Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$\lim_{x\to -2} \frac{x^2-2x+1}{x+3}$

$= \frac{(-2)^2-2(-2)+1}{-2+3}$

$= \frac{9}{1}$

9

$(-2, 9)$

$\lim_{x\to 3}(16-x^2)$

$= 16-(3)^2$

$= 7$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

  - $\Rightarrow$ Factor
  - $\Rightarrow$ Rationalize
  - $\Rightarrow$ Expand
  - $\Rightarrow$ Find Common Denominators

Examples:

$\lim_{x\to 3} \frac{x^2-6x+9}{9-x^2} = \frac{0}{0}$

$\lim_{x\to 3} \frac{(x-3)(x-3)}{(3-x)(3+x)}$ $\quad -1$

$= \frac{-1(0)}{6}$

$= 0$

$\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} \left(\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right)$

$\lim_{h\to 0} \frac{(4+h)-4}{h(\sqrt{4+h}+2)}$

$\lim_{h\to 0} \frac{h}{h(\sqrt{4+h}+2)}$

$= \frac{1}{\sqrt{4+0}+2} = \frac{1}{4}$

Try these...remember to use your algebra skills to try and eliminate the **indeterminate form**.

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{x(x+3)}{[(x+2)-(x-2)][(x+2)+(x-2)]}$$

$$\lim_{x\to 0}\frac{x(x+3)}{(4)(2x)}$$

$$= \frac{3}{8}$$

$$\lim_{x\to -2}\frac{x^4-16}{x^3+8}$$

$$\lim_{x\to -2}\frac{(x^2-4)(x^2+4)}{(x+2)(x^2-2x+4)}$$

$$\lim_{x\to -2}\frac{(x-2)(x+2)(x^2+4)}{(x+2)(x^2-2x+4)}$$

$$=\frac{(-4)(8)}{12}=\frac{-32}{12}=-\frac{8}{3}$$

$$\lim_{x\to 2}\frac{(x+2)^2-16}{x^2-4}$$

$$\frac{3}{5}$$

$$7$$

$$\lim_{x\to 2}\frac{(x+2-4)(x+2+4)}{(x-2)(x+2)}$$

$$\frac{3}{5}\times\frac{1}{7}$$

$$\lim_{x\to 2}\frac{(x-2)(x+6)}{(x-2)(x+2)}$$

$$= \frac{8}{4}$$

$$= 2$$

$$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$$

$$\lim_{x\to 2}\left(\frac{2-x}{2x}\right)\cdot\frac{1}{x-2}$$

$$= \frac{-1}{2(2)} = -\frac{1}{4}$$

Homework...

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#1, 4, 5, 6, 9

$$\frac{\left(\frac{7}{x+2}\right)}{\left(\frac{3x}{1}\right)}$$

$$\frac{7}{x+2} \cdot \frac{1}{3x}$$