# Review of 2-Dimension Coordinate Geometry

'**AKA**... Numbers, Relations and Functions 10'

---

P262 #9,13

step 1 → declare, state restrictions

step 2 → constraints & inequalities

step 3 → Objective Function

step 4 → Graph & identify vertices

step 5 → "what point is your max/min?"

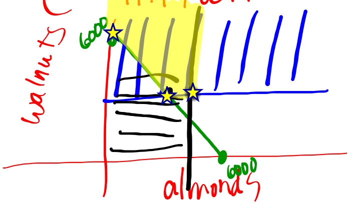
① almonds → a $\quad a \geq 0,\ a \in R$

walnuts → w $\quad w \geq 0 \quad w \in R$

② $w \geq 3000 \qquad w + a \geq 6000$

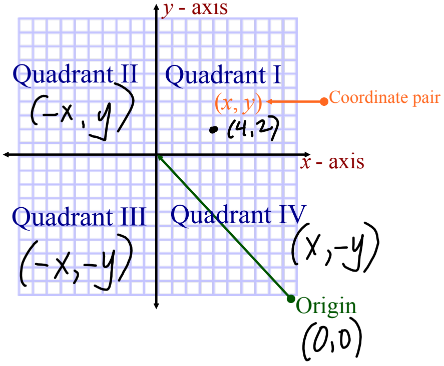
$a \leq 5000 \qquad w \geq -a + 6000$

③ $C = 11.19a + 13.10w$



---

## Cartesian Plane



Associates each point with a pair of numbers (**ordered pair**).

---

a =

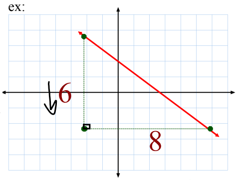
## Calculating Slope

**#1. Graph**

**Slope = $\frac{\text{Rise}}{\text{Run}}$**

$m = \frac{-6}{8} = \frac{-3}{4}$

ex:



**#2. Two Points** delta

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

ex: (-3, 5) & (1, -7)

$x_1\ y_1 \quad x_2\ y_2$

$m = \frac{-7-5}{1+3} = \frac{-12}{4} = -3$

**#3. Equation**

$y = mx + b$

slope

ex: Determine the slope of...

$3x - 2y - 6 = 0$

$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{6}{-2}$

$y = \frac{3}{2}x - 3$

Example...

Find the value for $x$ if the line segment joining the points (x, 0) and (-2, 4) has a slope value of $-\frac{2}{3}$

$x_1\ y_1 \quad x_2\ y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$-\frac{2}{3} = \frac{4-0}{-2-x}$

$\frac{-2}{3} = \frac{4}{(-2-x)}$

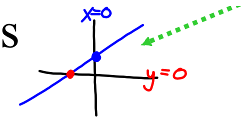
$-2(-2-x) = 4(3)$

$+4 + 2x = 12$

$2x = 8$

$x = 4$

# Intercepts



## x intercept

Where does it cross the x - axis?  (Let y = 0)

## y intercept

Where does it cross the y - axis? (Let x = 0)

Ex.  2x - 3y = 12

x-int let y=0

$2x-3(0) = 12$

$2x = 12$

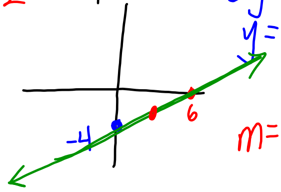
$x=6$

y-int let x=0

$2(0)-3y=12$

$-3y=12$

$y=-4$



$m=\frac{4}{6}=\frac{2}{3}$

## Graphing Linear Functions

NOTES - Graphing Linear Relationships.docx

**Method #1 - Table of Values (must have at least 3 points)**

ex:  3x - 6y + 18 = 0

$3(-1)-6y+18=0$

$-3-6y+18=0$

$\frac{-6y}{-6}=\frac{-15}{-6}$

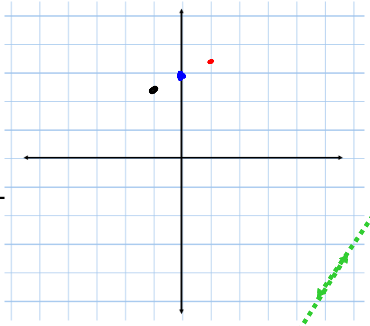
$y=5/2$

$3(0)-6y+18=0$

$-6y=-18$

$y=3$

| x | y |
|---|---|
| -1 | 2.5 |
| 0 | 3 |
| 1 | |

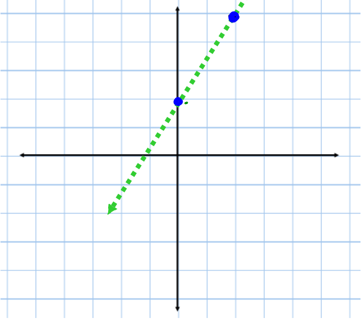


Method #2 - Using the slope/y intercept form of the equation

- put equation in the form.

$y = mx + b$ ← y-int

- plot the y intercept
- use slope = Rise/Run to plot other points.



ex:  3x - 2y = -4

$\frac{-2y}{-2}=\frac{-3x}{-2}-\frac{4}{-2}$

$y=\frac{3}{2}x+\boxed{2}$

Method #3 - Using x / y intercepts

ex:  x - 5y - 10 = 0

x-int let y=0

$x-5(0)-10=0$

$x-10=0$

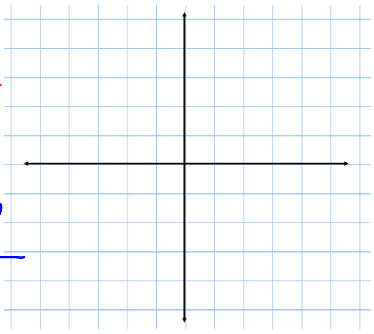
$x=10$

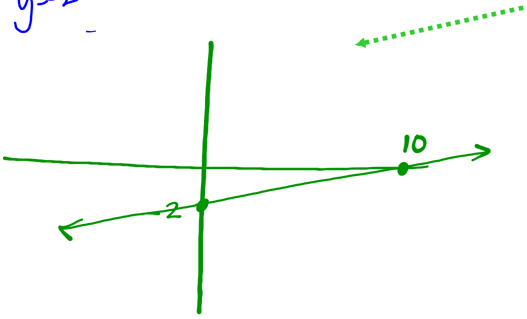
y-int let x=0

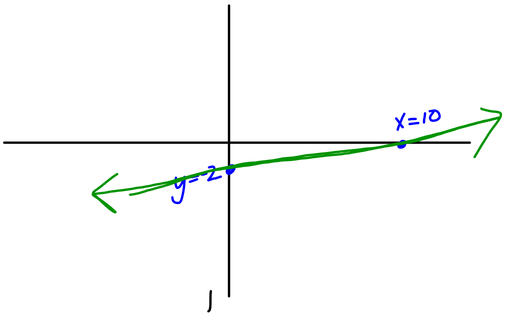
$0-5y-10=0$

$-5y=10$

$y=-2$







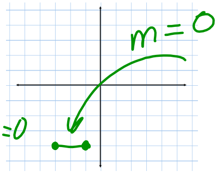
What about vertical versus horizontal lines???

