Test: $\cos^2 x^5$ $(\cos x^5)^2$

(b) $$y' = \frac{\left[2(\cos x^5)^1(-\sin x^5 (5x^4)) + \sec\sqrt{x}\tan\sqrt{x}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)\right](\text{Den}) - (\text{Num})\left[3(3x-\cot x^7)^2(3+\csc^2 x^7(7x^6))\right]}{(\text{Den})^2}$$

2. $f(x)=\sqrt{x^3-4}$ $f''(2)$

$$f'(x)=\underbrace{\frac{1}{2}(x^3-4)^{-\frac{1}{2}}}\underbrace{(3x^2)}$$

$$f''(x)=-\frac{1}{4}(x^3-4)^{-\frac{3}{2}}(3x^2)(3x^2)+\frac{1}{2}(x^3-4)^{-\frac{1}{2}}(6x) *$$

$$f''(2)=-\frac{1}{4}(4)^{-\frac{3}{2}}(12)(12)+\frac{1}{2}(4)^{-\frac{1}{2}}(12)$$

$$=-\frac{1}{\cancel{4}}\left(\frac{1}{8}\right)(\cancel{12}^{3})(12)+\cancel{\frac{1}{2}}\cancel{\left(\frac{1}{2}\right)}(\cancel{12}^{3})$$

$$=-\frac{36}{8}+\frac{3}{1}$$

$$=-\frac{36}{8}+\frac{24}{8}$$

$$=-\frac{12}{8}=-\frac{3}{2}$$

$$3x^2y^4 + x^3(4y^3)\frac{dy}{dx} - 15 = 3\frac{dy}{dx} + 10x$$

$$(4x^3y^3 - 3)\frac{dy}{dx} = 10x - 3x^2y^4 + 15$$

at $(2,-1)$

$$\frac{dy}{dx} = \frac{10x - 3x^2y^4 + 15}{4x^3y^3 - 3}$$

$$m = \frac{20 - 12 + 15}{-32 - 3}$$

$$m = -\frac{23}{35}$$

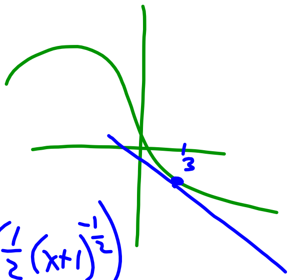
$y = mx + b$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -\frac{23}{35}(x - 2)$$

$$35y + 35 = -23x + 46$$

$$\boxed{23x + 35y - 11 = 0}$$



$x=3$

$$f'(x)=\frac{2x\sqrt{x+1}-x^2\left(\frac{1}{2}(x+1)^{-\frac{1}{2}}\right)}{x+1}$$

$$f'(3)=\frac{6(2)-9\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{4}$$

$$=\frac{\frac{12}{1}-\frac{9}{4}}{4}$$

slope

$$=\frac{39}{4}\cdot\frac{1}{4}=\frac{39}{16}$$

Point: $x=3$

$$f(3)=\frac{9}{\sqrt{3+1}}=\frac{9}{2}$$

$\left(3,\frac{9}{2}\right)$

$$y-y_1=m(x-x_1)$$

$$(16)\quad y-\overset{(16)}{\frac{9}{2}}=\frac{39}{\cancel{16}}\overset{(\cancel{16})}{}(x-3)$$

$$16y-72=39x-117$$

$$0=39x-16y-45$$

5. $y'=6x^2+12x-12$

$$\frac{6}{6}=\frac{6x^2}{6}+\frac{12x}{6}-\frac{12}{6}$$

$$1=x^2+2x-2$$

$$0=x^2+2x-3$$

$$0=(x+3)(x-1)$$

$x=-3,\ 1$

Sub. $y=\ldots$

$y=37$ $\quad$ $y=-3$

$(-3,37)$ $\quad$ $(1,-3)$

6.

$$f'(x) = 12x^2(8-x)^4 + 4x^3\left[4(8-x)^3(-1)\right]$$

$$0 = 12x^2(8-x)^4 - 16x^3(8-x)^3$$

$$0 = 4x^2(8-x)^3\left[3(8-x)^1 - 4x\right]$$

$$0 = 4x^2(8-x)^3(24-7x)$$

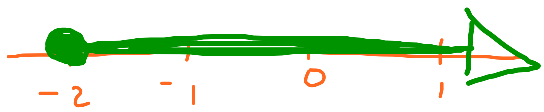
$4x^2 = 0$ $\quad$ $(8-x)^3 = 0$ $\quad$ $24 - 7x = 0$

