Warm Up

Divide using long and synthetic division:

1

$$\left(x^3 + 4x^2 + 3x - 5\right) \div \left(x + 3\right)$$

Synthetic

$$\begin{array}{r|rrrr} 3 & 1 & 4 & 3 & -5 \\ & & 3 & 3 & 0 \\ \hline & 1 & 1 & 0 & -5 \end{array}$$

$= (x^2 + x)$ Rem $= {}^{-}5$

$$\begin{array}{r} x^2 + x \\ x+3 \overline{\smash{\big)}\, x^3 + 4x^2 + 3x - 5} \\ \underline{x^3 + 3x^2} \\ x^2 + 3x \\ \underline{x^2 + 3x} \end{array}$$

R = -5

Factor Theorem

Factor Theorem

(x-n) is a factor of f(x) **if and only if** f(n) = 0.

Ex. Use the Factor Theorem to determine whether $(x-1)$ is a factor of $(2x^4+3x^2-5x+7)$.

Sub. x=1 . . .

$2(1)^4+3(1)^2-5(1)+7=7$

Not a factor

Remainder

Is $(x+2)$ a factor of $(x^3+3x^2-4x-12)$?

$x=-2$

$-8+12+8-12=0$

$\therefore (x+2)$ is a factor

24

6 4

2 3

$24 \div 6$

$x^3+3x^2-4x-12$

$x+2$?

x^2+x-6

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 2 & -12 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$(x+2)(x^2+x-6)$

$(x+2)(x+3)(x-2)$

EXAMPLE...

Factor completely $4x^3 - 16x^2 + 19x - 6$

$\pm 1, \pm 2, \pm 3$
± 6

$x = 1$
$4 - 16 + 19 - 6 \neq 0$

$x = -1$
$-4 - 16 - 19 - 6 \neq 0$

$x = 2$
$32 - 64 + 38 - 6 = 0$

$\therefore (x-2)$ is a factor

$$\begin{array}{r|rrrr} -2 & 4 & -16 & 19 & -6 \\ & & -8 & 16 & -6 \\ \hline & 4 & -8 & 3 & 0 \end{array}$$

$(x-2)(4x^2 - 8x + 3)$

12

$4x^2 - 6x - 2x + 3$

$2x(2x-3) - 1(2x-3)$

$\boxed{(x-2)(2x-3)(2x-1)} = 0$

$x = 2, \frac{3}{2}, \frac{1}{2}$

$3.5 / 4$

Warm Up

#1. When a polynomial f(x) is divided by $(3x+2)$ the quotient is $(4x^2-2x+3)$ with a remainder of 6. What is f(x)?

$4x^2-2x+3$

$3x+2\overline{)\ f(x)=??}$

6

$(3x+2)(4x^2-2x+3)+6=f(x)$

$12x^3-6x^2+9x+8x^2-4x+6+6=f(x)$

$\underline{12x^3+2x^2+5x+12=f(x)}$

#2. Find k if the polynomial $f(x)=(x^3-kx^2-5x+3k)$ is divisible by $(x+3)$

$-5-(-3k-9)$

$$\begin{array}{r|rrrr} 3 & 1 & -k & -5 & 3k \\ & & 3 & -3k-9 & 9k+12 \\ \hline & 1 & -k-3 & 3k+4 & 0 \end{array}$$

$\rightarrow 3k-(9k+12)=0$

$-6k-12=0$

$-6k=12$

$\underline{k=-2}$

Factoring Polynomial Expressions

REMEMBER:

Factor Theorem

(x-n) is a factor of f(x) **if and only if** f(n) = 0.

Ex. $6x^3 + 23x^2 + 29x + 12$

$\longrightarrow$ **cubic equations can only have 3 roots!**

Test values that are factors of the constant term:

$x = -1$

$-6 + 23 - 29 + 12 = 0$

$(x+1)$ is a factor

Use division to find another factor:

$$\begin{array}{r|rrrr} 1 & 6 & 23 & 29 & 12 \\ & & 6 & 17 & 12 \\ \hline & 6 & 17 & 12 & 0 \end{array}$$

Factor further (if possible):

$(x+1)(6x^2+17x+12)$ 72

$6x^2+9x+8x+12$

$3x(2x+3)+4(2x+3)$

$$\boxed{(x+1)(3x+4)(2x+3)}$$

$$(x+7)^{*}(x-3)=0$$

$x = {}^{-}7, 3$

Solving Polynomial Equations

The Factor Theorem can be used to identify points where the graph of a function crosses the x-axis.

Solve $16x^3 + 48x^2 - x - 3 = 0$

$x=-3$ $P(-3)=0$

$\therefore (x+3)$ is a factor

$$\begin{array}{r|rrrr} 3 & 16 & 48 & -1 & -3 \\ & & 48 & 0 & -3 \\ \hline & 16 & 0 & -1 & 0 \end{array}$$

$(x+3)(16x^2-1)=0$

$(x+3)(4x-1)(4x+1)=0$

Roots $\rightarrow x=-3, \pm\frac{1}{4}$

Ex. Identify the points where the graph of the function $f(x) = x^3 + 6x^2 - 2x - 5$ crosses the x-axis.

Pg. 125
#14, 15

Pg. 134
#5, 6, 7, 15

$x = 1$

$(x-1)$ is factor

$$\begin{array}{r|rrrr} -1 & 1 & 6 & -2 & -5 \\ & & -1 & -7 & -5 \\ \hline & 1 & 7 & 5 & 0 \end{array}$$

$$(x-1)(x^2+7x+5) = 0$$

$x = 1$

$$x = \frac{-7 \pm \sqrt{49 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-7+\sqrt{29}}{2}, \frac{-7-\sqrt{29}}{2}$$

#14) $mx^3 - 3x^2 + nx + 2$

$\div (x+3)$ $R = -1$

$m(-3)^3 - 3(-3)^2 + n(-3) + 2 = -1$

$-27m - 27 - 3n + 2 = -1$

$\boxed{-27m - 3n = 24}$

$\div (x-2)$ $R = -4$