## Graphs of Special Lines

- horizontal lines - slope value of zero

ex: (3, -4) & (-1, -4)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4+4}{-1-3} = \frac{0}{-4} = 0$

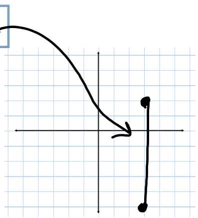


- vertical lines - slope value is **undefined**

ex: x = 2

(3,-5) (3,2)

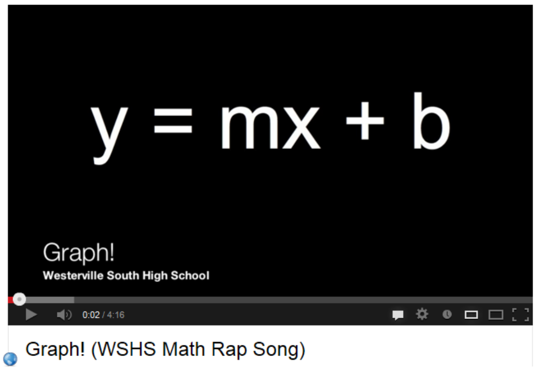
$m = \frac{2--5}{3-3} = \frac{2+5}{0} = \frac{7}{0}$



$\frac{7}{0}$ undefined

$\frac{0}{7} = 0$

WHY WE CAN'T DIVIDE BY ZERO...


y = mx + b

Graph!

Westerville South High School

0:02 / 4:16

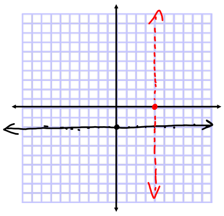
Graph! (WSHS Math Rap Song)

# HOMEWORK...

Puzzle Worksheet - Graphing Lines.docx

---

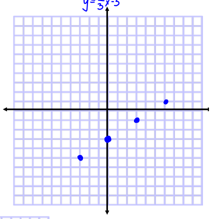
QUESTIONS FROM THE HOMEWORK???



$2x-3y=9$

$-3y=-2x+9$

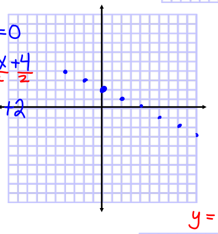
$y=\frac{2}{3}x-3$



$x+2y-4=0$

$\frac{2y}{2}=\frac{-x}{2}+\frac{4}{2}$

$y=-\frac{1}{2}x+2$



$\frac{-1}{2}=\frac{1}{-2}$

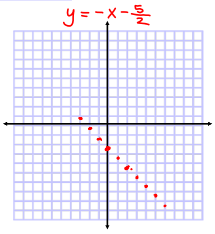
⑨

$-2x=2y+5$

$\frac{-2x}{2}-\frac{5}{2}=\frac{2y}{2}$

$-x-\frac{5}{2}=y$

$y=-x-\frac{5}{2}$

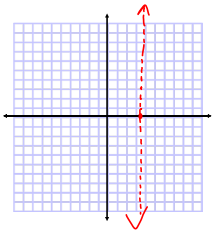


⑧

$2x-7=0$

$2x=7$

$x=7/2=3.5$



---

## WARM-UP: Let's Review...
## PRIOR KNOWLEDGE???

***WORDS You Need to Communicate Effectively*** Warm Up - Prior Knowledge for Coordinate Geometry.docx

**1.** Match each term with the best example or description on the right.

iv **a)** linear equation
**b)** $x$- and $y$-intercepts
**c)** slope
**d)** linear inequality
**e)** dependent variable
**f)** domain
**g)** range
**h)** discrete
**i)** continuous
**j)** independent variable
**k)** quadrant I

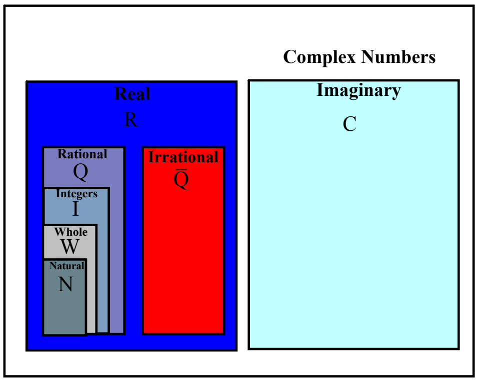
**i)** the value 3 in the equation $y = 3x + 1$
**ii)** $\{1, 2, 3\}$ in the solution set $\{(1, 5), (2, 6), (3, 7)\}$
**iii)** in a relationship, the variable graphed on the $y$-axis
~~**iv)** $2y = 3x + 7$~~
**v)** $3 \leq x + 5$
**vi)** term used to describe a solution set from the set of real numbers
**vii)** $\left(\frac{5}{4}, 0\right)$ and $(0, -5)$ for the graph of $y = 4x - 5$
**viii)** $\{5, 6, 7\}$ in the solution set $\{(1, 5), (2, 6), (3, 7)\}$
**ix)** in a relationship, the variable graphed on the $x$-axis
**x)** term used to describe a solution set from the set of integers
**xi)** the part of the coordinate plane where $x > 0$ and $y > 0$

**Answers**

**1.**

---

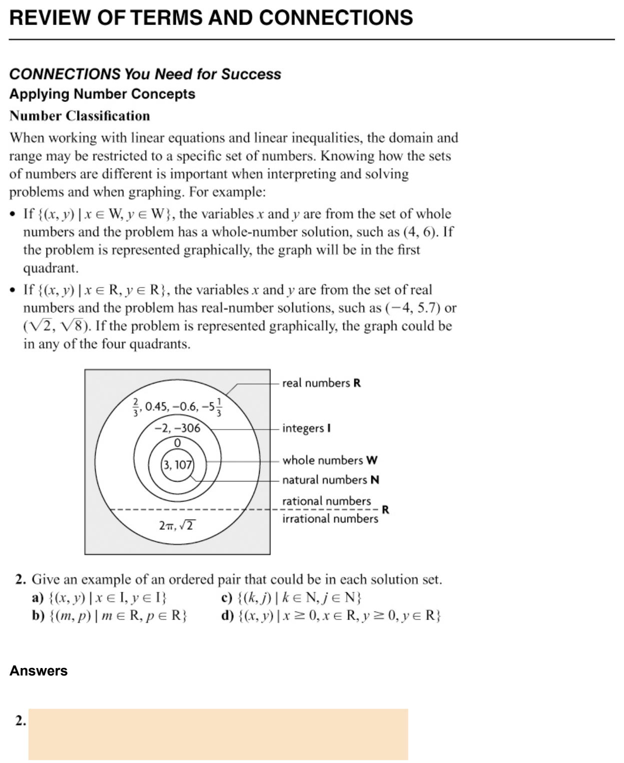
## STORYTIME: "The Complete Number System"



⑧ $2x-2y+5=0$

$\frac{-2y}{-2}=\frac{-2x}{-2}\frac{-5}{-2}$

$y=1x+2.5$



## REVIEW OF TERMS AND CONNECTIONS

***CONNECTIONS* You Need for Success**

**Applying Number Concepts**

**Number Classification**

When working with linear equations and linear inequalities, the domain and range may be restricted to a specific set of numbers. Knowing how the sets of numbers are different is important when interpreting and solving problems and when graphing. For example:

- If $\{(x, y) \mid x \in W, y \in W\}$, the variables $x$ and $y$ are from the set of whole numbers and the problem has a whole-number solution, such as (4, 6). If the problem is represented graphically, the graph will be in the first quadrant.
- If $\{(x, y) \mid x \in R, y \in R\}$, the variables $x$ and $y$ are from the set of real numbers and the problem has real-number solutions, such as $(-4, 5.7)$ or $(\sqrt{2}, \sqrt{8})$. If the problem is represented graphically, the graph could be in any of the four quadrants.

**2.** Give an example of an ordered pair that could be in each solution set.

**a)** $\{(x, y) \mid x \in I, y \in I\}$
**b)** $\{(m, p) \mid m \in R, p \in R\}$
**c)** $\{(k, j) \mid k \in N, j \in N\}$
**d)** $\{(x, y) \mid x \geq 0, x \in R, y \geq 0, y \in R\}$

**Answers**

**2.**

??? ?? [redacted]equality sign - could be one of the following...

