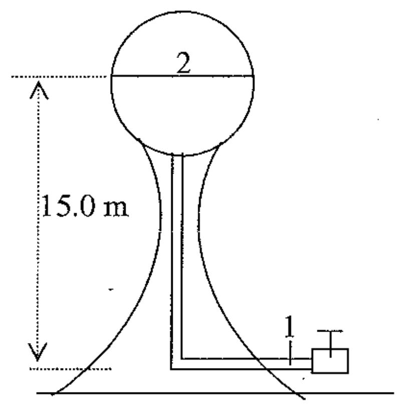
Science 122

Problems – Continuity and Bernoulli Equations

1. The radius of the aorta is about 1.0 cm and the blood passing through it has a speed of about 30 cm/s. A typical capillary has a radius of about 4.0×10^{-4} cm, and blood flows through it at a speed of about 5.0×10^{-4} m/s. Estimate how many capillaries there are in the body. (3.8×10^{9})

2. What are the dimensions of a square heating duct if air moving at 3.0 m/s along it can replenish air every 15 minutes in a room of 300 m^3 volume? Assume the air's density remains constant. (33 cm x 33 cm)

3. Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6 cm diameter pipe on the second floor 5.0 m above? Let the basement be point 1. (1.2 m/s, 2.5×10^{5} N/m^2)



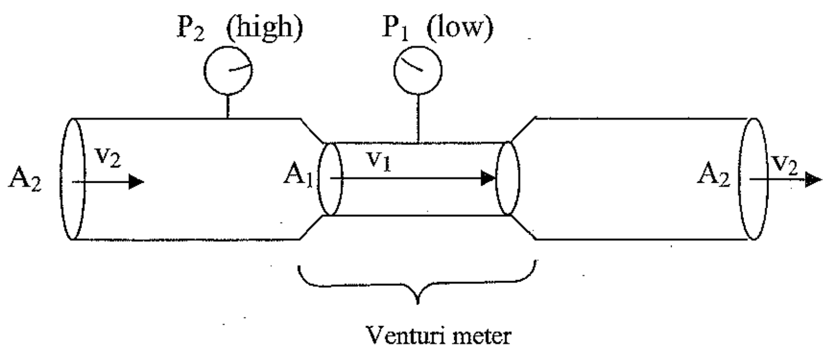
4. The water tower in the drawing is drained by a pipe that extends to the ground. The flow is nonviscous. The top surface of the water at point 2 is at atmospheric pressure.
   a) What is the absolute pressure at point 1 if the valve is closed? (2.48×10^{5} Pa)
   b) Pressure at point 1 becomes equal to atmospheric pressure when the valve is opened. Assuming the effective cross-sectional area of the valve opening is 2.00×10^{-2} m^2, find the volume flow rate at point 1. (0.342 m^3/s)

5. The construction of a flat rectangular roof (5.0 m x 6.3 m) allows it to withstand a maximum net outward force of 22 000 N. The density of the air is 1.29 kg/m^3. Assuming the top and bottom surfaces of the roof are at the same height, at what wind speed will this roof blow outward? (33 m/s)

6. A liquid is flowing through a horizontal pipe whose radius is 0.0200 m. The pipe bends straight upward through a height of 10.0 m and joins another horizontal pipe whose radius is 0.0400 m. What volume flow rate will keep the pressures in the two horizontal pipes the same? (1.81×10^{-2} m^3/s)

7. A Venturi meter is a device for measuring the speed of a fluid within a pipe. The drawing shows a gas flowing at speed v_2 through a horizontal section of pipe whose cross-sectional area is $A_2 = 0.0700\ m^2$. The gas has a density of $\rho = 1.30\ kg/m^3$. The Venturi meter has a cross-sectional area of $A_1 = 0.0500\ m^2$ and has been substituted for a section of the larger pipe. The pressure difference between the two sections is $P_2 - P_1 = 120$ Pa.

   a) Find the speed v_2 of the gas in the larger original pipe. (14 m/s)
   b) Find the volume flow rate Q of the gas. ($0.98\ m^3/s$)



[Fluids - Continuity + Bernoulli Equations ~~Extra Practice~~]

