 4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$(x-1)(x^2-4)$

$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x^2-4)}$

$=\frac{1+1+1}{1-4}$

$=\frac{3}{-3}=-1$

$2a$

4. $\lim_{h\to 0} \frac{(a+h)^2-a^2}{h}$

$\lim_{h\to 0} \frac{[(a+h)-a][(a+h)+a]}{h}$

$\lim_{h\to 0} \frac{\cancel{(h)}(2a+h)}{\cancel{h}}$

$=2a+0$

$=2a$

9.

b) $\lim\limits_{x\to 2} \dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1}\left(\dfrac{\sqrt{6-x}+2}{\sqrt{6-x}+2}\right)\left(\dfrac{\sqrt{3-x}+1}{\sqrt{3-x}+1}\right)$

$\lim\limits_{x\to 2} \dfrac{[(6-x)-4]\,[\sqrt{3-x}+1]}{[(3-x)-1]\,(\sqrt{6-x}+2)}$

$\lim\limits_{x\to 2} \dfrac{\cancel{(2-x)}(\sqrt{3-x}+1)}{\cancel{(2-x)}(\sqrt{6-x}+2)}$

$= \dfrac{\sqrt{3-2}+1}{\sqrt{6-2}+2} = \dfrac{2}{4}$

$= \dfrac{1}{2}$

5. e) $\lim\limits_{h\to 0} \dfrac{\sqrt{9+h}-3}{h}\left(\dfrac{\sqrt{9+h}+3}{\sqrt{9+h}+3}\right)$

$\lim\limits_{h\to 0} \dfrac{\overset{h}{\cancel{(9+h)-9}}}{\cancel{(h)}(\sqrt{9+h}+3)}$

$= \dfrac{1}{\sqrt{9+0}+3}$

$= \dfrac{1}{6}$

Recall from our prior discussions that ...

$y=\sqrt{x}$

1 Theorem $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$

Let's look at a couple of unique functions:

1) $\lim_{x \to 1} \sqrt{x-1}$ DNE

$\lim_{x \to 1^-} \sqrt{0.999\ldots - 1}$

$\sqrt{-0.000\ldots 1}$

$\varnothing$

DNE

$\lim_{x \to 1^+} \sqrt{1.00\ldots 1 - 1}$

$= \sqrt{0.00\ldots 1}$

$= 0$

* Watch for even indexed radicals in isolation!!!

2) $\lim\limits_{x\to -2} \dfrac{|x+2|}{x+2}$

$$\lim_{x\to -2^-} \frac{|-2.000\ldots1+2|}{-2.000\ldots1+2} = \frac{|-0.0000\ldots1|}{-0.000\ldots1} = -1$$

$$\lim_{x\to -2^+} \frac{|-1.99\ldots+2|}{-1.99\ldots+2} = \frac{|0.000\ldots1|}{0.000\ldots1} = 1$$

$$\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$$

$\therefore$ <u>DNE</u>

Piecewise Defined Functions

Definition:
- **Functions defined by different formulas in different parts of their domains**

Example:

$$f(x) = \begin{cases} x+3 & if \;\; x \le 2 \\ x^2 - 2 & if \;\; x > 2 \end{cases}$$

1) Determine *f(1)* , *f(3),* and *f(2)*.
2) Sketch *f(x)*.

① $f(1) = 4$ $\quad f(3) = 3^2 - 2 = 7$ $\quad f(2) = 2+3 = 5$

2/ $y = x+3$

x	y
2	5
0	3

$y = x^2 - 2$

x	y
2	2
3	7

$V(0,-2)$