Quiz

1. (b) $t_n = 3n + 5\boxed{t_{n-1}}$ $t_1 = {}^-4$

t_1

$t_2 = 3(2) + 5(-4)$
$= {}^-14$

$t_3 = 3(3) + 5(-14)$
$= -61$

$t_4 = 3(4) + 5(-61)$
$= -293$

6/ $4y+1,\ y+4,\ 10-y$

$$\frac{y+4}{4y+1} = \frac{10-y}{y+4}$$

$$(y+4)^2 = (4y+1)(10-y)$$

$$y^2 + 8y + 16 = 40y - 4y^2 + 10 - y$$

30

$$5y^2 - 31y + 6 = 0$$

$$5y^2 - 30y - 1y + 6 = 0$$

$$5y(y-6) - 1(y-6) = 0$$

$$(y-6)(5y-1) = 0$$

$$y = 6,\ \frac{1}{5}$$

$25, 10, 4$ $\frac{9}{5}, \frac{21}{5}, \frac{49}{5}$

Infinite Sequences and Series

Finite Sequence: A sequence that eventually comes to an end.

ex. $t_1, t_2, t_3, t_4, \ldots, t_n$

Infinite Sequence: A sequence that continues indefinitely.

ex. $t_1, t_2, t_3, t_4, \ldots$

Another example of an infinite sequence would be

$$t_n = \frac{n}{2n+1}, \ where\ n \in N$$

$t_{100} = \frac{100}{201}$

$= 0.49\ldots$

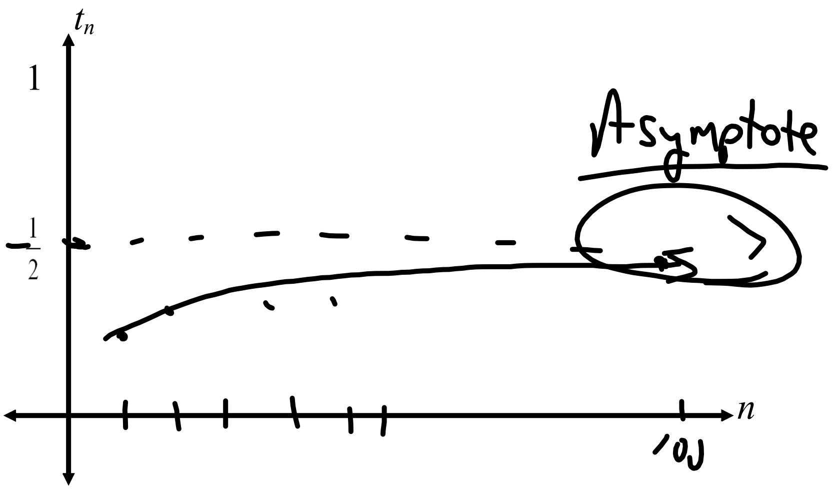
Let's look at the first few terms of this infinite sequence...

$$t_1 = \frac{1}{3},\ t_2 = \frac{2}{5},\ t_3 = \frac{3}{7},\ t_4 = \frac{4}{9}$$

$0.\bar{3}$ $\quad 0.\bar{4}$ $\quad 0.\overline{43}$ $\quad 0.\bar{4}$

As n increases what is happening to t_n ?

What would the graph of this sequence look like?



This result is expressed mathematically as follows:

$$\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}$$

"The limit of $\frac{n}{2n+1}$ as n approaches infinity is $\frac{1}{2}$."

Infinite Series

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \ldots + a_n + \ldots$$

• The first thing to understand about an infinite series is that it is not a true sum. For example we can add a finite number of 2's together and get a real number, but if we add an infinite number of 2's together we do not get a real number at all.

Partial Sum: The sum of a finite number of terms of a series.

Example: $\underbrace{-2} + 0 + 2 + 4 + 6 + 8 + 10 + \ldots$

$-2 \rightarrow$ 1st partial sum = -2

$-2+0$ 2nd partial sum = -2

$-2+0+2$ 3rd partial sum = 0

$-2+0+2+4$ 4th partial sum = 4

etc.

