Science 122

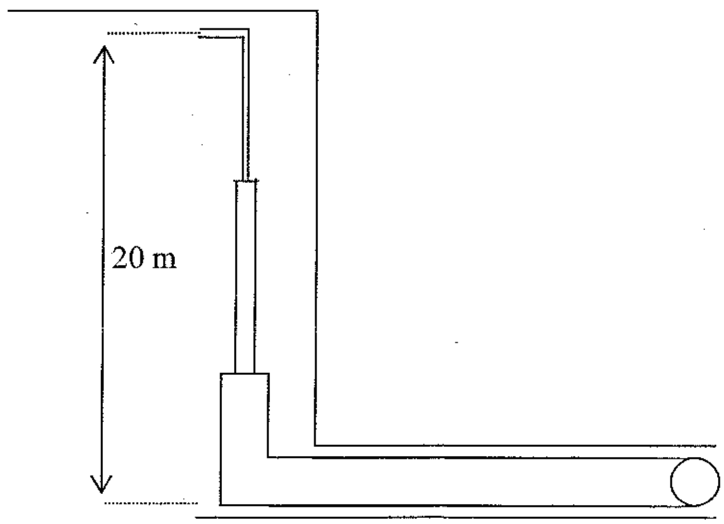
Fluids – Continuity and Bernoulli Equation #2

1. A typical mass flow rate for the Mississippi River is 1.8×10^7 kg/s.
   a) Find the volume flow rate. (1.8×10^4 m^3/s)
   b) Find the flow speed in a region where the river is 2.0 km wide and an average of 6.1 m deep. (1.5 m/s)

2. A 17 cm radius air duct is used to replenish the air of a room 9.2 m x 5.0 m x 4.5 m every 10 minutes. How fast does the air flow in the duct? (3.8 m/s)

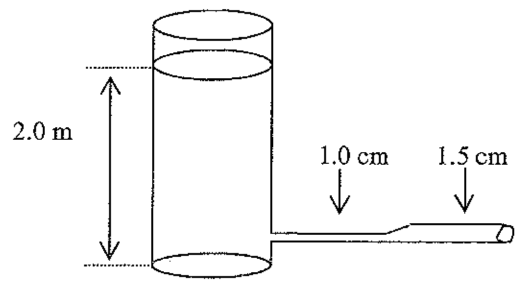
3. If wind blows at 30 m/s over your house, what is the net force on the flat roof if its area is 240 m^2? (1.4×10^5 N)

4. Water at a pressure of 3.8 atm at street level flows into an office building at a speed of 0.60 m/s through a pipe 5.0 cm in diameter. The pipe tapers down to 2.6 cm in diameter by the top floor, 20 m above. Calculate the flow velocity and the pressure in such a pipe on the top floor. Ignore viscosity. (2.2 m/s, 1.8 atm)



5. A 1.0 cm diameter Venturi flowmeter is inserted in a 2.0 cm diameter pipe carrying water. The pressure difference between the flowmeter and pipe is 17 kPa.
   a) What is the flow speed in the pipe? (1.5 m/s)
   b) What is the volume flow rate? (4.7×10^{-4} m^3/s)

6. A 1.0 m diameter tank is filled with water to a depth of 2.0 m and is open to the atmosphere at the top. The water drains through a 1.0 cm diameter pipe at the bottom; that pipe then joins a 1.5 cm diameter pipe open to the atmosphere. Find the flow speed in the 1.0 cm section of pipe. (14 m/s)



1. $\frac{m}{t} = 1.8 \times 10^7 \, kg/s$

a) $Q = \frac{V}{t} = Av$

$$\frac{m}{t} = \rho A v$$

$$Av = \frac{m}{\rho t}$$

$$Av = \frac{1.8\times10^7 \, kg/s}{1000}$$

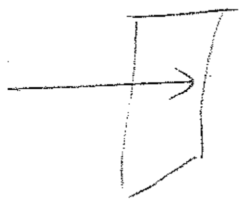
$$\boxed{Av = 1.8 \times 10^4 \frac{m^3}{s} \Rightarrow Q}$$

b) $Q = Av$

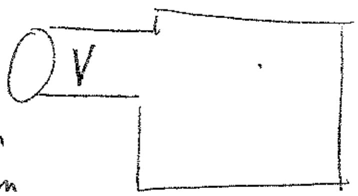
$$v = \frac{Q}{A}$$

$$v = \frac{1.8\times10^4 \frac{m^3}{s}}{(2.0\times10^3 \times 6.1 m)}$$

$$\boxed{v = 1.5 m/s}$$



2.