## Linear Inequalities:

| > | < | ≥ | ≤ | ≠ |
|---|---|---|---|---|
| GREATER THAN | LESS THAN | GREATER THAN OR EQUAL TO | LESS THAN OR EQUAL TO | NOT EQUAL TO |

When solving an in-equation, all the steps are the same EXCEPT when it comes to isolating...

4 $\boxed{<}$ 11 , fill in the box.

Now divide both by -1

-4 $\boxed{>}$ -11, fill in the box.

RULE: If you multiply or divide by a negative, reverse the inequality sign!!!

$2x-5=7$

$\frac{2x}{2}=\frac{12}{2}$

$x=6$

$2x-5>7$

$\frac{2x}{2}>\frac{12}{2}$

$x>6$

$4-3x=19$

$\frac{-3x}{-3}=\frac{15}{-3}$

$x=-5$

$4-3x\leq19$

$\frac{-3x}{-3}\leq\frac{15}{-3}$

$x\geq-5$

$2x+6=10 \quad -6$

$\frac{2x}{2}=\frac{4}{2}$

$x=2$

$2x+6\geq10$

$\frac{2x}{2}\geq\frac{4}{2}$

$x\geq2$

$4-3\cancel{x}=16$

$-3x=16-4$

$\frac{-3x}{-3}=\frac{12}{-3}$

$x=-4$

$4-3x<16$

$\frac{-3x}{-3}<\frac{12}{-3}$

$x>-4$

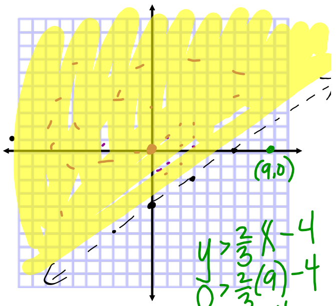
$y = \frac{2}{3}x - 4$

$y > \frac{2}{3}x - 4$

$0 > \frac{2}{3}(0) - 4$

$0 > -4$

True



$y > \frac{2}{3}x - 4$

$0 > \frac{2}{3}(9) - 4$

$0 > 6 - 4$

$0 > 2$

$y = -\frac{4}{3}x + 2$

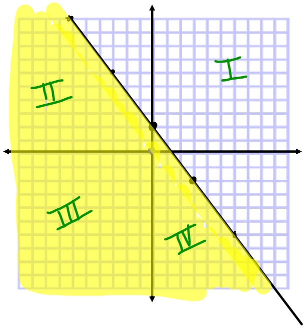
$y \leq -\frac{4}{3}x + 2$

(0,0)

$0 \leq -\frac{4}{3}(0) + 2$

$0 \leq 2$

True



## Rearranging Linear Inequalities

| | |
|---|---|
| ex $4x - 3y + 12 = 0$<br>$\frac{'2}{3}$ | *new<br>$4x - 3y + 12 > 0$<br>$-2$ $'1x - 12$ |

Try

a) $2x + y \geq 7$

$y \geq -2x + 7$

b) $x \boxed{-5y} + 10 \leq 0$

$\frac{-5y}{-5} \leq \frac{-x}{-5} \frac{-10}{-5}$

$y \geq \frac{1}{5}x + 2$

$\frac{x}{5} + \frac{10}{5} \leq \frac{+5y}{5}$

$\frac{1}{5}x + 2 \leq y$ )

$\boxed{x>2}$ $2<x$

*

$4^{+2} < 11^{+2}$

$6^{-5} < 13^{-5}$

$1^{\times 10} < 8^{\times 10}$

$10^{\div 2} < 80^{\div 2}$

$5^{\times(-3)} < 40^{\times(-3)}$

$-15 > -120$

Try these:

① $-5+2x \le 9$

$\frac{2x}{2} \le \frac{14}{2}$

$x \le 7$

② $10-4x \le -14$

$\frac{-4x}{-4} \le \frac{-24}{-4}$

$x \ge 6$

Rearrange "y mx+b"

ex $-3x+2y-12>0$

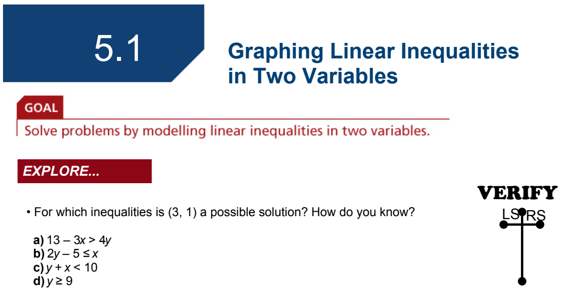
$\frac{2y}{2} > \frac{3x}{2} + \frac{12}{2}$

$y > \frac{3}{2}x+6$

$4x-3y \ge 9$

$\frac{\cancel{-3}y}{\cancel{-3}} \ge \frac{-4x}{-3} + \frac{9}{-3}$

$y \le \frac{4}{3}x - 3$



## 5.1 Graphing Linear Inequalities in Two Variables

**GOAL**

Solve problems by modelling linear inequalities in two variables.

**EXPLORE...**

- For which inequalities is (3, 1) a possible solution? How do you know?

**a)** $13 - 3x > 4y$
**b)** $2y - 5 \le x$
**c)** $y + x < 10$
**d)** $y \ge 9$

**VERIFY**

Let's VERIFY...

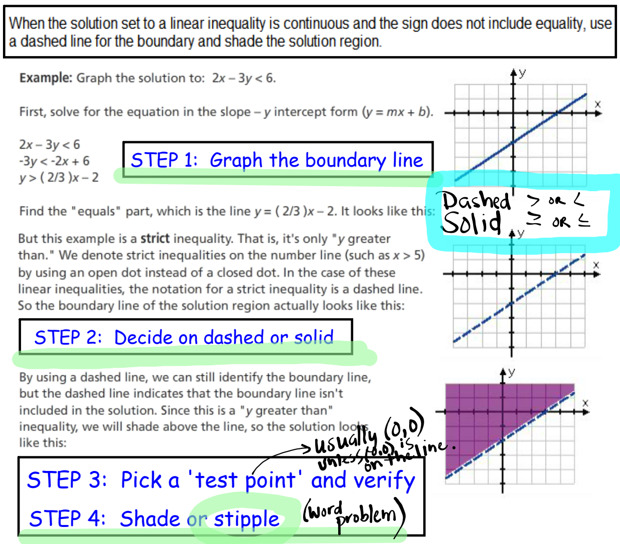
a) $13-3(3) > 4(1)$
$4 > 4$
False!

b) $2(1)-5 \le 3$
$2-5$
$-3 \le 3$
True

c) $1+3 < 10$
$4 < 10$
True

d) $1 \ge 9$
False

NOTES - Graphing a Linear Inequation.docx



When the solution set to a linear inequality is continuous and the sign does not include equality, use a dashed line for the boundary and shade the solution region.

**Example:** Graph the solution to: $2x - 3y < 6$.

First, solve for the equation in the slope – $y$ intercept form ($y = mx + b$).

$2x - 3y < 6$
$-3y < -2x + 6$
$y > (2/3)x - 2$

STEP 1: Graph the boundary line

Find the "equals" part, which is the line $y = (2/3)x - 2$. It looks like this:

Dashed $>$ OR $<$
Solid $\geq$ OR $\leq$

But this example is a **strict** inequality. That is, it's only "$y$ greater than." We denote strict inequalities on the number line (such as $x > 5$) by using an open dot instead of a closed dot. In the case of these linear inequalities, the notation for a strict inequality is a dashed line. So the boundary line of the solution region actually looks like this:

STEP 2: Decide on dashed or solid

By using a dashed line, we can still identify the boundary line, but the dashed line indicates that the boundary line isn't included in the solution. Since this is a "$y$ greater than" inequality, we will shade above the line, so the solution looks like this:

usually (0,0) unless (0,0) is on the line.

