Sequences and Series

Sequence:
A pattern of numbers in a definite order that follow a certain rule.

Examples of sequences:
1) 1, 2, 3, 4, 5, 6, 7, ... *add 1 to the preceding term*
2) 2, 4, 7, 11, 16, 23, 31. *add 2 to the preceding term, add 3 to the next term, etc*
3) 1, 1, 2, 3, 5, 8, 13, 21, 34,... *add the two preceding terms together*

Series:
The sum of the terms in a sequence.

Using the above sequences, we have the following series:
1) 1 + 2 + 3 + 4 + 5 + 6 + 7 +...
2) 2 + 4 + 7 + 11 + 16 + 23 + 31.
3) 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 +...

Finite Sequence or Series:
Comes to a definite end

2 + 4 + 7 + 11 + 16 + 23 + 31

Infinite Sequence or Series:
Continue indefinitely

1, 1, 2, 3, 5, 8, 13, 21, 34,...

Notation of Sequences and Series:

$-5, 0, 5, 10, 15, 20, \ldots$ t_1

- Each element of a sequence or series is referred to as a term.

$t_1, t_2, t_3, t_4, t_5, t_6, \ldots, t_{n-1}, t_n$ ← Referred to as the n^{th} term or the General Term.

General Term:
An equation or formula used to determine the values of the terms in a sequence.

Example: $t_n = 5n - n^2$ What is the 15th term in this sequence?

$t_{15} = 5(15) - (15)^2$
$= 75 - 225$
$= -150$

What are the first 3 terms?
$t_1 = 5(1) - (1)^2$
$= 4$
$t_2 = 6$ $t_3 = 6$

$4, 6, 6, \ldots$

Recursive Sequence:
A sequence that uses the previous term to come up with each successive term.
- Must be given the first term to develop a recursive sequence.

Example: $t_1 = 4$
$t_n = -2n + 4(t_{n-1} + 1)$

What is the 5th term in this sequence?

$t_2 = -2(2) + 4(4 + 1)$
$= -4 + 20$
$= 16$

$t_3 = -2(3) + 4(16 + 1)$
$= -6 + 68$
$= 62$

$t_4 = -2(4) + 4(62 + 1)$
$= -8 + 252$
$= 244$

$t_5 = -2(5) + 4(244 + 1)$
$= -10 + 980$
$= 970$

Examples:

1. Determine the fifth and tenth terms of the sequence defined by the following general term: $t_n = 20n - 5(1 - n^2)$ ← Non-Recursive

$t_5 = 20(5) - 5(1 - (5)^2)$
$= 100 - (-120)$
$= 220$

$t_{10} = 20(10) - 5(1 - 10^2)$
$= 200 + 495$
$= 695$

2. Determine the eighth term of the recursive sequence with an initial term of -5 and defined by the general term: $t_n = 5t_{n-1} + 2n$ ← Recursive

$t_2 = 5(-5) + 2(2)$
$= -21$

$t_3 = 5(-21) + 2(3)$
$= -99$

$t_4 = 5(-99) + 2(4)$
$= -487$

$t_5 = 5(-487) + 2(5)$
$= -2425$

$t_6 = 5(-2425) + 2(6)$
$= -12\,113$

$t_7 = 5(-12113) + 2(7)$
$= -60\,551$

$t_8 = 5(-60551) + 2(8)$
$= -302\,739$

$t_1 = -1,\ t_2 = 3$

$\boxed{t_4 = ?}$

$t_n = -3(n+1) - 2t_{n-2} + (t_{n-1})^2$

$t_3 = -3(3+1) - 2(-1) + (3)^2$
$= -12 + 2 + 9$
$= -1$

$t_4 = -3(4+1) - 2(3) + (-1)^2$
$= -15 - 6 + 1$
$= -20$

Famous Sequence...

Fibonacci Numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

$$F_n = F_{n-1} + F_{n-2}$$

(ie. The 8th Fibonacci number would be the sum of the 6th and 7th Fibonacci numbers, etc.)