1. $r_a = 1.0\,cm = 0.010\,m$

$V_a = 30\,\frac{cm}{s} \times \frac{10^{-2}\,m}{1\,cm} = 0.30\,\frac{m}{s}$

$r_c = 4.0\times10^{-4}\,cm \times \frac{10^{-2}\,m}{cm} = 4.0\times10^{-6}\,m$

$V_c = 5.0\times10^{-4}\,m/s$

$$A_a v_a = n A_c v_c$$

$$n = \frac{A_a v_a}{A_c v_c}$$

$$n = \frac{\pi r_a^2 v_a}{\pi r_c^2 v_c}$$

$$n = \frac{(0.010\,m)^2(0.30\,m/s)}{(4.0\times10^{-6}\,m)^2(5.0\times10^{-4}\,m/s)}$$

$$\boxed{n = 3.8\times10^{9}}$$

2. $V = 300\,m^3$

$t = 15\text{ minutes} \times \frac{60\,s}{1\,min} = 900\,s.$

$A = ?$

$v = 3.0\,m/s$

room $\left\{\frac{V}{t} = Av\right\}$ duct

$$A = \frac{V}{vt}$$

$$x^2 = \frac{V}{vt}$$

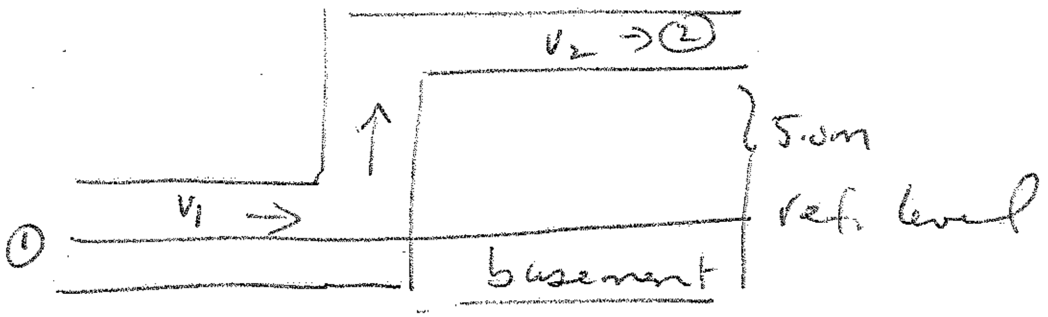
$$x = \sqrt{\frac{V}{vt}}$$

$$x = \sqrt{\frac{300\,m^3}{(3.0)(900)}}$$

$$x = 0.33\,m$$

$$\boxed{33\,cm \times 33\,cm}$$

3.



$P_1 = 3.0 \text{ atm} = 3.03 \times 10^5 \text{ Pa}$

$d_1 = 4.0 \text{ cm} = 0.040 \text{ m} \rightarrow r_1 = 0.020 \text{ m}$

$v_1 = 0.50 \text{ m/s}$

$d_2 = 2.6 \text{ cm} = 0.026 \text{ m} \rightarrow r_2 = 0.013 \text{ m}$

fluid - water $\rho = 1000 \text{ kg/m}^3$

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$v_2 = \frac{\pi r_1^2 v_1}{\pi r_2^2}$$

$$v_2 = \frac{(0.020)^2 (0.50)}{(0.013)^2}$$

$$v_2 = 1.2 \text{ m/s}$$ ← flow speed 2nd floor

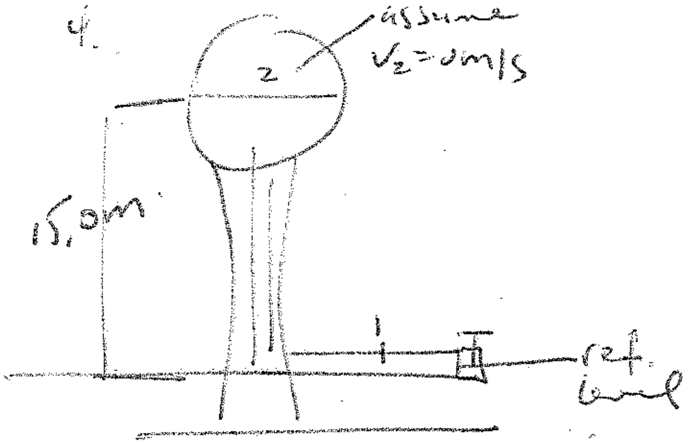
$$P_1 + \cancel{\rho g y_1} + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \rho g y_2 - \frac{1}{2}\rho v_2^2$$

$$P_2 = 3.03 \times 10^5 + \frac{1}{2}(1000)(0.50)^2 - (1000)(9.80)(5.0) - \frac{1}{2}(1000)(1.2)^2$$

$$P_2 \approx 2.5 \times 10^5 \text{ Pa}$$

4.