STEP 3: Pick a 'test point' and verify

STEP 4: Shade or stipple (word problem)

VIDEO - Graphing Inequalities

Click HERE to watch the video!!!

## WORK that NEEDS to be DONE at HOME...

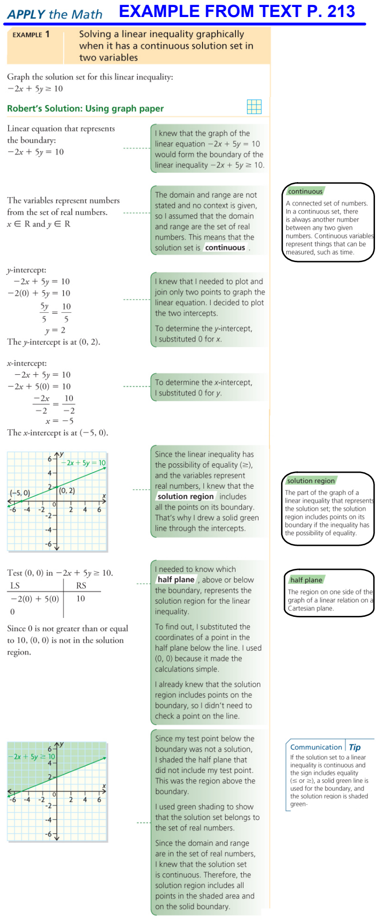
(also known as 'Homework')

1) READ Given Notes on Graphing Inequalities
2) Watch Video Link - lesson is on the website
3) Complete logic problem and pass it in!

**DUE MONDAY...**

Logic - Three Little Pigs.doc

*APPLY the Math* EXAMPLE FROM TEXT P. 213



EXAMPLE 1 Solving a linear inequality graphically when it has a continuous solution set in two variables

Graph the solution set for this linear inequality:
$-2x + 5y \geq 10$

**Robert's Solution: Using graph paper**

Linear equation that represents the boundary:
$-2x + 5y = 10$

I knew that the graph of the linear equation $-2x + 5y = 10$ would form the boundary of the linear inequality $-2x + 5y \geq 10$.

The variables represent numbers from the set of real numbers.
$x \in \mathrm{R}$ and $y \in \mathrm{R}$

The domain and range are not stated and no context is given, so I assumed that the domain and range are the set of real numbers. This means that the solution set is **continuous**.

**continuous**
A connected set of numbers. In a continuous set, there is always another number between any two given numbers. Continuous variables represent things that can be measured, such as time.

$y$-intercept:
$$-2x + 5y = 10$$
$$-2(0) + 5y = 10$$
$$\frac{5y}{5} = \frac{10}{5}$$
$$y = 2$$
The $y$-intercept is at (0, 2).

I knew that I needed to plot and join only two points to graph the linear equation. I decided to plot the two intercepts.

To determine the $y$-intercept, I substituted 0 for $x$.

$x$-intercept:
$$-2x + 5y = 10$$
$$-2x + 5(0) = 10$$
$$\frac{-2x}{-2} = \frac{10}{-2}$$
$$x = -5$$
The $x$-intercept is at $(-5, 0)$.

To determine the $x$-intercept, I substituted 0 for $y$.

Since the linear inequality has the possibility of equality ($\geq$), and the variables represent real numbers, I knew that the **solution region** includes all the points on its boundary. That's why I drew a solid green line through the intercepts.

**solution region**
The part of the graph of a linear inequality that represents the solution set; the solution region includes points on its boundary if the inequality has the possibility of equality.

Test (0, 0) in $-2x + 5y \geq 10$.

| LS | RS |
|---|---|
| $-2(0) + 5(0)$ | 10 |
| 0 | |

Since 0 is not greater than or equal to 10, (0, 0) is not in the solution region.

I needed to know which **half plane**, above or below the boundary, represents the solution region for the linear inequality.

To find out, I substituted the coordinates of a point in the half plane below the line. I used (0, 0) because it made the calculations simple.

I already knew that the solution region includes points on the boundary, so I didn't need to check a point on the line.

**half plane**
The region on one side of the graph of a linear relation on a Cartesian plane.

Since my test point below the boundary was not a solution, I shaded the half plane that did not include my test point. This was the region above the boundary.

I used green shading to show that the solution set belongs to the set of real numbers.

Since the domain and range are in the set of real numbers, I knew that the solution set is continuous. Therefore, the solution region includes all points in the shaded area and on the solid boundary.

Communication *Tip*
If the solution set to a linear inequality is continuous and the sign includes equality ($\leq$ or $\geq$), a solid green line is used for the boundary, and the solution region is shaded green.

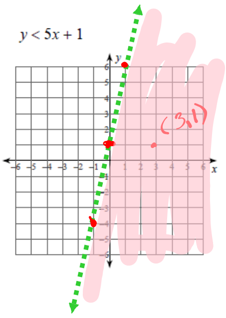
$$4x - 3y = 12$$
$$\frac{-3y}{-3} = \frac{-4x}{-3} + \frac{12}{-3}$$
$$y = \frac{4}{3}x - 4$$

$$4x - 3y \leq 12$$
$$\frac{-3y}{-3} \leq \frac{-4x}{-3} \frac{+12}{-3}$$
$$y \geq \frac{4}{3}x - 4$$
$$\frac{4}{3}x - 4 \leq y$$

$x > 2$

$2 < x$

**EXAMPLE #2:**



$y < 5x + 1$

Test (3,1)

$y < 5x+1$

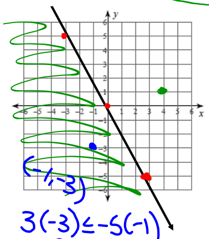
$1 < 5(3)+1$

$1 < 16$

**EXAMPLE #3:**

$3y \le -5x$

$y \le -\frac{5}{3}x$



$3(-3) \le -5(-1)$

True! $-9 \le 5$

TestPoint

(4,1)

$3(1) \le 5(4)$

$3 \le -20$

④ $3x + 2y < 6$

$\frac{2y}{2} < \frac{-3x}{2} + \frac{6}{2}$

$y < -\frac{3}{2}x + 3$

slope ②

y-int ①

(0,0)

$0 < -\frac{3}{2}(0) + 3$

$0 < 3$

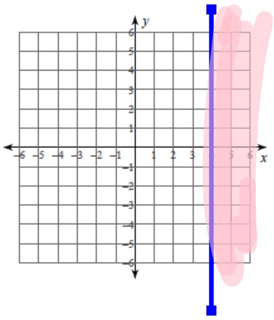
True!

⑩ $2(x-y) \ge 5$

$2x - 2y \ge 5$

**EXAMPLE #4:**

$x \geq 4$



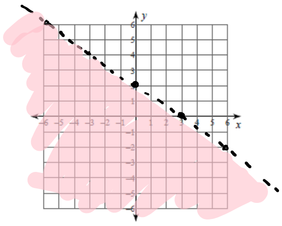
Test (0,0)

$0 \geq 4$

False!

**EXAMPLE #5...**

$2x + 3y - 6 < 0$



$3y < -2x + 6$

$y < -\frac{2}{3}x + 2$

Test (0,0)

$0 < -\frac{2}{3}(0) + 2$

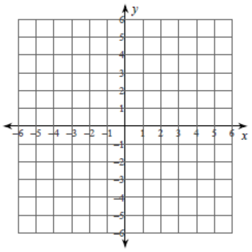
$0 < 2$ True

HOMEWORK...

