Factoring Methods Covered...

- Greatest Common Factor
- Simple Trinomials
- Hard Trinomials
- Perfect Square Trinomials
- Difference of Squares

- Sum & Diff. of Cubes

New Factoring...

Sum & Difference of Cubes

$x^3 - y^3$

$(x - y)(x^2 + xy + y^2)$

1st, 2nd, square 1st, product of 1st x 2nd, opposite sign, square the 2nd

cube root of 1st

$(a^6 + b^9)$

$(a^2 + b^3)(a^4 - a^2b^3 + b^6)$

$(a^2)^2 \qquad (b^3)^2$

ex.

$\sqrt[3]{x^{27}} = (x^{27})^{\frac{1}{3}}$

① $8x^{27} - 27y^{12}$

$(2x^9 - 3y^4)(4x^{18} + 6x^9y^4 + 9y^8)$

② $w^{12} - 1$

Cubes

$(w^4 - 1)(w^8 + w^4 + 1)$

$(w^2 - 1)(w^2 + 1)(w^8 + w^4 + 1)$

$(w - 1)(w + 1)(w^2 + 1)(w^8 + w^4 + 1)$

Start with Diff. of Squares

$(w^6 - 1)(w^6 + 1)$

$(w^3 - 1)(w^3 + 1)(w^6 + 1)$

$(w - 1)(w^2 + w + 1)(w + 1)(w^2 - w + 1)(w^2 + 1)(w^4 - w^2 + 1)$

M/C hoice

$$S = \frac{a}{1-r}$$

9. $8 - 24 + 72 - 216 + \cdots$ (-3)

$r = -3 \quad a = 8$

$$S_9 = \frac{8((-3)^9 - 1)}{-3-1}$$

$$= \frac{8(-19683-1)}{(-3-1)} = \underline{39368}$$

12/

$14 + 70 + 350 + \cdots + 43750$ $n = ??$

$$S_n = \frac{a(r^n - 1)}{r-1} \qquad a = 14 \quad r = 5$$

$$S_6 = \frac{14(5^6 - 1)}{5-1}$$

$= 54684$

$$\frac{43750}{14} = \frac{14(5)^{n-1}}{14}$$

$$3125 = (5)^{n-1}$$

$$5^5 = 5^{n-1}$$

$$5 = n - 1$$

$$\underline{6 = n}$$

II. Open Response: All work must be shown for each of the following in the space provided.

1. Determine the fourth term in the sequence defined by the recursive formula: [3]

$$t_1 = 4,\quad t_n = -5n^2 + 2(n - t_{n-1})$$

-90

2. Determine the exact sum of each of the following finite series: [8]

(a) $7 + 19 + 31 + 43 + \ldots + 799$

$n = 67$

$S_{67} = 27001$

(b) $-\frac{2}{5} + \frac{2}{15} - \frac{2}{45} + \ldots + \frac{2}{7971615}$

$\left(-\frac{5}{\cancel{2}}\right)\left(\frac{\cancel{2}}{7971615}\right) = \left(-\frac{\cancel{2}}{5}\right)\left(-\frac{1}{3}\right)^{n-1}\left(-\frac{5}{\cancel{2}}\right)$

$-\frac{1}{1594323} = \left(-\frac{1}{3}\right)^{n-1}$

$\left(-\frac{1}{3}\right)^{13} = \left(-\frac{1}{3}\right)^{n-1}$

$13 = n - 1$

$n = 14$

$S_{14} = \frac{-\frac{2}{5}\left(\left(-\frac{1}{3}\right)^{14} - 1\right)}{-\frac{1}{3} - 1}$

$= -0.299\overline{9}$

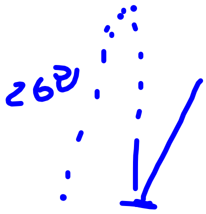
3. A super bouncy ball is fired 268 m into the air. The ball falls, rebounds to 80% of its original height, and then falls to the ground again. This pattern of rebounding to 80% of the height from the previous bounce continues.

(a) How high will the ball bounce in the air after it bounces for the 15th time? [3]

$214.4 \xrightarrow[\times 0.8]{}$

$t_{15} = 214.4(0.8)^{15}$

$= \underline{9.43\,m}$



(b) Determine the total distance travelled by the ball the instant it hits the ground for the 16th bounce? [4]

268 268

$268(2),\ \frac{214.4}{(2)},\ 171.52 \ldots\ldots$ Bounce 16

536,

$S_{16} = \frac{268(0.8^{16}-1)}{0.8-1} = 1302.28$

$\times 2$

$\underline{2604.56\,m}$

4. The sixth and eleventh terms of an arithmetic sequence are 2 and -23, respectively. Determine the **sum** of the first 37 terms of this sequence. [6]

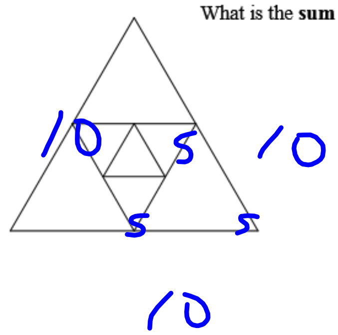
$a+(n-1)d = t_n$

$a+5d=2 \qquad a+10d=-23$

$\left.\begin{array}{l} d=-5 \\ a=27 \end{array}\right\} \quad S_{37} = \frac{n}{2}[2(a)+(n-1)d]$

$= \underline{\underline{-2331}}$

5. One side of an equilateral triangle is 10 cm. The midpoints of its sides are joined to form an inscribed equilateral triangle, and this process is continued, as shown in the diagram.



What is the **sum of the perimeters** of the triangles if this process is continued without end? [4]

$30 + 15 + 7.5 + \cdots$

$r = \frac{1}{2}$ $\quad S = \frac{30}{1 - \frac{1}{2}} = \underline{\underline{60}}$

6. For a geometric sequence, $t_2 + t_3 = -24$ and $t_7 + t_8 = -5832$. Find the sum of the first 12 terms of this sequence. [4]

$ar + ar^2 = -24$

$ar^6 + ar^7 = -5832$

$ar(1+r) = -24 \qquad ar^6(1+r) = -5832$

$\frac{\cancel{a}r^6\cancel{(1+r)}}{\cancel{a}r\cancel{(1+r)}} = \frac{-5832}{-24}$

$\sqrt[5]{r^5} = \sqrt[5]{243}$

$\underline{r = 3}$

$a(3) + a(3)^2 = -24$

$3a + 9a = -24$

$12a = -24$

$\underline{a = -2}$

$S_{12} = \frac{-2(3^{12} - 1)}{3 - 1}$

$= \underline{-531440}$