History

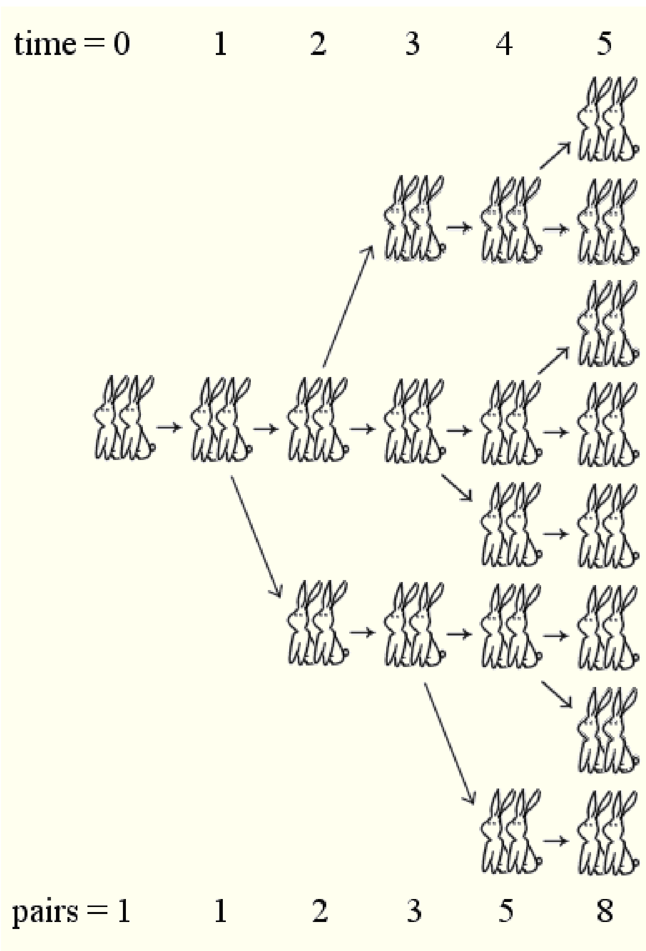
Fibonacci was known in his time and is still recognized today as the "greatest European mathematician of the middle ages." born in the 1170's and died in the 1240's and there is now a statue commemorating him located at the Leaning Tower end o cemetery next to the Cathedral in Pisa.

His full name was Leonardo of Pisa, or Leonardo Pisano in Italian since he was born in Pisa. He called himself Fibonacci w short for Filius Bonacci, standing for "son of Bonacci", which was his father's name.

In Fibonacci's day, mathematical competitions and challenges were common. For example, in 1225 Fibonacci took part in a tournament at Pisa ordered by the emperor himself, Frederick II.

It was in just this type of competition that the following problem arose:

Beginning with a single pair of rabbits, if every month each *productive* pair bears a new pair, which becomes productive when they are 1 month old, how many rabbits will there be after *n* months?



Homework

Worksheet

Check Up

1. Determine the 5th term for each of the following sequences:

(a) $t_n = 2(n-3)^2 - 2n^3$

$t_5 = 2(5-3)^2 - 2(5)^3$

$= 2(4) - 250$

$= -242$

t_{n-2}

$t_{5-2} = t_3$

(b) $t_1 = -4$, $t_2 = 1$ $\rightarrow$ t_{3-2} t_1

$t_n = 5(n - t_{n-2}) + 4t_{n-1}$

$t_3 = 5(3-(-4)) + 4(1)$

$= 5(7) + 4$

$= 39$

$t_4 = 5(4 - t_2) + 4t_3$

$= 5(4-1) + 4(39)$

$= 15 + 156$

$= 171$

$t_5 = 5(5-39) + 4(171)$

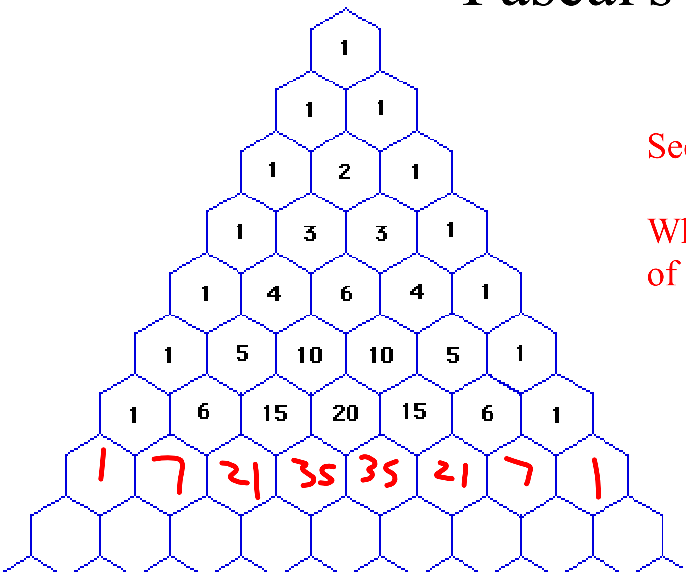
n

$= 5(-34) + 684$

$= 514$

Another famous number pattern...

