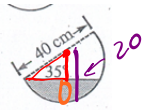
HOMEWORK QUESTIONS...

12. Water in a hemispherical bowl begins to pour out when the bowl is tilted through an angle of 35°. How deep is the water in the bowl?

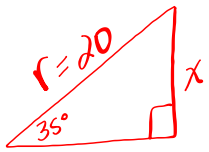


$\frac{x}{20} = \sin 35°$

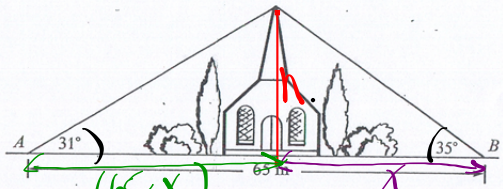
$x = 11.47$

Depth $= 20 - 11.47$

8.5 cm



15. To an observer at A, the angle of elevation of a church spire is 31°. To an observer at B, the angle of elevation is 35°. If the observers are 65 m apart with the spire directly between them, what is the height of the spire?



$\tan 31° = \frac{h}{(65-x)}$

$(65-x)\tan 31° = h$

$\tan 35° = \frac{h}{x}$

$x \tan 35° = h$

$(65-x)\tan 31° = x \tan 35°$

$65 \tan 31° - x \tan 31° = x \tan 35°$

$65 \tan 31° = x \tan 35° + x \tan 31°$

$65 \tan 31° = x(\tan 35° + \tan 31°)$

$\frac{65 \tan 31°}{(\tan 35° + \tan 31°)} = x$

$30 = x$

So $\tan 35° = \frac{h}{x}$

$30 \tan 35° = h$

$21 = h$

REVIEW... Angle Properties & Trigonometry

- constructing angles...compass, protractor, ruler - not on test
- bearings and directions [N/S/E/W]
- geometry theorem...see the notes!
- angle properties...see assignment - justify for test!
- parallel theorems [corresponding, alternate interior, co interior]
- Pythagorean theorem [review, word problems, triples]
- similar triangles
- SOH CAH TOA [find side/angle/solve/word problems]

PROBLEM #1

From a boat on the water, the angle of elevation to the top of the cliff at Alcatraz Island is 31°. From a point 300 m closer to the cliff, the angle of elevation is 33°. **Find the height of the cliff.**

PROBLEM #2

From the top of the 22 metre lighthouse at Peggy's Cove, the angles of depression of two channel buoys in the same line of sight on the water are 13° and 15° **How far apart are the buoys?**

PROBLEM #3

A Golden Gate Bridge in San Francisco is 150 metres above the water. From the ends of the bridge, the angles of depression of a buoy anchored in the water directly below the bridge are 32° and 47°. **Find the length of the bridge.**

PROBLEM #4

At the Sussex Balloon Festival, you had an opportunity to go up in a hot air balloon. You measured the angle of depression to your vehicle to be 35°. From a point 200 m higher vertically, the angle of depression to your vehicle was 54°. **Find the distance from your vehicle to the point in the field directly beneath the balloon.**

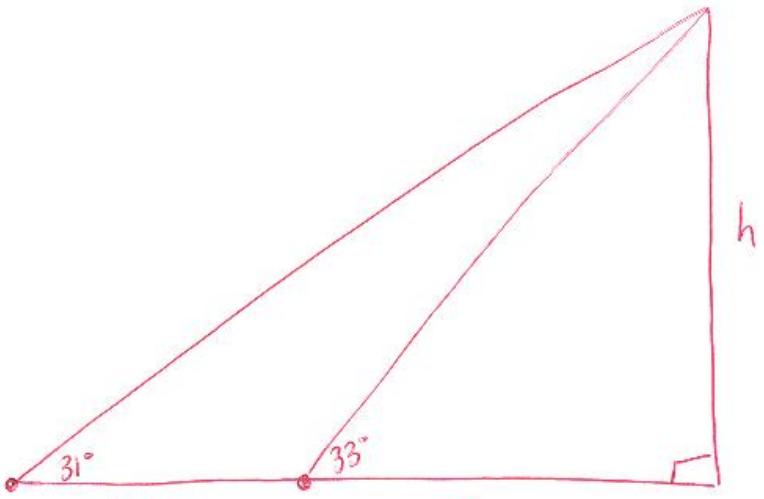
PROBLEM #5

From the roof of MVHS, the angles of depression of the top and bottom of the 10 m hydro pole are 33° and 52°. **Find the height of MVHS.**

SOLUTIONS!!!

