Let's simplify things...

A function whose graph has holes or breaks is considered discontinuous at these particular points.

If you have to lift your pencil from the page to sketch the graph, it is discontinuous anywhere you lift your pencil

Examples:

Given the function
$$f(x)=\begin{cases}3-x & , \quad \text{if } x<-1\\ 4 & , \quad \text{if } -1\le x<2\\ 1 & , \quad \text{if } x=2\\ 8-x^2 & , \quad \text{if } x>2\end{cases}$$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.

(a) $x=-1$

$\boxed{f(-1)}=4$

$\boxed{\lim_{x\to -1^-} f(x)}$ $= 3-(-1)$ $=4$

$\boxed{\lim_{x\to -1^+} f(x)}$ $=4$

$f(-1)=\lim_{x\to -1} f(x)$

$\therefore$ continuous

(b) $x=2$

$f(2)=1$

$\lim_{x\to 2^-} f(x) = 4$

$\lim_{x\to 2^+} f(x) = 8-(2)^2 = 4$

$f(2)\neq \lim_{x\to 2} f(x)$

$\therefore$ $f(x)$ is discontinuous at $x=2$

$$f(x)=\begin{cases}3-x & , \quad \text{if } x<-1\\ 4 & , \quad \text{if } -1\le x<2\\ 1 & , \quad \text{if } x=2\\ 8-x^2 & , \quad \text{if } x>2\end{cases}$$

$y=3-x$

x	y
-1	4
-2	5

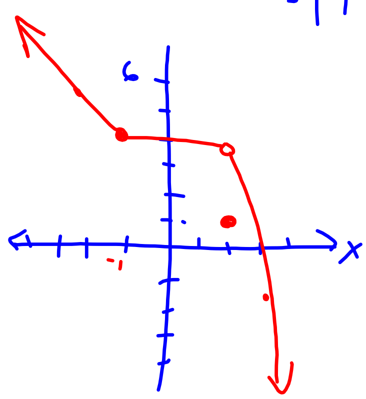
$y=4$

$(2,1)$

$y=8-x^2$

x	y
2	4
3	-1

$V(0,8)$



Given the function $f(x)=\begin{cases}2-x^2 & if\ x<1\\ 3 & if\ x=1\\ 2x-1 & if\ 1<x\le 3\\ (x-4)^2 & if\ x>3\end{cases}$

$(x+1)^2-7$

$V(-1,-7)$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.

(a) $\underline{x=1}$

$f(1)=\boxed{3}$

$\lim_{x\to 1^-} f(x) = 2-(1)^2 = \boxed{1}$ $\lim_{x\to 1^+} f(x) = 2(1)-1 = \boxed{1}$

$f(1)\neq \lim_{x\to 1} f(x)$

$\therefore$ discontinuous at $x=1$

(b) $\underline{x=3}$

$f(3)=\boxed{5}$

$\lim_{x\to 3^-} f(x) = \boxed{5}$ $\lim_{x\to 3^+} f(x) = \boxed{1}$

$\lim_{x\to 3} f(x)$ D.N.E.

$\therefore f(x)$ is discontinuous at $x=3$

$f(x)=\begin{cases}2-x^2 & if\ x<1\\ 3 & if\ x=1\\ 2x-1 & if\ 1<x\le 3\\ (x-4)^2 & if\ x>3\end{cases}$

① $y=2-x^2$

$y=-x^2+2$

$(0,2)$

x	y
1	1 ← open

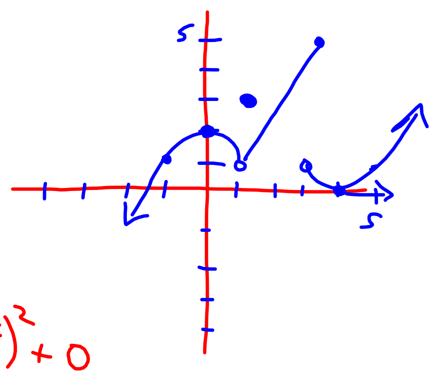
$y=a(x-h)^2+k$

$V(h,k)$

② $(1,3)$

③ $y=2x-1$

x	y
1	1
3	5



④ $y=(x-4)^2+0$

$V(4,0)$

x	y
3	1

Homework

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Warm Up

Given the function...

$$h(x) = \begin{cases} -(x+2)^2+1 & \text{if } x<-1 \\ 2 & \text{if } x=1 \\ \sqrt{x+1} & \text{if } -1<x\le 3 \\ |x-4|+1 & \text{if } 3<x<5 \\ 7-x & \text{if } x>5 \end{cases}$$

(1) Examine $h(x)$ for any points of discontinuity. Provide a mathematical reason for any point of discontinuity.

