Number of "Things"

* There are 2 "things" in a pair.
There are 12 "things" in a dozen.
There are 20 "things" in a score.
There are 500 "things" in a ream.

The Mole

There are 6.02×10^{23} "things" in a mole.

N_A

If we let n be the number of moles and N the number of things, the number of moles is given by:

$$\boxed{n = \frac{N}{N_A}}$$

Also, n can be determined using:

$$\boxed{n = \frac{m}{M}}$$

g

$\frac{g}{mol}$

NOTE:

$$\boxed{m_{particle} = \frac{M}{N_A}}$$

$$\frac{6.02 \times 10^{23}}{mol}$$

$$[6.02 \times 10^{23}]\, mol^{-1}$$

Kinetic Theory of Gases

Kinetic theory makes several assumptions about the behavior of molecules in a gas.

- Molecules move in continuous, random motion.
- There are an exceedingly large number of molecules in any container of gas.
- The separation between individual molecules is large.
- Molecules do not act on one another at a distance; they do not exert electrical or gravitational forces on other molecules.
- All collisions between molecules, or between a molecule and the walls of a container, are elastic (ie/ kinetic energy is not lost in these conditions).

Two results of kinetic theory are particularly important. They are derived using Newtonian mechanics and the assumptions above.

1. The relationship between the internal energy of a gas, U, and its temperature is:

$$\boxed{U = \frac{3}{2}PV = \frac{3}{2}nRT = \frac{3}{2}Nk_BT}$$

$$*PV = nRT = Nk_BT$$

2. The average kinetic energy per particle of gas is given by:

$$\boxed{\overline{E}_k = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT}$$

$$\overline{KE} = \overline{E}_k$$

$$\boxed{v_{rms} = \sqrt{\frac{2\overline{E}_k}{m}}} \qquad \boxed{v_{rms} = \sqrt{\frac{3k_BT}{m}}}$$

*v_{rms} is the root-mean-square (rms) speed of a gas which can be considered to be the average speed of the molecules in a gas.

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Example 6 - The Speed of Molecules in Air (Cutnell - Page 402)

Air is primarily a mixture of N_2 (28.0 g/mol) and O_2 (32.0 g/mol). Assume that each behaves as an ideal gas and determine the rms speeds of the nitrogen and oxygen molecules when the temperature of the air is 293 K.

T = 293 K

$$\boxed{\overline{E}_k = \frac{1}{2}mv_{rms}^2 = \frac{3k_BT}{2}}$$

$$\boxed{v_{rms} = \sqrt{\frac{2\overline{E}_k}{m}}} \qquad \boxed{v_{rms} = \sqrt{\frac{3k_BT}{m}}}$$

$$m_{oxygen} = \frac{M}{N_A}$$

$$m_{oxygen} = \frac{32.0\ g/mol}{6.02 \times 10^{23}\ mol^{-1}}$$

O_2 2(16.0g/mol)

O 16.0 g/mol

$$m_{oxygen} = 5.316 \times 10^{-23}\ g \text{ or } 5.316 \times 10^{-26}\ kg$$

$$m_{nitrogen} = 4.651 \times 10^{-23}\ g \text{ or } 4.651 \times 10^{-26}\ kg$$

oxygen

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

$$v_{rms} = 478\ m/s$$

nitrogen

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

$$v_{rms} = 511\ m/s$$

$$\overline{E}_k = \frac{3k_BT}{2}$$

$$\overline{E}_k = \frac{3(1.38 \times 10^{-23} J/K)(293)}{2}$$

$$\overline{E}_k = 6.065 \times 10^{-21}\ J$$

oxygen

$$v_{rms} = \sqrt{\frac{2\overline{E}_k}{m}}$$

$$v_{rms} = \sqrt{\frac{2(6.065 \times 10^{-21}\ J)}{5.316 \times 10^{-26}\ kg}}$$

$$v_{rms} = 478\ m/s$$

nitrogen

$$v_{rms} = \sqrt{\frac{2\overline{E}_k}{m}}$$

$$v_{rms} = \sqrt{\frac{2(6.065 \times 10^{-21}\ J)}{4.651 \times 10^{-26}\ kg}}$$

$$v_{rms} = 511\ m/s$$

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#28. 415 m/s
#30. 746 K
#31. 1.2×10^4 m/s
#32. 2.098
#33. 3.87×10^5 J
#36. 5.0×10^1 s

1 hp = 746 W