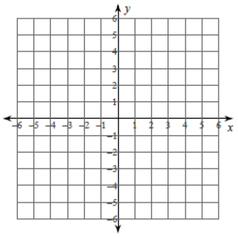
Puzzle Worksheet - Graphing Linear Inequalities with Two Variables.pdf

**WARM-UP: Graph each of the following...**

1) $y \leq -\frac{8}{3}x - 3$



2) $2x + 5y - 20 > 0$



## Graphs of Linear In-Equalities

Sometimes the domain and range are stated as being in the set of integers. This means that the solution set is **discrete** and consists of separate or distinct parts. Discrete variables represent things that can be counted, such as people in a room. This means that the solution region is not shaded but rather stippled with points.

So when interpreting the solution region for a linear inequality, consider the restriction on the domain and range of the variables.

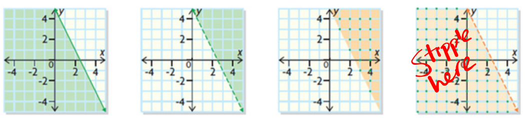
If the solution set is **continuous**, all the points in the solution region are in the solution set. (Shaded)

If the solution set is **discrete**, only specific point in the solution region are in the solution set. This is represented graphically by stippling.

Some solution sets may be restricted to specific quadrants. For example, most linear inequalities representing real-world problem situations have graphs that are restricted to the first quadrant.

Here are some examples:

$\{(x, y) \mid y \le -2x + 5, x \in R, y \in R\}$ $\{(x, y) \mid y > -2x + 5, x \in R, y \in R\}$ $\{(x, y) \mid y \ge -2x + 5, x \in I, y \in I\}$ $\{(x, y) \mid y < -2x + 5, x \in I, y \in I\}$



$$\{(x,y) \mid y > -2x+5, x\varepsilon\_, y\varepsilon\_\}$$

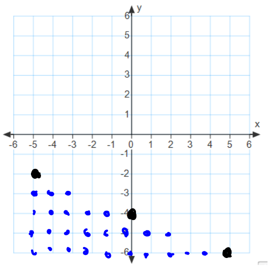
"set of" "such that" "belongs to"

## Let's do a couple more...

1) $\{(x,y) \mid 2x+5y \le -20, x\,\varepsilon\, I, y\,\varepsilon\, I\}$

$5y \le -2x-20$

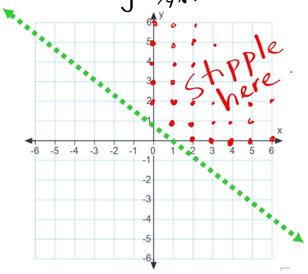
$y \le -\frac{2}{3}x-4$



2) $\{(x,y) \mid 3x+4y > 4, x\,\varepsilon\, W, y\,\varepsilon\, W\}$

$4y > -3x+4$

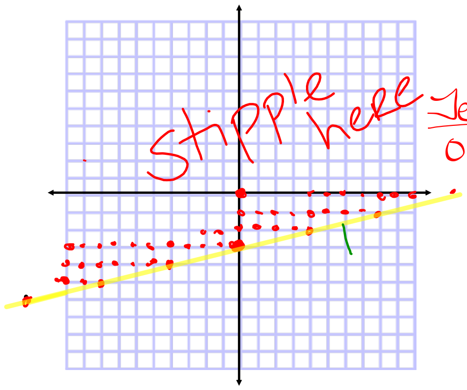
$y > -3/4x+1$



Try

a) $\{(x,y) \mid y \ge \frac{1}{4}x-3, x\varepsilon I, y\varepsilon I\}$

Sti



Test (0,0)

$0 \ge \frac{1}{4}(0)-3$

$0 \ge -3$

True

b) $\{(x, y) \mid 2x - 3y < -6, x \in W, y \in W\}$

$-3y < -2x - 6$

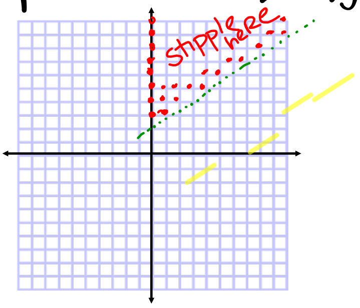
$y > \frac{2}{3}x + 2$

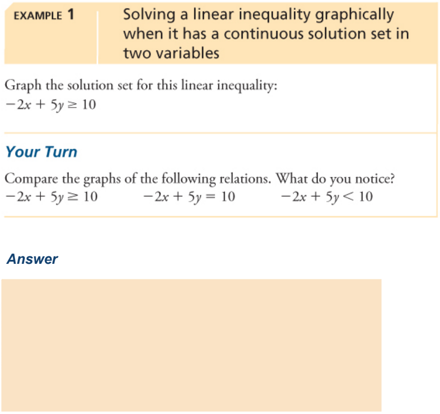
Test (0,0)

$2(0) - 3(0) < -6$

$0 < -6$

False





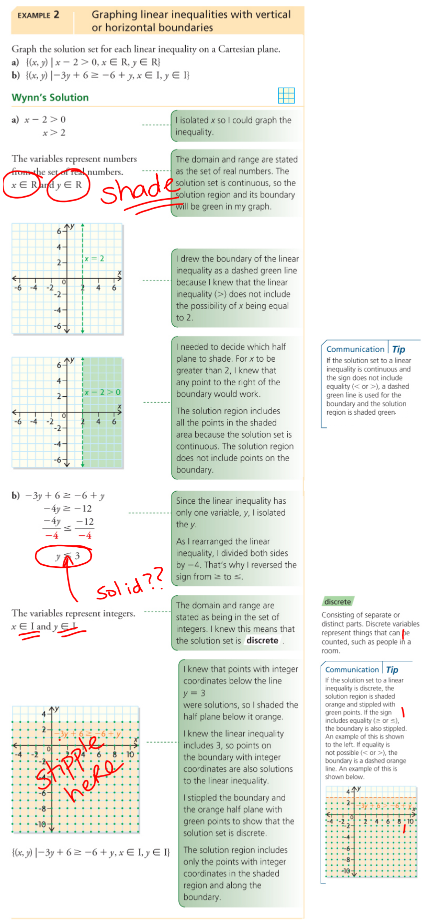
**EXAMPLE 1** Solving a linear inequality graphically when it has a continuous solution set in two variables

Graph the solution set for this linear inequality:
$-2x + 5y \geq 10$

*Your Turn*

Compare the graphs of the following relations. What do you notice?
$-2x + 5y \geq 10$ $-2x + 5y = 10$ $-2x + 5y < 10$

*Answer*



**EXAMPLE 2** Graphing linear inequalities with vertical or horizontal boundaries

Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{(x, y) \mid x - 2 > 0, x \in R, y \in R\}$

b) $\{(x, y) \mid -3y + 6 \geq -6 + y, x \in I, y \in I\}$

**Wynn's Solution**

a) $x - 2 > 0$

$x > 2$

I isolated $x$ so I could graph the inequality.

The variables represent numbers from the set of real numbers.
$x \in R$ and $y \in R$

shade

The domain and range are stated as the set of real numbers. The solution set is continuous, so the solution region and its boundary will be green in my graph.

I drew the boundary of the linear inequality as a dashed green line because I knew that the linear inequality (>) does not include the possibility of $x$ being equal to 2.

I needed to decide which half plane to shade. For $x$ to be greater than 2, I knew that any point to the right of the boundary would work.

The solution region includes all the points in the shaded area because the solution set is continuous. The solution region does not include points on the boundary.

**Communication** ***Tip***
If the solution set to a linear inequality is continuous and the sign does not include equality (< or >), a dashed green line is used for the boundary and the solution region is shaded green.

b) $-3y + 6 \geq -6 + y$

$-4y \geq -12$

$\frac{-4y}{-4} \leq \frac{-12}{-4}$

$y \leq 3$

solid??