$x = 0$ $\quad$ $x = 8$ $\quad$ $24 = 7x$

$$\frac{24}{7} = \frac{7x}{7}$$

$$x = \frac{24}{7}$$

Bracket Notation



$(-1, 2]$

$x \geq -2$

$[-2, \infty)$

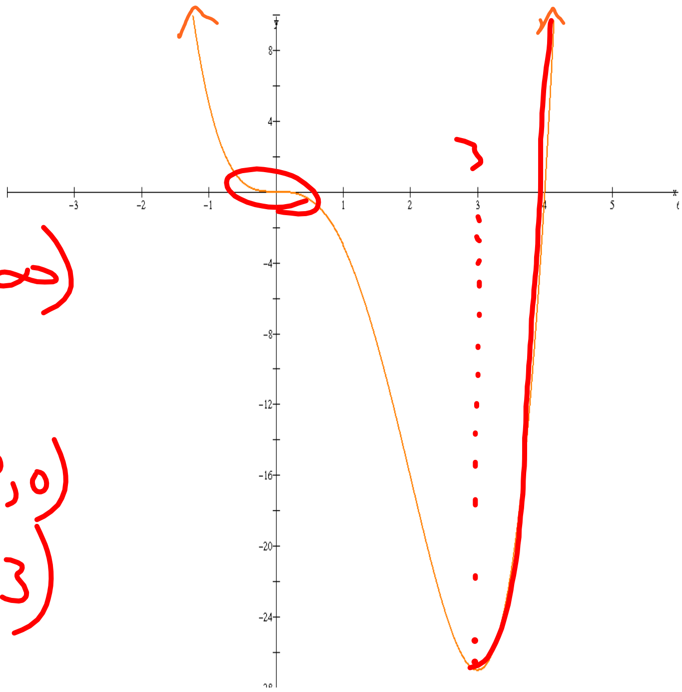
$(-\infty, 3)$

$(2, 3)$

Intervals of Increase and Decrease...

Given the function $f(x) = x^4 - 4x^3$, use the graph below to determine the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing.

Graph...

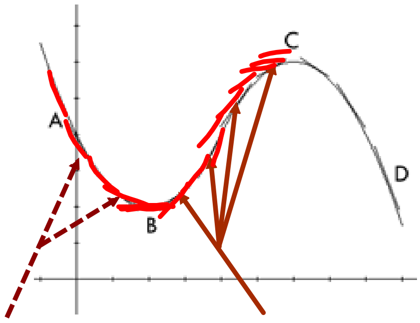


Inc: $(3, \infty)$

Dec: $(-\infty, 0)$
$(0, 3)$

The Calculus of Intervals of Increase and Decrease

- Examine this graph for intervals of increase and decrease...



What do you notice about the slopes of these tangents?

What do you notice about the slopes of these tangents?

Intervals of increase?

$f'(x) > 0$

Intervals of decrease?

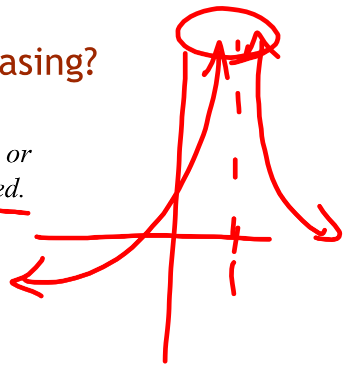
$f'(x) < 0$

Critical Value(s):

Any value of x such that $f'(x) = 0$ or $f'(x)$ is undefined.

Where does $f(x)$ switch from increasing to decreasing?
How would this tie in with Calculus?

- *At the point where a function switches from increasing to decreasing, or decreasing to increasing, the derivative must be equal to 0 or undefined.*

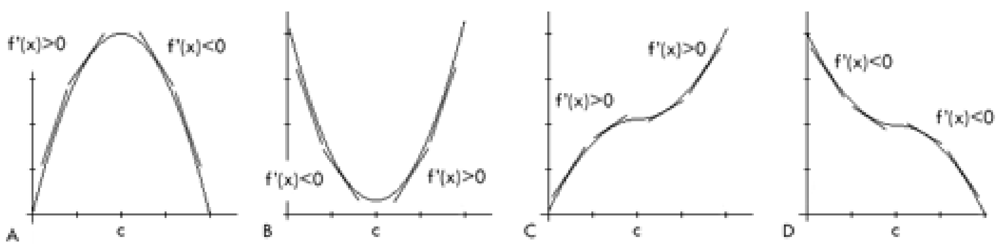


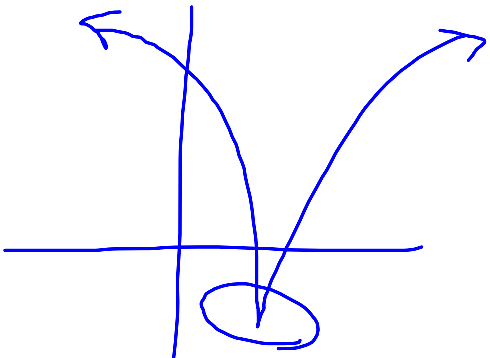
Let's summarize how Calculus could be used to identify regions of increase or decrease...

