

WARM UP

Solve the following quadratic equation by... (a) factoring

$$6x^2 - 13x - 5 = 0$$

Factoring,
 $(\frac{x-5}{2})(\frac{3x+1}{2}) = 0$
 $(2x-5)(3x+1) = 0$
 $2x-5=0 \quad 3x+1=0$
 $2x=5 \quad 3x=-1$
 $x=\frac{5}{2} \quad x=-\frac{1}{3}$
 $x=2.5$

Completing the Square

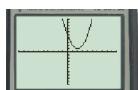
$$\begin{aligned} 6x^2 - 13x - 5 &= 0 \\ 6x^2 - 13x &= 5 \\ x^2 - \frac{13}{6}x &= \frac{5}{6} \\ x^2 - \frac{13}{6}x + \left(\frac{13}{12}\right)^2 &= \frac{5}{6} + \left(\frac{13}{12}\right)^2 \\ \left(x - \frac{13}{12}\right)^2 &= \frac{5}{6} + \frac{169}{144} \\ \left(x - \frac{13}{12}\right)^2 &= \frac{289}{144} \\ x - \frac{13}{12} &= \pm \frac{17}{12} \\ x_1 &= \frac{13}{12} + \frac{17}{12} = \frac{30}{12} = 2.5 \\ x_2 &= \frac{13}{12} - \frac{17}{12} = \frac{-4}{12} = -\frac{1}{3} \end{aligned}$$

Non-Real Roots

- What if it is not possible to factor a quadratic equation and you cannot use completing the square or quadratic formula because there is a negative under the radical sign???

???

EXAMPLE: $y = x^2 - 4x + 5$



Look, it doesn't cross the x-axis

What happens if I try to use the quadratic formula?

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} \\ a &= 1 \\ b &= -4 \\ c &= +5 \\ x &= \frac{4 \pm \sqrt{16-20}}{2} \\ x &= \frac{4 \pm \sqrt{-4}}{2} \end{aligned}$$

error!

The Nature of the Roots

- any quadratic equation can be solved using the quadratic formula.
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Discriminant: $D = b^2 - 4ac$
- discriminant determines the "Nature of the Roots" without actually finding the roots.

- there are THREE cases.

CASE #1: Real and Unequal Roots

This happens when Discriminant is **positive**.

The quadratic will have two real and unequal roots.

factored

If the discriminant is a perfect square then the roots will be **RATIONAL**.

Otherwise, the roots will be **IRRATIONAL**.

EXAMPLE:

$$2x^2 - x - 6 = 0$$

Calculate the discriminant value

$$D = b^2 - 4ac$$

$$D = (-1)^2 - 4(2)(-6)$$

$$D = 1 + 48 = 49$$

$$D = 7^2$$

$$D = 49$$

CASE #2: Real and Equal Roots

This happens when Discriminant = 0.

The quadratic will have one real root (one real root).

EXAMPLE: $x^2 - 8x + 16 = 0$

Calculate the discriminant value

$$D = b^2 - 4ac$$

$$D = (-8)^2 - 4(1)(16)$$

$$D = 64 - 64 = 0$$

$$D = 0$$

CASE #3: Non-Real and Unequal Roots

This happens when Discriminant < 0.

The quadratic will have non-real and unequal roots (imaginary/complex roots)

EXAMPLE: $-3x^2 - 6x - 5 = 0$

Calculate the discriminant value

$$D = b^2 - 4ac$$

$$D = (-6)^2 - 4(-3)(-5)$$

$$D = 36 - 60 = -24$$

$$D = -24$$

$$D = 24i$$

$$D = \sqrt{-24}$$

$$D = \sqrt{24}i$$

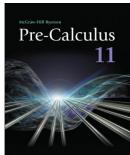
$$D = 2\sqrt{6}i$$

SUMMARY: Nature of the Roots

Value of the Discriminant	Real or Non-real	Equal or Unequal	Rational or Irrational
$D = b^2 - 4ac$	Real or Non-real	Equal or Unequal	Rational or Irrational
1. $D > 0$ but not a perfect square	Real	Unequal 2 x int	Irrational
2. $D > 0$ and is a perfect square	Real	Equal 2 + int	Rational
3. $D = 0$	Real	Equal 1 x int	Rational
4. $D < 0$	Non-real	Unequal no x int	n/a



Homework/Classwork



P254 #1, 2 a, c, e

#5b, e #7d

| a) $D = 33$
 Real, unequal, irrational
 c)

$$\#5b \quad 6h^2 + \frac{h}{6} - \frac{1}{2} = 0$$

$$6h^2 + h - 3 = 0$$