r=17cm
r=0.17m

t=10 minutes

$$Q = \frac{V}{t} = Av$$

$$A_d v_d = \frac{V_{room}}{t}$$

$$v_d = \frac{V_{room}}{t A_d}$$

$$v_d = \frac{9.2 \times 5.0 \times 4.5}{\left(10 \times \frac{60s}{1m}\right)\left(\pi (0.17)^2\right)}$$

$$\boxed{v_d = 3.8 m/s}$$

$P = \frac{F}{A}$

3.

out $v_2 = 30\,m/s$, P_2

in $v_1 = 0\,m/s$, P_1 ref. level

$\left[P_1 - P_2\right] = \frac{F_{net}}{A}$

$F_{net} = A(P_1 - P_2)$

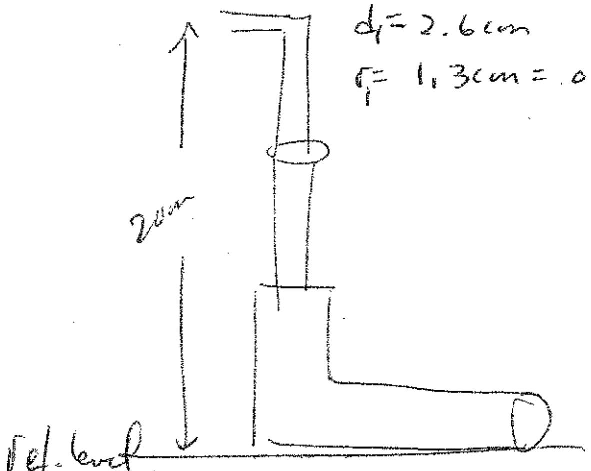
$P_1 + \cancel{\frac{1}{2}\rho v_1^2} + \cancel{\rho g y_1} = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}$

$P_1 - P_2 = \frac{1}{2}\rho v_2^2$

$F_{net} = (240)\left(\frac{1}{2}(1.29)(30\,m/s)^2\right)$

$\boxed{F_{net} = 1.4\times10^{5}\,N}$

4.



$d_1 = 2.6\,cm$

$r_1 = 1.3\,cm = 0.013\,m$

$P_1 = ?$

$v_1 = ?$

$v_2 = 0.60\,m/s$

$d_2 = 5.0\,cm$

$r_2 = 2.5\,cm = 0.025\,m$

$P_2 = 3.8\,atm = 3.8\times(1.01\times10^5)\,Pa$

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}$

$2P_1 + \rho v_1^2 + 2\rho g y_1 = 2P_2 + \rho v_2^2$

$2P_1 = 2P_2 + \rho v_2^2 - 2\rho g y_1 - \rho v_1^2$

$P_1 = \frac{2P_2 + \rho v_2^2 - 2\rho g y_1 + \rho v_1^2}{2}$

$A_1 v_1 = A_2 v_2$

$v_1 = \frac{A_2 v_2}{A_1}$

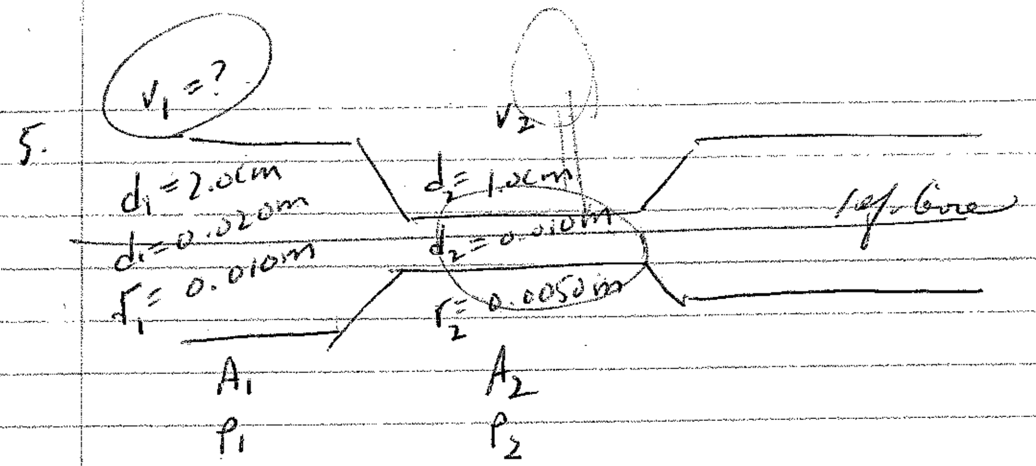
$v_1 = \frac{\pi(0.025)^2(0.60)}{\pi(0.013)^2}$

$\boxed{v_1 = 2.2\,m/s}$

$P_1 = \frac{2(3.8\times1.01\times10^5) + (1000)(0.60)^2 - 2(1000)(9.80)(20) + (1000)(2.2)^2}{2}$

$P_1 = 185560\,Pa \times \frac{1\,atm}{1.01\times10^5} = \boxed{1.8\,atm}$

5.