If $f'(x) > 0$ ------→ Increasing

If $f'(x) < 0$ ------→ Decreasing

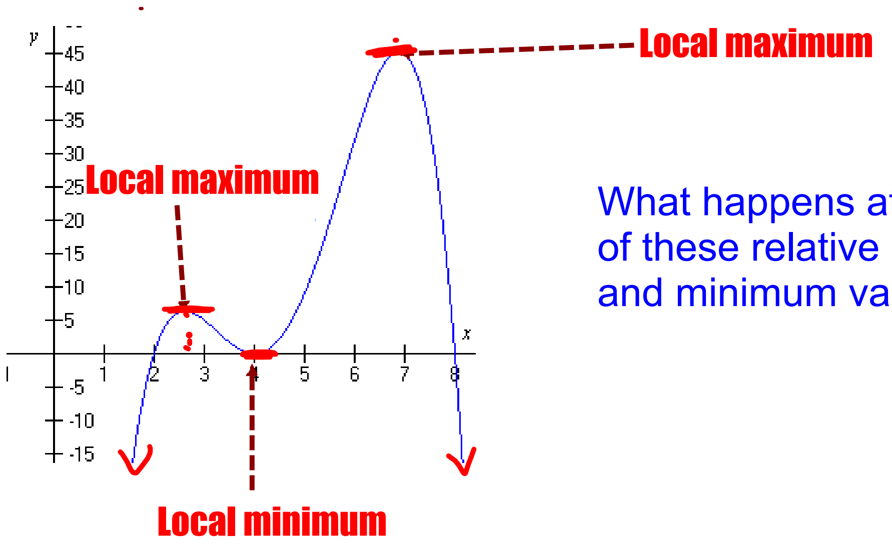
The graphs below illustrate the first derivative test.





$f'(x)$ is undefined

Relative Extrema: (local maximum or local minimum)



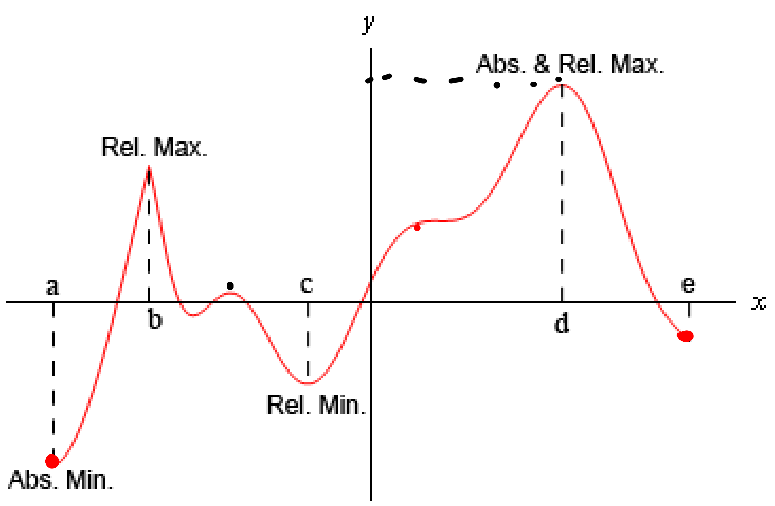
What happens at each of these relative maximum and minimum values?

We say that *f(x)* has a **relative (or local) maximum** at $x = c$ if $f(x) \le f(c)$ for every x in some open interval around $x = c$.

- function switches from increasing to decreasing.

We say that *f(x)* has a **relative (or local) minimum** at $x = c$ if $f(x) \ge f(c)$ for every x in some open interval around $x = c$.

- function switches from decreasing to increasing.



First Derivative Sign Table:

Intervals of increase and decrease can be organized using a first derivative sign table...

Example:

Determine the intervals of increase and decrease for the function ...

$$f(x) = x^4 - 4x^3 + 2$$

① Critical Values:

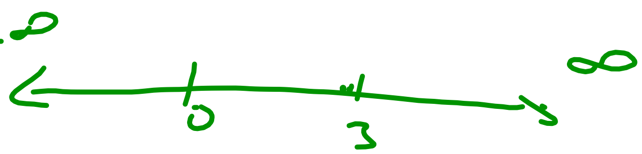
$f'(x) = 4x^3 - 12x^2$

$0 = 4x^3 - 12x^2$

f'

$0 = (4x^2)(x-3)$

$x = 0, 3$



Sign Table

Intervals	$4x^2$	$x-3$	f'	f
$(-\infty, 0)$	+	−	−	Dec
$(0, 3)$	+	−	−	Dec
$(3, \infty)$	+	+	+	Inc

Local Max.

None

Local Min.: $x=3$