a) $P_1 = P_2 + \rho g h$

$P_1 = 1.01\times10^5 + (1000)(9.80)(15.0)$

$P_1 = 2.48\times10^5\ Pa$

b) $Q_1 = A_1 v_1$, $P_1 = P_2$

$\cancel{P_1} + \rho g y_1 + \frac{1}{2}\rho v_1^2 = \cancel{P_2} + \rho g y_2 + \frac{1}{2}\cancel{\rho v_2^2}$

$\frac{1}{2}\rho v_1^2 = \cancel{\rho} g y_2$

$v_1^2 = 2 g y_2$

$v_1 = \sqrt{2 g y_2}$

$v_1 = \sqrt{2(9.80)(15.0m)}$

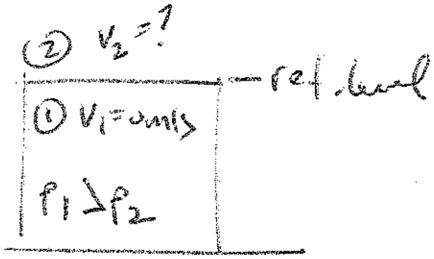
$v_1 = 17.15\ m/s$

$Q_1 = A_1 v_1$

$Q_1 = (2.00\times10^{-2})(17.15)$

$Q_1 = 0.343\ m^3/s$

5.



$F_{net} = 22\,000\ N$

$\rho_{air} = 1.29\ kg/m^3$

$(5.0m \times 8.3m)$

* $P = F/A$

* $\Delta P = \frac{F_{net}}{A}$

$P_1 + \cancel{\rho g y_1} + \frac{1}{2}\cancel{\rho v_1^2} = P_2 + \cancel{\rho g y_2} + \frac{1}{2}\rho v_2^2$

$P_1 = P_2 + \frac{1}{2}\rho v_2^2$

$P_1 - P_2 = \frac{1}{2}\rho v_2^2$

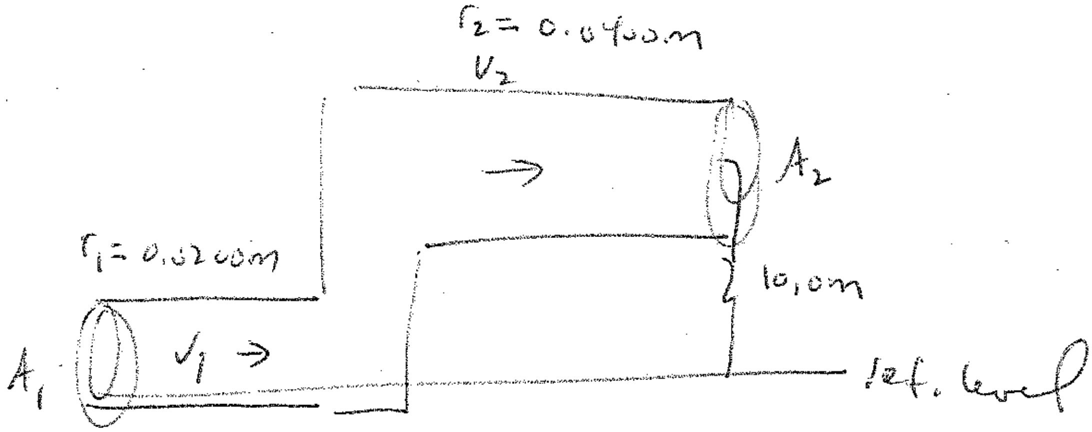
$v_2^2 = \frac{2(\Delta P)}{\rho}$

$v_2 = \sqrt{\frac{2\Delta P}{\rho}}$

$v_2 = \sqrt{\frac{2}{\rho}\left(\frac{F_{net}}{A}\right)}$

$v_2 = 33\ m/s$

6.



$A_1 < A_2$

$V_1 > V_2$

In this case, $P_1 = P_2$ because of volume flow rate.