PROBLEM #1

From a boat on the water, the angle of elevation to the top of the cliff at Alcatraz Island is 31°. From a point 300 m closer to the cliff, the angle of elevation is 33°. **Find the height of the cliff.**



$\tan 31° = \frac{h}{300+x}$

$\tan 31°(300+x) = h$

$\tan 33° = \frac{h}{x}$

$x \tan 33° = h$

$\tan 31°(300+x) = x \tan 33°$

$\tan 31°(300) + x \tan 31° = x \tan 33°$

$300 \tan 31 = x \tan 33 - x \tan 31$

$300 \tan 31 = x(\tan 33 - \tan 31)$

$\frac{300 \tan 31}{\tan 33 - \tan 31} = x$

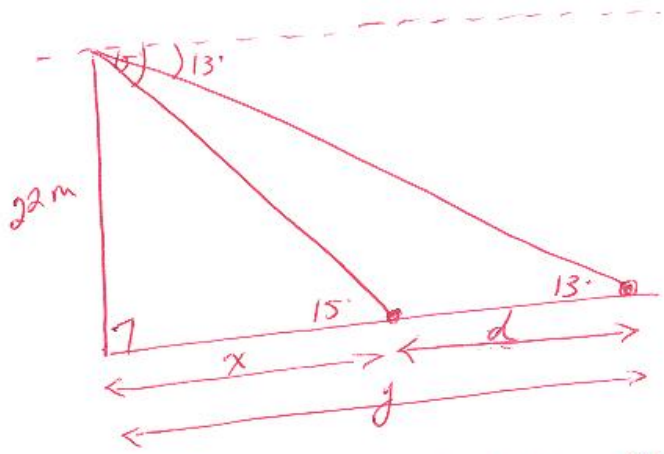
$3713.1 \text{ m} = x$

$h = 3713.1 \tan 33°$

$= \boxed{2411.3 \text{ m}}$

PROBLEM #2

From the top of the 22 metre lighthouse at Peggy's Cove, the angles of depression of two channel buoys in the same line of sight on the water are 13° and 15° **How far apart are the buoys?**



$\frac{x \tan 15}{\tan 15} = \frac{22 \cdot x}{x \tan 15}$

$x = 82.105$

$\frac{y \tan 13}{\tan 13} = \frac{22 \cdot y}{y \tan 13}$

$y = 95.292$

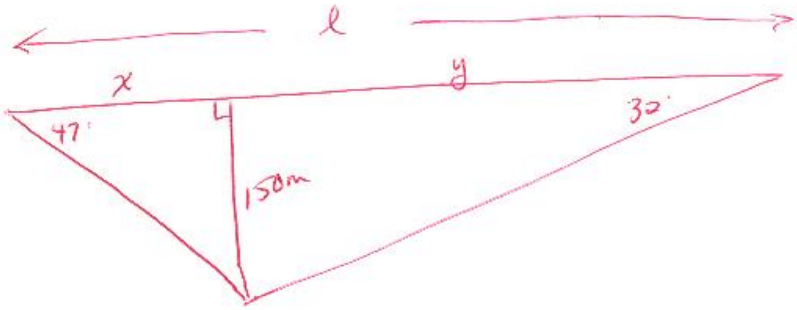
$d = y - x$

$= 95.292 - 82.105$

$= \boxed{13.2\ m}$

PROBLEM #3

A Golden Gate Bridge in San Francisco is 150 metres above the water. From the ends of the bridge, the angles of depression of a buoy anchored in the water directly below the bridge are 32° and 47°. **Find the length of the bridge.**



