1. If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 $\boxed{\text{(B) } \frac{1}{6}}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

$$\lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\left(\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}\right)$$

$$\lim_{x\to 2} \frac{(2x+5)-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$$

$$\lim_{x\to 2} \frac{\cancel{x-2}\ 1}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})}$$

$$= \frac{1}{\sqrt{9}+\sqrt{9}}$$

$$= \frac{1}{6}$$

Extra . . .

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

$$\lim_{x\to 1} \frac{x^{1/2}-1}{x^{1/3}-1}$$ ← Diff. of cubes ← " " squares

$$\lim_{x\to 1} \frac{\cancel{(x^{1/6}-1)}(x^{2/6}+x^{1/6}+1)}{\cancel{(x^{1/6}-1)}(x^{1/6}+1)}$$

$$= \frac{1+1+1}{1+1}$$

$$= \frac{3}{2}$$

Limits at Infinity

What exactly is infinity?

- It is the ***process*** of making a value arbitrarily large or small

$+\infty$ ⟶ Positive Infinity...process of becoming arbitrarily large

$-\infty$ ⟶ Negative Infinity...process of becoming arbitrarily small

$\boxed{4}$ **Definition** Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

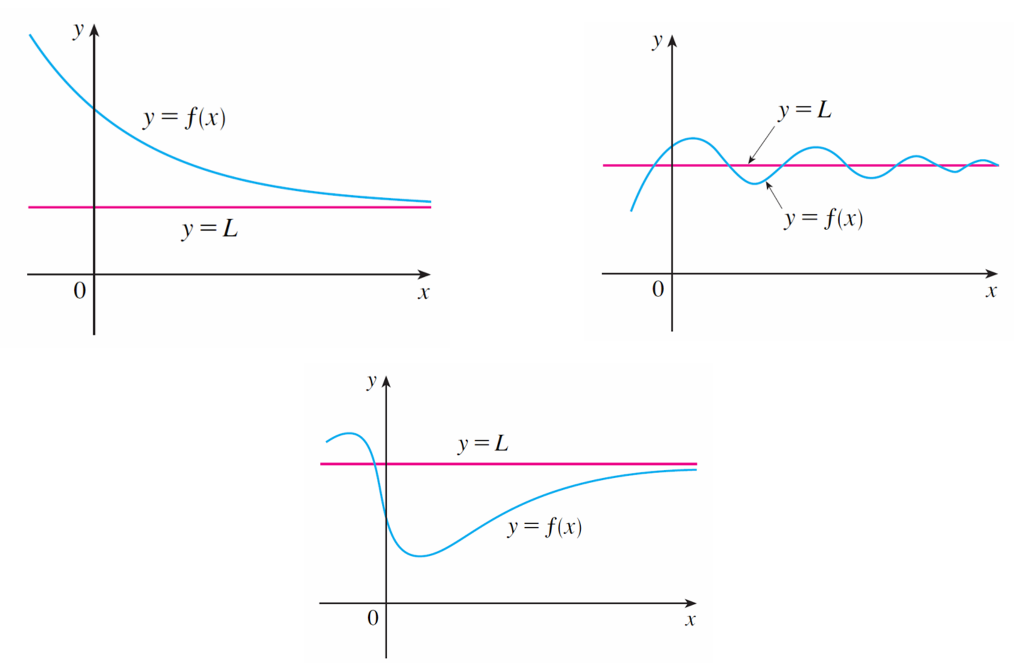
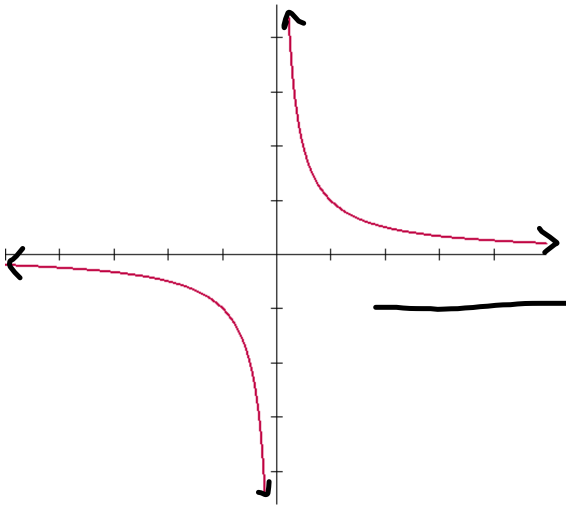


FIGURE 9
Examples illustrating $\lim_{x \to \infty} f(x) = L$

Have a look at these limits...

$$\lim_{x\to\infty}\frac{1}{x} = \frac{1}{99999....} = 0 \qquad \lim_{x\to-\infty}\frac{1}{x} = \frac{1}{-99999...} = 0$$



In general...