$P_1 > P_2$

$\rho A v$

a) $P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g y_1} = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}$

$$2P_1 + \rho v_1^2 = 2P_2 + \rho v_2^2$$

$$A_1 v_1 = A_2 v_2$$

$$\cancel{v_1 = \frac{A_2 v_2}{A_1}}$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$v_2 = \frac{\pi(0.010)^2 v_1}{\pi(0.0050)^2}$$

$$v_2 = 4v_1$$

$$2P_1 + \rho v_1^2 = 2P_2 + \rho(4v_1)^2$$

$$2P_1 - 2P_2 = \rho(4v_1)^2 - \rho v_1^2$$

$$2(P_1 - P_2) = 16\rho v_1^2 - \rho v_1^2$$

$$v_1^2 = \frac{2(P_1 - P_2)}{16\rho - \rho}$$

$$v_1 = \sqrt{\frac{2(17 \times 10^3)}{16(1000) - 1000}}$$

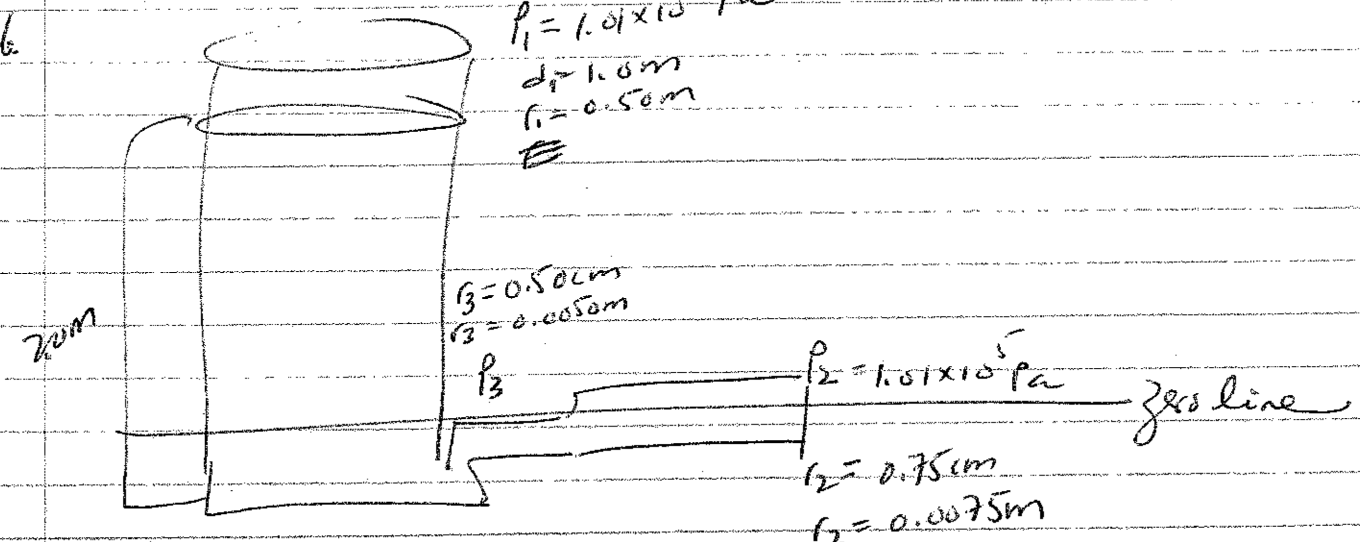
$$v_1 = 1.5 \, m/s$$

b) $Q_1 = A_1 v_1$

$Q_1 = (\pi)(0.010)^2(1.5)$

$Q_1 = 4.7 \times 10^{-4} \frac{m^3}{s}$

6.



$$\cancel{P_1} + \cancel{\tfrac{1}{2}\rho v_1^2} + \cancel{\rho} g y_1 = \cancel{P_2} + \tfrac{1}{2}\cancel{\rho} v_2^2 + \cancel{\rho g y_2}$$

$$g y_1 = \frac{1}{2} v_2^2$$

$$\sqrt{2 g y_1} = v_2$$

$$v_2 = \sqrt{2(9.80)(2.0)}$$

$$v_2 = 6.26 \text{ m/s}$$

$$A_3 v_3 = A_2 v_2$$

$$v_3 = \frac{A_2 v_2}{A_3}$$

$$v_3 = \frac{\cancel{\pi} r_2^2 v_2}{\cancel{\pi} r_3^2}$$

$$v_3 = \frac{(0.0075)^2 (6.26)}{(0.0050)^2}$$

$$v_3 = 14 \text{ m/s}$$