~~(2) Sketch $h(x)$~~

Solution:

$x=-1$

$f(-1)=2$

$\lim_{x\to -1^-} f(x) = -(-1+2)^2+1 = 0$

$\lim_{x\to -1^+} f(x) = \sqrt{-1+1} = 0$

$\lim_{x\to -1} f(x) = 0$

$\therefore$ discontinuous...

$\lim_{x\to -1} f(x) \ne f(-1)$

$x=5$

$f(5)$ DNE

$\therefore$ discontinuous

$x=3$

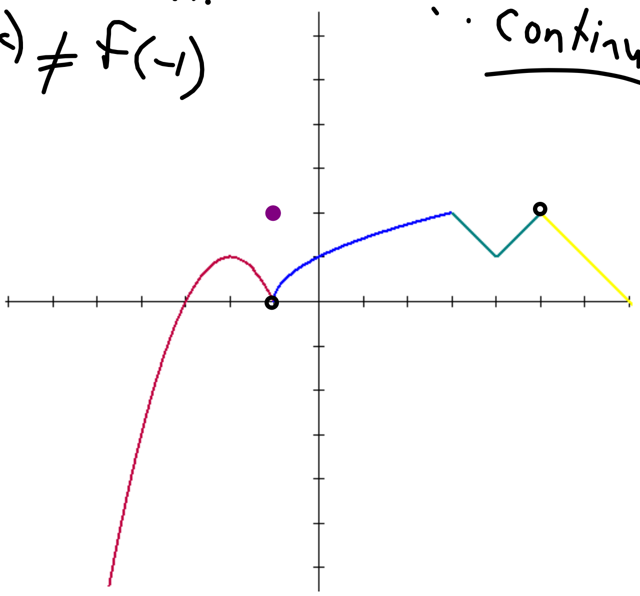
$h(3)=\sqrt{3+1}$
$=2$

$\lim_{x\to 3^-} h(x) = \sqrt{3+1} = 2$

$\lim_{x\to 3^+} h(x) = |3-4|+1 = 2$

$\therefore h(3) = \lim_{x\to 3} h(x)$

$\therefore$ continuous



Warm-Up...

$$f(x)=\begin{cases} -(x+2)^2+1 & if\ x<-1 \\ x+1 & if\ -1\le x<2 \\ 1 & if\ x=2 \\ 3 & if\ x>2 \end{cases}$$

(a) Using the three conditions for continuity examine *f(x)* for any points of discontinuity. At any point(s) of discontinuity clearly indicate a mathematical reason to support why the function is discontinuous at that particular point. **[4]**

(b) Draw a sketch of *f(x)* to support what you have found in part (a). **[4]**

(a) $\underline{x=-1}$

$f(-1)=-1+1$
$=0$

$\lim_{x\to -1^-} f(x)$
$=-(-1+2)^2+1$
$=\underline{0}$

$\lim_{x\to -1^+} f(x)$
$=-1+1$
$=\underline{\underline{0}}$

$\therefore$ continuous

$\underline{x=2}$

$f(2)=1$

$\lim_{x\to 2^-} f(x)$
$=2+1$
$=3$

$\lim_{x\to 2^+} f(x)$
$=3$

$\lim_{x\to 2} f(x)\neq f(2)$

$\therefore$ discontinuous

Extra . . .

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$$

Attachments

- Worksheet - Sketching Angles in Radians.doc
- Warm-Up - Intro to Limits.docx
- Review - Factoring.pdf
- Worksheet - Factoring Review.doc