Since the linear inequality has only one variable, $y$, I isolated the $y$.

As I rearranged the linear inequality, I divided both sides by −4. That's why I reversed the sign from ≥ to ≤.

The variables represent integers.
$x \in I$ and $y \in I$

The domain and range are stated as being in the set of integers. I knew this means that the solution set is **discrete**.

**discrete**
Consisting of separate or distinct parts. Discrete variables represent things that can be counted, such as people in a room.

stipple here

I knew that points with integer coordinates below the line
$y = 3$
were solutions, so I shaded the half plane below it orange.

I knew the linear inequality includes 3, so points on the boundary with integer coordinates are also solutions to the linear inequality.

I stippled the boundary and the orange half plane with green points to show that the solution set is discrete.

The solution region includes only the points with integer coordinates in the shaded region and along the boundary.

$\{(x, y) \mid -3y + 6 \geq -6 + y, x \in I, y \in I\}$

**Communication** ***Tip***
If the solution set to a linear inequality is discrete, the solution region is shaded orange and stippled with green points. If the sign includes equality (≥ or ≤), the boundary is also stippled. An example of this is shown to the left. If equality is not possible (< or >), the boundary is a dashed orange line. An example of this is shown below.

# HOMEWORK...

p. 221: #3, #4, #6

not told

Shade

N, W, I

stipple

R

Shade

6b $\{(x,y)\,|\,\boxed{x+6y-14<0}, x\in I, y\in I\}$

$\frac{6y}{6}<\frac{-x}{6}+\frac{14}{6}$

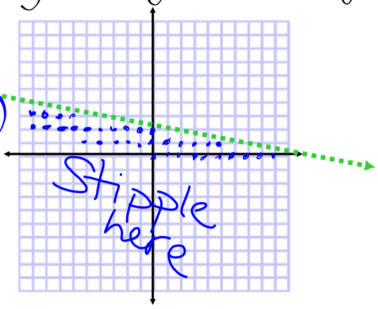
$y<-\frac{1}{6}x+\boxed{\frac{7}{3}}$

Test (0,0)

$\cancel{0+(6)(0)}-14<0$

$-14<0$

True



## *INVESTIGATE* the Math

Amir owns a health-food store. He is making a mixture of nuts and raisins to sell in bulk. His supplier charges \$25/kg for nuts and \$8/kg for raisins.

? What quantities of nuts and raisins can Amir mix together if he wants to spend less than \$200 to make the mixture?

**A.** Suppose that Amir wants to spend exactly \$200 to make the mixture. Work with a partner to create an equation that represents this situation.

**B.** To what set of numbers does the domain and range of the two variables in your equation belong? Use this information to help you graph the equation on a coordinate plane.

**C.** Explain why the graph is a line segment, not a ray or a line.

**D.** What region of the coordinate plane includes points representing quantities of nuts and raisins that Amir could use if he wants to spend less than \$200? How do you know?

**E.** There are many possible solutions to Amir's problem. Plot at least three points that represent reasonable solutions to Amir's problem. Explain why you chose these points.

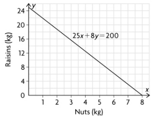
### Answers

**A.** Let $x$ represent the number of kilograms of nuts, and let $y$ represent the number of kilograms of raisins:

$25x + 8y = 200$

**B.** The domain and range belong to the set of real numbers greater than or equal to 0.

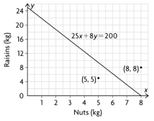
Raisins and Nuts



**C.** The graph is a line segment because it has endpoints at the $x$-axis and $y$-axis.

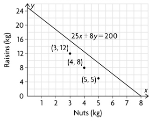
**D.** The region below the line segment includes points representing quantities of nuts and raisins that Amir could use. Answers will vary, e.g., I picked a point in the region above and below the line and tested both algebraically:

Raisins and Nuts



**E.** Answers will vary, e.g., I chose these points because they are in the region below the line. I know that points in the region below the line will solve the problem. However, even though points such as (0, 0), (7, 0) and (0, 24) are below the line, they wouldn't make very good mixtures because they either involve only one quantity or no quantities at all.

Raisins and Nuts



| Test a point above the line. | Test a point below the line. |
|---|---|
| (8, 8), or 8 kg of nuts and 8 kg of raisins:<br>$25x + 8y \rightarrow 25(8) + 8(8) = 264$<br>So, 8 kg of nuts and 8 kg of raisins would cost \$264, which is more than \$200. | (5, 5), or 5 kg of nuts and 5 kg of raisins:<br>$25x + 8y \rightarrow 25(5) + 8(5) = 165$<br>So, 5 kg of nuts and 5 kg of raisins would cost \$165, which is less than \$200. |

Line Segment vs Line vs Ray

## Reflecting

**F.** Discuss and then decide whether the **solution set** for Amir's problem is represented by

- **i)** points in the region above the line segment.
- **ii)** points in the region below the line segment.
- **iii)** points on the line segment.

**G.** Why might the line segment be considered a boundary of the solution set?

**H.** Why might you use a dashed line segment for this graph instead of a solid line segment?

solution set
The set of all possible solutions.

### Answers

**F. i)** Points in the region above the line segment are not part of the solution set because they result in costs greater than \$200.

**ii)** Points in the region below the line segment represent the solution set because they result in costs less than \$200.

**iii)** Points on the line segment are not part of the solution set because they result in costs that are exactly \$200.

**G.** The line segment might be considered a boundary because the solution set is completely on one side of it. It separates the points that are in the solution set from the points that are not.

**H.** A dashed line segment shows that the points on the line segment are not part of the solution set.

## Applications...Apply your skills to a context

**EXAMPLE #2:** HANDOUT - Application of a Linear Inequality.docx

Malia and Lainey are competing in a spelling quiz. Malia gets a point for every word she spells correctly. Lainey is younger than Malia, so she gets 3 points for every word she spells correctly plus one bonus point. What combination of correctly spelled words for Malia and Lainey result in Malia spelling more? Choose two combinations that make sense and explain why.

---

Step 1: Declare variables

$m \rightarrow$ # of words for Malia

$l \rightarrow$ # of words for Lainey

Step 2: State restrictions

$m \geq 0, m \in W$

$l \geq 0, l \in W$

Step 3: Develop the inequation

$m > 3l + 1$

$(y > 3x + 1)$

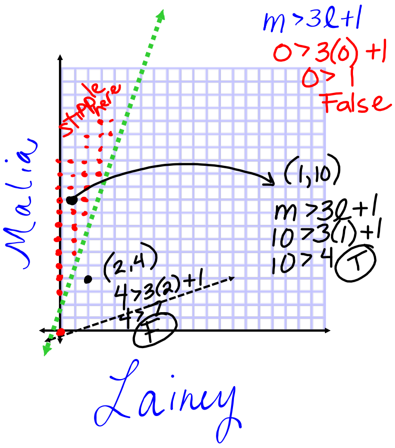
Step 4: Graph the solution set (MUST include labels/scales)

$m > 3l + 1$

$0 > 3(0) + 1$

$0 > 1$

False



$m > 3l + 1$

$10 > 3(1) + 1$

$10 > 4$ $\boxed{T}$

$4 > 3(2) + 1$

$4 > 7$ $\boxed{F}$

**EXAMPLE 3** p. 218 — Solving a real-world problem by graphing a linear inequality with discrete whole-number solutions

A sports store has a net revenue of \$100 on every pair of downhill skis sold and \$120 on every snowboard sold. The manager's goal is to have a net revenue of more than \$600 a day from the sales of these two items. What combinations of ski and snowboard sales will meet or exceed this daily sales goal? Choose two combinations that make sense, and explain your choices.