Sometimes with an infinite series the sequence of partial sums, all of which are true sums, approaches a finite limit S:

$$\lim_{n\to\infty} \sum_{k=1}^{n} a_k = \lim_{n\to\infty} \left(a_1 + a_2 + \ldots + a_n\right) = S$$

- If this is the case then we say that the series converges to S, and it would make sense to define S as the sum of the infinite series.

- If the limit of the partial sums does not exist, then the series diverges and has no sum.

Example:

Which of the following series converge?

(a) 0.1 + 0.01 + 0.001 + 0.0001 + ... converges

(b) 10 + 20 + 30 + 40 + 50 + 60 + ... diverges

(c) 1 - 1 + 1 - 1 + 1 - 1 + 1 -... diverges

$S_1 = 1$

$S_2 = 0$

$S_3 = 1$

$S_4 = 0$

Convergence of Geometric Series

$\sum_{k=1}^{\infty} ar^{k-1}$ will converge if and only if $|r|<1$.

If it does converge, the sum of the series will be $\frac{a}{1-r}$.

Why?

- Have a look at what happens if you take a fraction that is less than 1 and raise it to large powers.

$$\left(\frac{1}{3}\right)^{100} \approx 0 \quad \left(\frac{1}{3}\right)^{1000} \approx 0$$

- Now use what you just found to look at the limit of the sum of a geometric series as n approaches infinity

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\lim_{n\to\infty} \frac{a(\cancel{1-r^n})}{1-r}$$

$$= \frac{a(\cancel{1-0})}{1-r}$$

$$= \frac{a}{1-r}$$

Example:

Determine whether each of the following series converges. If it converges, give the sum of the series.

$a r^{n-1}$

$$1)\sum_{k=1}^{\infty} 3(0.75)^{k-1}$$

(<1)

$\Rightarrow r = 0.75 \therefore$ convergent

$$S = \frac{3}{1-\frac{3}{4}} = \frac{3}{\frac{1}{4}} = 12$$

$$2)\sum_{n=0}^{\infty}\left(-\frac{4}{5}\right)^{n}$$

$$= 1 + -\frac{4}{5} + \frac{16}{25} - \frac{64}{125} + \ldots$$

$$r = \left|-\frac{4}{5}\right| < 1$$ converges

$$S = \frac{1}{1+\frac{4}{5}} = \frac{5}{9}$$

$$3)\sum_{n=1}^{\infty}\left(\frac{\pi}{2}\right)^{n}$$

$$r = \frac{3.14}{2} > 1$$

divergent

$$4)\, 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

$r = \frac{1}{2}$ converges

$$S = \frac{1}{1-\frac{1}{2}} = 2$$

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5. a) $S_n = 33$ $\quad t_n = 48$ $\quad r = -2$

$$\frac{a(r^n - 1)}{r-1} = 33 \qquad ar^{n-1} = 48$$

$$\frac{a(\boxed{(-2)^n} - 1)}{-3} = 33 \qquad a(-2)^{n-1} = 48$$

$$(-2)^{n-1} = \frac{48}{a}$$

$$a((-2)^n - 1) = -99 \qquad \frac{(-2)^n}{(-2)^1} = \frac{48}{a}$$

$$a\left(\frac{-96}{a} - 1\right) = 99 \qquad (-2)^n = \frac{-96}{a}$$

$$-96 - a = 99$$

$$3 = a$$

$$S_n = \frac{a(r^n - 1)}{r-1} \qquad t_n = ar^{n-1}$$

$$S_n = \frac{ar^n - a}{r-1} \qquad t_n = \frac{ar^n}{r}$$

$$rt_n = ar^n$$

$$\boxed{S_n = \frac{rt_n - a}{r-1}}$$

Practice Pg. 63

1, 2, 5, 6, 7, 9, 11, 17