$\frac{x \tan 47°}{\tan 47°} = \frac{150x}{x \tan 47}$

$x = 139.877$

$\frac{y \tan 32}{\tan 32} = \frac{150y}{y \tan 32}$

$y = 240.050$

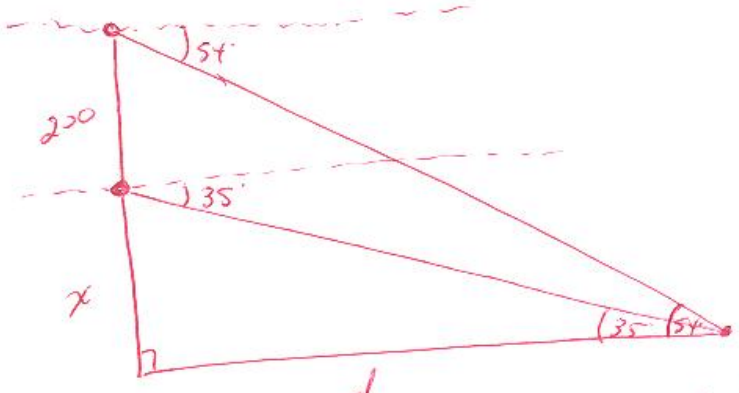
$l = x + y$

$= 139.877 + 240.050$

$= \boxed{379.9\text{m}}$

PROBLEM #4

At the Sussex Balloon Festival, you had an opportunity to go up in a hot air balloon. You measured the angle of depression to your vehicle to be 35°. From a point 200 m higher vertically, the angle of depression to your vehicle was 54°. **Find the distance from your vehicle to the point in the field directly beneath the balloon.**



$\frac{d \cdot \tan 35^\circ}{\tan 35^\circ} = \frac{x \cdot d}{d \tan 35^\circ}$ $\quad \frac{d \tan 54^\circ}{\tan 54^\circ} = \frac{(200+x)d}{d \tan 54^\circ}$

$d = \frac{x}{\tan 35^\circ}$ $\quad d = \frac{200+x}{\tan 54^\circ}$

$\frac{x}{\tan 35^\circ} = \frac{(200+x)}{\tan 54^\circ}$

$x \tan 54 = (200+x)\tan 35^\circ$

$x \tan 54 = 200 \tan 35^\circ + x \tan 35^\circ$

$x \tan 54 - x \tan 35^\circ = 200 \tan 35$

$x(\tan 54 - \tan 35) = 200 \tan 35$

$\frac{x(\tan 54 - \tan 35)}{\tan 54 - \tan 35} = \frac{200 \tan 35}{\tan 54 - \tan 35}$

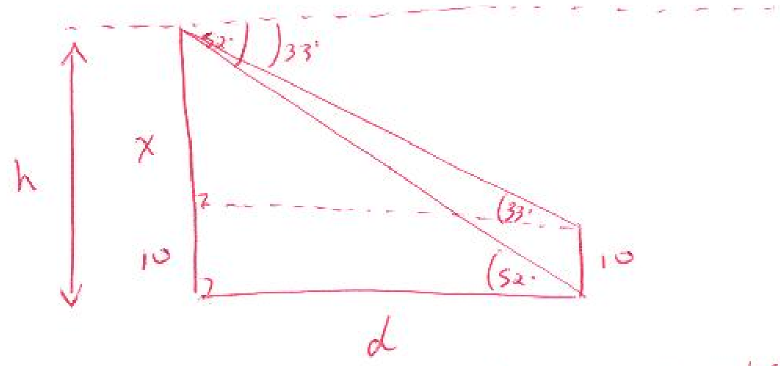
$x = 207.109$

$\rightarrow d = \frac{207.109}{\tan 35^\circ}$

$d = \boxed{295.8 \text{ m}}$

PROBLEM #5

From the roof of MVHS, the angles of depression of the top and bottom of the 10 m hydro pole are 33° and 52°. **Find the height of MVHS.**



$$\frac{d\tan 33^\circ}{\tan 33^\circ} = \frac{x \cdot d}{d \tan 33}$$

$$d = \frac{x}{\tan 33^\circ}$$

$$\frac{d\tan 52}{\tan 52^\circ} = \frac{(x+10)d}{d\tan 52^\circ}$$

$$d = \frac{x+10}{\tan 52}$$

$$\frac{x}{\tan 33} = \frac{x+10}{\tan 52^\circ}$$

$$x\tan 52^\circ = (x+10)\tan 33^\circ$$

$$x\tan 52 = x\tan 33 + 10\tan 33$$

$$x\tan 52 - x\tan 33^\circ = 10\tan 33$$

$$\frac{x(\tan 52 - \tan 33)}{\tan 52 - \tan 33} = \frac{10\tan 33^\circ}{\tan 52 - \tan 33}$$

$$x = 10.299$$

$$h = x + 10$$

$$= 10.299 + 10$$

$$= \boxed{20.3\text{ m}}$$

MORE PRACTICE???

#1. An airplane approaches an airport. At a certain time, it is 939 m high. Its angle of elevation measured from the airport is 19.5°. To the nearest metre, how far is the plane from the airport?