Step 1: Declare variables

let $x \rightarrow$ # of skis sold

$y \rightarrow$ # of snowboards sold

Step 2: State restrictions

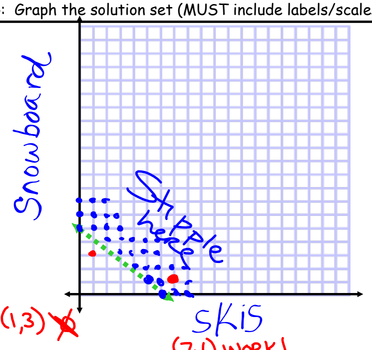
$x \geq 0, x \in W$

$y \geq 0, y \in W$

Step 3: Develop the inequation

$100x + 120y \geq 600$

Step 4: Graph the solution set (MUST include labels/scales)



$100x + 120y > 600$

$120y \geq -100x + 600$

$y \geq \frac{-100}{120}x + \frac{600}{120}$

$y \geq \frac{-5}{6}x + 5$

Test (0,0)

$100(0) + 120(0) \geq 600$

$0 \geq 600$

(1,3) ✗

$100(1) + 120(3) \geq 600$

$100 + 360$

$460 \geq 600$

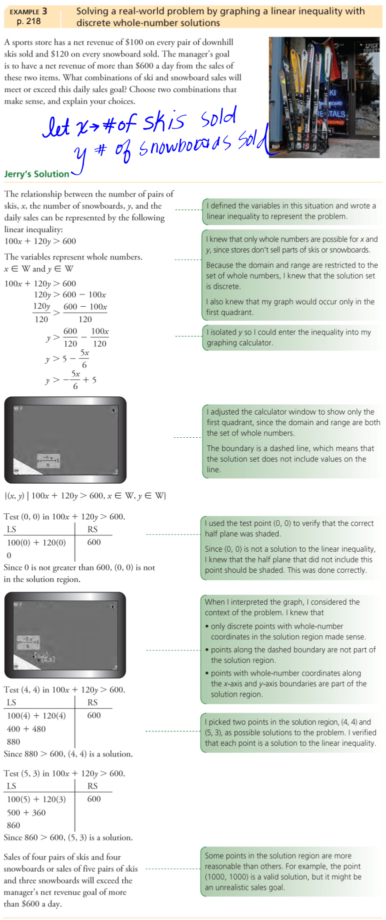
False

(7,1) WORK!

$7(100) + 1(120) \geq 600$

$700 + 120$

$820 \geq 600$



**EXAMPLE 3** p. 218 — Solving a real-world problem by graphing a linear inequality with discrete whole-number solutions

A sports store has a net revenue of \$100 on every pair of downhill skis sold and \$120 on every snowboard sold. The manager's goal is to have a net revenue of more than \$600 a day from the sales of these two items. What combinations of ski and snowboard sales will meet or exceed this daily sales goal? Choose two combinations that make sense, and explain your choices.

let $x \rightarrow$ # of skis sold

$y$ # of snowboards sold

**Jerry's Solution**

The relationship between the number of pairs of skis, $x$, the number of snowboards, $y$, and the daily sales can be represented by the following linear inequality:

$100x + 120y > 600$

I defined the variables in this situation and wrote a linear inequality to represent the problem.

The variables represent whole numbers.

$x \in W$ and $y \in W$

I knew that only whole numbers are possible for $x$ and $y$, since stores don't sell parts of skis or snowboards.

Because the domain and range are restricted to the set of whole numbers, I knew that the solution set is discrete.

I also knew that my graph would occur only in the first quadrant.

$$100x + 120y > 600$$
$$120y > 600 - 100x$$
$$\frac{120y}{120} > \frac{600 - 100x}{120}$$
$$y > \frac{600}{120} - \frac{100x}{120}$$
$$y > 5 - \frac{5x}{6}$$
$$y > -\frac{5x}{6} + 5$$

I isolated $y$ so I could enter the inequality into my graphing calculator.

I adjusted the calculator window to show only the first quadrant, since the domain and range are both the set of whole numbers.

The boundary is a dashed line, which means that the solution set does not include values on the line.

$\{(x, y) \mid 100x + 120y > 600, x \in W, y \in W\}$

Test (0, 0) in $100x + 120y > 600$.

| LS | RS |
|---|---|
| $100(0) + 120(0)$ | 600 |
| 0 | |

Since 0 is not greater than 600, (0, 0) is not in the solution region.

I used the test point (0, 0) to verify that the correct half plane was shaded.

Since (0, 0) is not a solution to the linear inequality, I knew that the half plane that did not include this point should be shaded. This was done correctly.

When I interpreted the graph, I considered the context of the problem. I knew that

- only discrete points with whole-number coordinates in the solution region made sense.
- points along the dashed boundary are not part of the solution region.
- points with whole-number coordinates along the $x$-axis and $y$-axis boundaries are part of the solution region.

Test (4, 4) in $100x + 120y > 600$.

| LS | RS |
|---|---|
| $100(4) + 120(4)$ | 600 |
| $400 + 480$ | |
| 880 | |

Since $880 > 600$, (4, 4) is a solution.

Test (5, 3) in $100x + 120y > 600$.

| LS | RS |
|---|---|
| $100(5) + 120(3)$ | 600 |
| $500 + 360$ | |
| 860 | |

Since $860 > 600$, (5, 3) is a solution.

I picked two points in the solution region, (4, 4) and (5, 3), as possible solutions to the problem. I verified that each point is a solution to the linear inequality.

Sales of four pairs of skis and four snowboards or sales of five pairs of skis and three snowboards will exceed the manager's net revenue goal of more than \$600 a day.

Some points in the solution region are more reasonable than others. For example, the point (1000, 1000) is a valid solution, but it might be an unrealistic sales goal.

### *Your Turn*

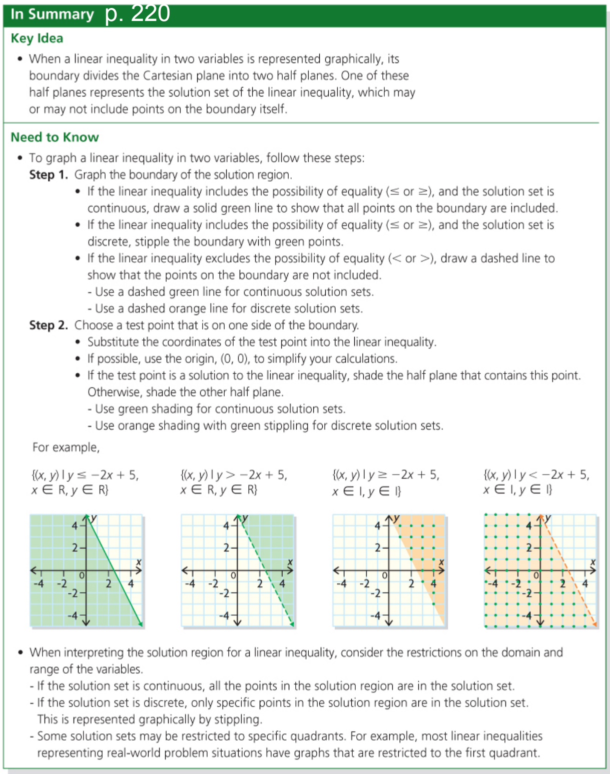
a) Would raising the daily sales goal to at least \$1000 change the graph that models this situation? Explain.

b) State two combinations of ski and snowboard sales that would meet or exceed this new daily sales goal.

