

Sheet: geometric

$$8/ \quad 220\,000 = t_3$$

$$t_6 = 292\,820$$

$$ar^2 = 220\,000$$

$$ar^5 = 292\,820$$

$$\frac{ar^5}{ar^2} = \frac{292\,820}{220\,000}$$

$$\sqrt[3]{r^3} = \sqrt[3]{1.331}$$

$$r = 1.1 \implies a(1.1)^2 = 220\,000$$

$$t_{10} = \left[ \frac{220\,000}{(1.1)^2} \right] (1.1)^9$$

$$a = \frac{220\,000}{(1.1)^2}$$

$$\underline{t_{10} = 428\,717}$$

# Series

## Summation Notation:

The capital Greek letter sigma  $\sum$  is used to provide a shorthand notation for a summation.

In summation notation, the sum of the terms of the sequence  $\{a_1, a_2, a_3, \dots, a_n\}$

is denoted  $\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$

which is read "the sum of  $a_k$  from  $k = 1$  to  $n$ "

Sigma notation is actually more widely used than the above definition suggests.

See if you can evaluate each of the following summations:

$$\textcircled{1} \sum_{k=1}^5 3k$$

$$= 3(1) + 3(2) + 3(3) + 3(4) + 3(5)$$

$$= 3 + 6 + 9 + 12 + 15$$

$$= 45$$

$$\textcircled{2} \sum_{k=4}^9 (k-2)$$

$$= (4-2) + (5-2) + (6-2) + (7-2) + (8-2) + (9-2)$$

$$= 2 + 3 + 4 + 5 + 6 + 7$$

$$= 27$$

$$\textcircled{3}$$

$$\textcircled{3} \sum_{k=1}^5 (k^2 - 3k)$$

$$= [1^2 - 3(1)] + [2^2 - 3(2)] + [3^2 - 3(3)] + [4^2 - 3(4)] + [5^2 - 3(5)]$$

$$= -2 + -2 + 0 + 4 + 10$$

$$= 10$$

Arithmetic Series:

- The summation of the terms of an arithmetic sequence

Formula  $\dashrightarrow S_n = \frac{n}{2} [2a + (n-1)d]$

Let's derive this formula that can be used to determine the sum of "n" terms in any arithmetic series...

n terms

$$S = a + (a+d) + (a+2d) + (a+3d) + \dots + [a+(n-2)d] + [a+(n-1)d]$$

$$S = \underline{a + (n-1)d + a + (n-2)d + \dots + a + d + a}$$

$$2S = 2a + (n-1)d + 2a + (n-1)d + \dots + 2a + (n-1)d + 2a + (n-1)d$$

$$\frac{2S}{2} = \frac{n}{2} [2a + (n-1)d]$$

$$S = \frac{n}{2} [2a + (n-1)d]$$

Example:

Evaluate the following summation:

$$-2 + 7 + 16 + 25 + 34 + \dots + 1420 =$$

Arithmetic 2<sup>nd</sup>

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{159} = \frac{159}{2} [2(-2) + (158)(9)]$$

$$= \underline{\underline{112731}}$$

①  $t_n = a + (n-1)d$

$$1420 = -2 + (n-1)(9)$$

$$\frac{1422}{9} = n-1$$

$$n = \frac{1422}{9} + 1 = \underline{\underline{159}}$$

Example:

A corner section of a stadium has 8 seats along the front row. Each successive row has two more seats than the row preceding it. If the top row has 24 seats, how many seats are in the entire section?

$$8 + (10 + 12 + \dots + 24)$$

$$24 = 8 + (n-1)(2)$$

$$\frac{16}{2} + 1 = n$$

$$\underline{\underline{9 = n}}$$

$$S_9 = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{9}{2} (2(8) + (8)(2))$$

$$= \underline{\underline{144 \text{ seats}}}$$

Geometric Series:

- The summation of the terms of a geometric sequence

Formula  $\dashrightarrow S_n = \frac{a(1-r^n)}{1-r}$  OR  $S_n = \frac{a(r^n-1)}{r-1}$

Let's derive this formula that can be used to determine the sum of "n" terms in any geometric series...

$$S = a + ar + ar^2 + ar^3 + \dots + ar^{n-2} + ar^{n-1} \quad (n-2)+1$$

- Multiply both sides of this summation by  $r$  and then subtract the two equations

$$\begin{aligned} Sr &= ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n \\ S &= a + ar + ar^2 + \dots + ar^{n-1} + a \end{aligned}$$

$$Sr - S = ar^n - a$$

$$S(r-1) = a(r^n-1)$$

$$S = \frac{a(r^n-1)}{r-1}$$

Example:

Find the sum of each of the following series:

(a)  $S = -3 - 6 - 12 - 24 - \dots - 196608$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{17} = \frac{-3(2^{17} - 1)}{2 - 1}$$

$$S_{17} = -393213$$

$$t_n = ar^{n-1}$$

$$-196608 = -3(2)^{n-1}$$

$$65536 = 2^{n-1}$$

$$\log_2 65536 = n - 1$$

$$16 = n - 1$$

$$n = 17$$

(b)  $S = 5 - 10 + 20 - 40 + \dots + 5242880$

$$a = 5$$

$$r = -2$$

$$\frac{5242880}{5} = \frac{5(-2)^{n-1}}{5}$$

$$1048576 = (-2)^{n-1}$$

$$\log 1048576 = \log (2)^{n-1}$$

$$\frac{\log 1048576}{\log 2} = n - 1$$

$$n = 21$$

$$S_{21} = \frac{5((+2)^{21} - 1)}{-2 - 1}$$

$$= 3495255$$

20. The 15th term in an arithmetic sequence is 43 and the sum of the first 15 terms of the series is 120. Determine the first three terms of the series.

$$t_{15} = a + 14d = 43$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{15} = \frac{15}{2} [2a + 14d] = 120$$

$$2a + 14d = 16$$

$$2a + 14d = 16$$

$$a + 14d = 43$$

$$a = -27$$

$$-27 + 14d = 43$$

$$14d = 70$$

$$d = 5$$

$a$     $a+5$     $a+10$   
 $\downarrow$     $\downarrow$     $\downarrow$   
 $-27, -22, -17$

## Practice Questions...

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#4, 5, 6, 7, 8, 10, 11, 13, 20 ✓

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#2, 5, 6, 7, 8, 10, 16, 17

Quiz:  
Sequences