7 If n is a positive integer, then

Any constant

$$\lim_{x\to\infty}\frac{1}{x^n} = 0 \qquad \lim_{x\to-\infty}\frac{1}{x^n} = 0$$

$$\lim_{x\to\infty}\frac{1000\,000}{x}$$

Calculating limits at infinity without using a graph

• Rational Functions

Note: If every term in a rational expression is divided by the same value, the rational expression will still be equal to it's original value

$$\frac{\left(\frac{1}{2}\right)12+8}{\left(\frac{1}{2}\right)6-2} = \frac{20}{4}$$

Divde the numerator and denominator by 2

$$\frac{6+4}{3-1} = \frac{10}{2} = 5 = 5$$

This will be important when evaluating limits for rational functions approaching infinity...

$$\frac{(3x^3-1)(x^6+4)}{}$$

Look at the following example:

$$\boxed{\lim_{x\to\infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}} = \lim_{x\to\infty} \frac{\dfrac{3x^2 - x - 2}{x^2}}{\dfrac{5x^2 + 4x + 1}{x^2}} = \lim_{x\to\infty} \frac{3 - \dfrac{1}{x} - \dfrac{2}{x^2}}{5 + \dfrac{4}{x} + \dfrac{1}{x^2}}$$

Divide every term by the HIGHEST power that is present in either the numerator or denominator of the rational expression once they are expanded

$$= \frac{\lim_{x\to\infty}\left(3 - \dfrac{1}{x} - \dfrac{2}{x^2}\right)}{\lim_{x\to\infty}\left(5 + \dfrac{4}{x} + \dfrac{1}{x^2}\right)}$$

$$= \frac{\lim_{x\to\infty} 3 - \lim_{x\to\infty} \dfrac{1}{x} - 2\lim_{x\to\infty} \dfrac{1}{x^2}}{\lim_{x\to\infty} 5 + 4\lim_{x\to\infty} \dfrac{1}{x} + \lim_{x\to\infty} \dfrac{1}{x^2}}$$

$$= \frac{3 - 0 - 0}{5 + 0 + 0} \qquad \text{[by (7)]}$$

$$= \frac{3}{5}$$

lim

This graph below validates our solution:

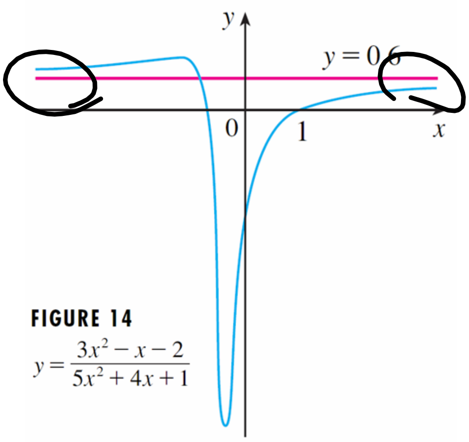


FIGURE 14
$y = \dfrac{3x^2 - x - 2}{5x^2 + 4x + 1}$

Evaluate the following limit:

$$\lim_{x\to\infty}\frac{-3(x^2-4)^2}{3-5x^2}$$

$$\lim_{x\to\infty}\frac{-3(x^4-8x^2+16)}{3-5x^2}$$

$$\lim_{x\to\infty}\frac{\frac{-3x^4}{x^4}+\frac{24x^2}{x^4}-\frac{48}{x^4}}{\frac{3}{x^4}-\frac{5x^2}{x^4}}$$

$$\frac{24}{x^2}$$

$$\lim_{x\to\infty}\frac{-3+0-0}{0-0}$$

undefined

$\therefore$ DNE

$$\lim_{x \to -\infty} \frac{(3x^5-2x)(x^7-2)}{(3-5x^6)^2}$$

$$\lim_{x \to -\infty} \frac{\frac{3x^{12}}{x^{12}} - \frac{6x^5}{x^{12}} - \frac{2x^8}{x^{12}} + \frac{4x}{x^{12}}}{\frac{9}{x^{12}} - \frac{30x^6}{x^{12}} + \frac{25x^{12}}{x^{12}}}$$

$$= \frac{3-0-0+0}{0-0+25}$$

$$= \frac{3}{25}$$

- Exponential Functions

$e = 2.7182\ldots$ (Euler's #)

$\lim_{x\to\infty} e^x$

$\to \infty$

$\therefore$ DNE

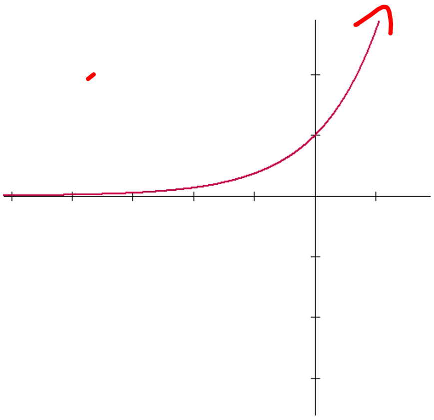
$3^{-2} = \frac{1}{3^2}$

$\lim_{x\to-\infty} e^x$

$(2.71\ldots)^{-\infty}$

$= \frac{1}{(2.71\ldots)^{\infty}}$

$= 0$



Try each of these...

1. $\lim_{x\to-\infty} 2^{3-x}$

$= 2^{3-(-\infty)}$

$= 2^{3+\infty}$ $(>1)^{\infty}$

$= 2^{\infty} \to \infty$

$\therefore$ DNE

2. $\lim_{x\to\infty} \frac{2^x}{5^x}$

$\left(\frac{2}{5}\right)^x$

$(<1)^{\infty} = 0$

3. $\lim_{x\to\infty} \frac{3^x}{4}$

$= \frac{3^{\infty}}{4}$

$= \frac{\infty}{4}$

$\to \infty$

$\therefore$ DNE

4. $\lim_{x\to-\infty} \frac{1}{5^x}$

$= \frac{1}{5^{-\infty}}$

$= 5^{\infty} \to \infty$

$\therefore$ DNE

Homework:

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#3

Attachments

Worksheet - Sketching Angles in Radians.doc

Warm-Up - Intro to Limits.docx

Review - Factoring.pdf

Worksheet - Factoring Review.doc