- Use Bernoulli's to get v_1 or v_2 then Q.

$$2\left(\cancel{P_1} + \rho g \cancel{y_1} + \frac{1}{2}\rho v_1^2 = \cancel{P_2} + \rho g y_2 + \frac{1}{2}\rho v_2^2\right)$$

$$\rho v_1^2 = 2\rho g y_2 + \rho v_2^2$$

$$v_1^2 = 2 g y_2 + v_2^2$$

↑ Use Continuity Eq. to get expression for v_2^2

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$v_2^2 = \frac{A_1^2 v_1^2}{A_2^2}$$

$$v_1^2 = 2 g y_2 + \frac{A_1^2 v_1^2}{A_2^2}$$

$$v_1^2 - \frac{A_1^2 v_1^2}{A_2^2} = 2 g y_2$$

$$v_1^2\left(1 - \frac{A_1^2}{A_2^2}\right) = 2 g y_2$$

$$v_1 = \sqrt{\frac{2 g y_2}{\left(1 - \frac{A_1^2}{A_2^2}\right)}}$$

$$v_1 = \sqrt{\frac{2(9.80)(10.0)}{1-\left(\frac{\pi^2 (r_1^2)^2}{\pi^2 (r_2^2)^2}\right)}} = 14.46\,m/s$$

$Q = A_1 v_1$

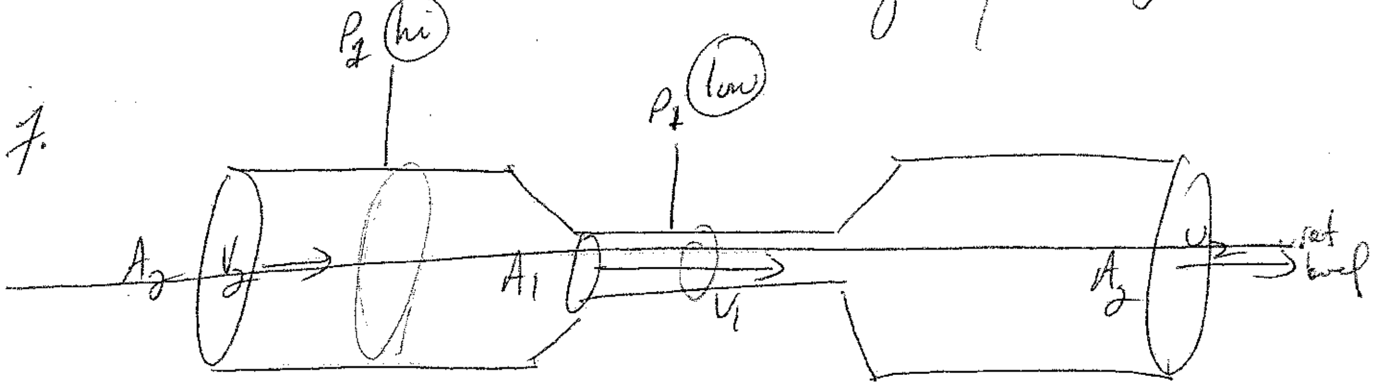
$Q = \pi r_1^2 v_1$

$Q = \pi(0.0200)^2(14.46)$

$Q = 1.82\times10^{-2}\ \frac{m^3}{s}$

gas $\rho = 1.30 \, kg/m^3$

7.



$A_2 = 0.0700 m^2$ $\qquad$ $A_1 = 0.0500 m^2$

$P_2 - P_1 = 120 \, Pa$

a) $P_1 + \cancel{\rho g y_1} + \frac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g y_2} + \frac{1}{2}\rho v_2^2$

$A_1 v_1 = A_2 v_2$ $\qquad$ $\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$

$v_1 = \frac{A_2 v_2}{A_1}$

$v_1 = \frac{0.0700}{0.0500} v_2$

$v_1 = 1.40 v_2$

$P_1 + \frac{1}{2}\rho(1.40 v_2)^2 = P_2 + \frac{1}{2}\rho v_2^2$

$2P_1 + \rho(1.40^2 v_2)^2 = 2P_2 + \rho v_2^2$

$1.40^2 \rho v_2^2 - \rho v_2^2 = 2P_2 - 2P_1$

$v_2^2 = \frac{2P_2 - 2P_1}{1.40^2 \rho - \rho}$

$v_2 = \sqrt{\frac{2(120)}{(1.30)(1.40^2 - 1)}}$

$v_2 = 14 \, m/s$

b) $Q = A_2 v_2 = (0.0700)(14) = 0.98 \, m^3/s$