9. 2813 m

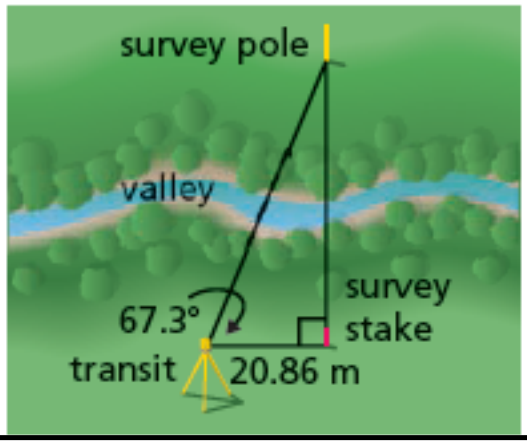
2.5 Using the Sine and Cosine Ratios to Calculate Lengths

#2. A ship is sailing off the north coast of the Queen Charlotte Islands. At a certain point, the navigator sees the lighthouse at Langara Point, due south of the ship. The ship then sails 3.5 km due east. The angle between the ship's path and the line of sight to the lighthouse is then 28.5°.
To the nearest tenth of a kilometre, how far is the ship from the lighthouse?

8. 4.0 km

2.5 Using the Sine and Cosine Ratios to Calculate Lengths

#3. A surveyor made the measurements shown in the diagram. How could the surveyor determine the distance from the transit to the survey pole to the nearest hundredth of a metre?



The distance from the transit to the survey pole is approximately 54.05 m.

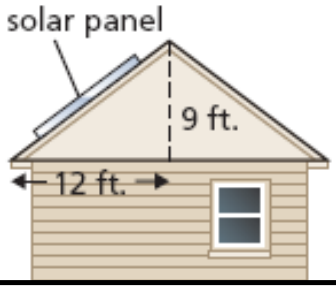
2.5 Using the Sine and Cosine Ratios to Calculate Lengths

#4. An observer is sitting on a dock watching a float plane in Vancouver harbour. At a certain time, the plane is 300 m above the water and 430 m from the observer. Determine the angle of elevation of the plane measured from the observer, to the nearest degree.

[Answer: approximately 44°]

#5: Using the Tangent Ratio to Determine an Angle of Inclination

The latitude of Fort Smith, NWT, is approximately 60°. Determine whether this design for a solar panel is the best for Fort Smith. Justify your answer.



SOLUTION
(erase to reveal)

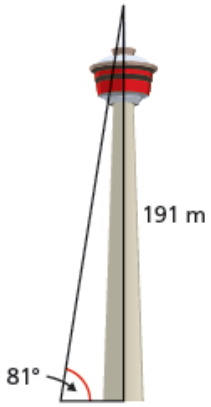
The angle of inclination of the solar panel is about 37°, which is not equal to the latitude of Fort Smith. So, this is not the best design.

#6. A birdwatcher sights an eagle at the top of a 20-m tree. The birdwatcher is lying on the ground 50 m from the tree. At what angle must he incline his camera to take a photograph of the eagle? Give the answer to the nearest degree.

14. 22°

2.1 The Tangent Ratio

#7. Claire knows that the Calgary Tower is 191 m high. At a certain point, the angle between the ground and Claire's line of sight to the top of the tower was 81°. To the nearest metre, about how far was Claire from the tower? Why is this distance approximate?

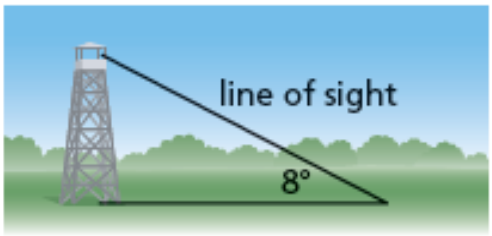


10. Approximately 30 m

2.2 Using the Tangent Ratio to Calculate Lengths

#8...

At a horizontal distance of 200 m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is 8°. How high is the tower to the nearest metre? The diagram is *not* drawn to scale.



[Answer: 28 m]

#9... Using the Tangent Ratio to Solve a Problem

A 10-ft. ladder leans against the side of a building with its base 4 ft. from the wall. What angle, to the nearest degree, does the ladder make with the ground?

SOLUTION (erase to reveal) The angle between the ladder and the ground is approximately 66°.

2.1 The Tangent Ratio