Pascal's Triangle



See the pattern???

What are the next two rows of Pascal's Triangle?

$(x+2)^7$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Arithmetic and Geometric Sequences

Arithmetic Sequence:
A pattern of numbers in which each term is found by adding or subtracting the preceding term by the same amount.

Common Difference (d) :
Difference between successive terms

Initial Term (a or t_1) :
First term in the sequence

Examples: t_{107}

1) -3, 1, 5, 9, 13, 17, ...

$d = 4$

$t_1 = -3$ or $a = -3$

2) 2, 0, -2, -4, -6, ...

$d = -2$

$t_1 = 2$

$t_{73} = a + 72d$

In general, any arithmetic sequence can be represented as follows...

t_1	t_2	t_3	t_4	t_5	t_6		t_n	
a ,	$a + d$,	$a + 2d$,	$a + 3d$,	$a + 4d$,	$a + 5d$,	...,	$a + (n - 1)d$,	...

General Term of any Arithmetic Sequence

$$t_n = \overset{t_1}{a} + (n - 1)d$$

Geometric Sequence:
A pattern of numbers in which each term is found by multiplying or dividing the preceding term by the same amount.

Common ratio (r) : $\Rightarrow$ calculate $\frac{t_2}{t_1}$
The multiplier between any two pairs of successive terms.

Initial Term (a or t_1) :
First term in the sequence

Examples: $(ar)(r)$

1) -3, 6, -12, 24, -48, 96, ...

$r = -2$

2) $9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

$r = \frac{1}{3}$

In general, any ~~arithmetic~~ geometric sequence can be represented as follows...

$\times r \cdots \rightarrow$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_n$

$a \overset{\times r}{,} ar \overset{\times r}{,} ar^2, ar^3, ar^4, ar^5, \ldots, ar^{n-1}, \ldots$

$t_{77} = ar^{76}$

General Term of any Geometric Sequence

$$t_n = ar^{n-1}$$

Examples:

1. Determine the general term for each of the following sequences:

(a) -7, -2, 3, 8, 13, ... Arithmetic

$-2-(-7)=5$

$3-(-2)=5$

$t_n=t_1+(n-1)d$

$t_n=-7+(n-1)(5)$

$t_n=-7+5n-5$

$\boxed{t_n=5n-12}$

$t_{478}=?$

$t_{478}=5(478)-12$

$=2378$

(b) $-\frac{1}{3}, \frac{1}{18}, -\frac{1}{108}, \frac{1}{648}, \ldots$ $r=-\frac{1}{6}$

$t_n=t_1r^{n-1}$

$t_n=-\frac{1}{3}\left(-\frac{1}{6}\right)^{n-1}$

2. How many terms would be in each of the following sequences?

(a) 3, 1, -1, -3, -5, ... , -1557 $t_{??}$ A $d=-2$ t_{781}

$t_n=t_1+(n-1)d$

$t_n=3+(n-1)(-2)$

$t_n=3-2n+2$

$t_n=5-2n$

$-1557=5-2n$

$\frac{-1562}{-2}=\frac{-2n}{-2}$

$n=781$

(b) -2, 6, -18, 54, ... , 39 366 G

$r=-3$

$t_n=-2(-3)^{n-1}$

$\frac{39366}{-2}=\frac{-2(-3)^{n-1}}{-2}$

$-19683=(-3)^{n-1}$

$3^{?}=19683$

$\log_3 19683=\frac{\log 19683}{\log 3}$

$=9$

$(-3)^9=(-3)^{n-1}$

$9=n-1$

$n=10$

3. (a) Determine the 455 th term of the following arithmetic sequence:

$t_{19} = 34$ and $t_{873} = 1742$ *(Hint: Systems of equations might help???)*

$t_{19} = t_1 + 18d = 34$

$t_{873} = t_1 + 872d = 1742$

$t_n = t_1 + (n-1)d$

(b) The fifth term of a geometric sequence is 40 and the eleventh term is $\frac{5}{8}$. Determine the 20th term in this sequence.

(Express your answer as a fraction!!)

Attachments

4.1 Page 206 Questions.pdf

Introductory worksheet.doc