### *Answers*

a)

b)



**In Summary** p. 220

**Key Idea**

- When a linear inequality in two variables is represented graphically, its boundary divides the Cartesian plane into two half planes. One of these half planes represents the solution set of the linear inequality, which may or may not include points on the boundary itself.

**Need to Know**

- To graph a linear inequality in two variables, follow these steps:

  **Step 1.** Graph the boundary of the solution region.
  - If the linear inequality includes the possibility of equality (≤ or ≥), and the solution set is continuous, draw a solid green line to show that all points on the boundary are included.
  - If the linear inequality includes the possibility of equality (≤ or ≥), and the solution set is discrete, stipple the boundary with green points.
  - If the linear inequality excludes the possibility of equality (< or >), draw a dashed line to show that the points on the boundary are not included.
    - Use a dashed green line for continuous solution sets.
    - Use a dashed orange line for discrete solution sets.

  **Step 2.** Choose a test point that is on one side of the boundary.
  - Substitute the coordinates of the test point into the linear inequality.
  - If possible, use the origin, (0, 0), to simplify your calculations.
  - If the test point is a solution to the linear inequality, shade the half plane that contains this point. Otherwise, shade the other half plane.
    - Use green shading for continuous solution sets.
    - Use orange shading with green stippling for discrete solution sets.

For example,

$\{(x, y) \mid y \leq -2x + 5, x \in R, y \in R\}$

$\{(x, y) \mid y > -2x + 5, x \in R, y \in R\}$

$\{(x, y) \mid y \geq -2x + 5, x \in I, y \in I\}$

$\{(x, y) \mid y < -2x + 5, x \in I, y \in I\}$

- When interpreting the solution region for a linear inequality, consider the restrictions on the domain and range of the variables.
  - If the solution set is continuous, all the points in the solution region are in the solution set.
  - If the solution set is discrete, only specific points in the solution region are in the solution set. This is represented graphically by stippling.
  - Some solution sets may be restricted to specific quadrants. For example, most linear inequalities representing real-world problem situations have graphs that are restricted to the first quadrant.

HOMEWORK...

p. 221: #2, ~~5bc~~, $\underbrace{7, 8, 9}$

1) Declare variables
2) State restrictions
3) Develop inequation
4) Graph solution set

5.3

## Graphing to Solve Systems of Linear Inequalities

**GOAL**

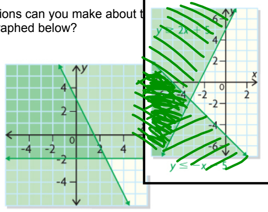
Solve problems by modelling system

**system of linear inequalities**

A set of two or more linear inequalities that are graphed on the same coordinate plane; the intersection of their solution regions represents the solution set for the system.

**EXPLORE...**

- What conclusions can you make about t inequalities graphed below?



**SAMPLE ANSWER**

Any or all of the following solutions are acceptable:

- It represents a system of two linear inequalities, each with a straight boundary and a solution region.
- One linear inequality is $y \leq -2x + 3$, and the horizontal inequality is $y \geq -2$. I determined $y \leq -2x + 3$ using the slope and $y$-intercept and the form $y = mx + b$, and I was able to identify $y \geq -2$ because it's a horizontal line through $-2$ on the $y$-axis.
- Both inequalities include the possibility of equality because the boundaries are solid.
- The solution set of the system is represented by the overlapping region because it's where the solution regions for the two linear inequalities overlap. The solution set includes points along the boundaries of the overlap.
- The domain and range are from the set of real numbers because the solution region is green and not stippled.
- All four quadrants are included so there are no restrictions on the set of real numbers.

## Solving Systems of Linear Inec

A **system of linear inequalities** is an extension of a system of linear equations and consists of two (or more) linear inequalities that have the same variables. For example, $2x + 3y < 4$ and $3x + 4y < 5$ constitute a system of inequalities if $x$ represents the same item in both equations, $y$ represents the same item in both equations, and both equations describe the same context.

Example #1:

Graph the following system and determine a possible solution

$\{(x,y) \mid y \leq 3x - 1, x\varepsilon R, y\varepsilon R\}$

Test (0,0)

$y \leq 3x - 1$

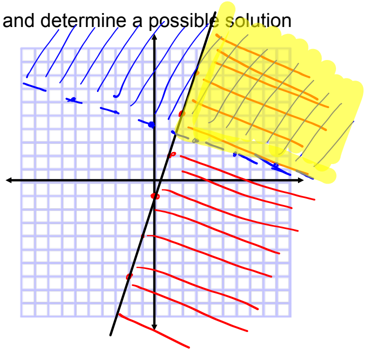
$0 \leq 3(0) - 1$

$0 \leq -1$ False

$\left\{(x,y) \mid y > -\frac{1}{3}x + 4, x\varepsilon R, y\varepsilon R\right\}$

Test (0,0)

$0 > 4$ False



b → teddy bears sold

t → tickets sold.

$b \geq 0$ $b \in W$

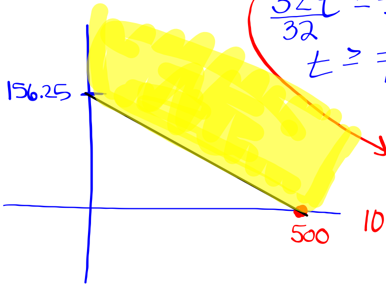
$t \geq 0$ $t \in W$

$10b + 32t \geq 5000$

$\frac{32t}{32} \geq \frac{-10b}{32} + \frac{5000}{32}$

$t \geq \frac{-5}{16}b + 156.25$

y-int



x-int
(b-int)

$10b + 32(0) = 5000$

$10b = 5000$

$b = 500$

EXAMPLE #2...

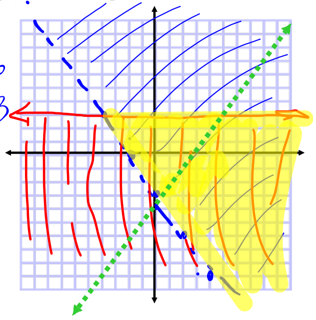
Graph the solution set for the following system of inequalities. Choose two possible solutions from the set.

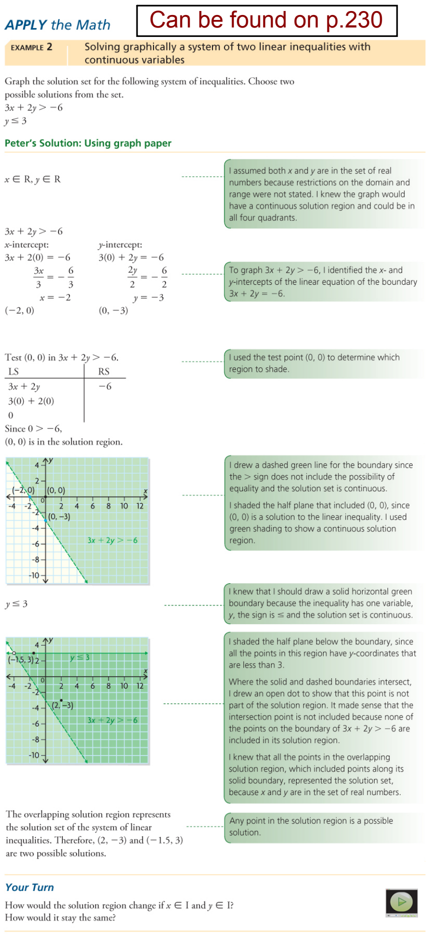
$3x + 2y > -6$

$y \leq 3$

$2y > -3x - 6$

$y > \frac{-3}{2}x - 3$





**APPLY** the Math

Can be found on p.230

**EXAMPLE 2** Solving graphically a system of two linear inequalities with continuous variables

Graph the solution set for the following system of inequalities. Choose two possible solutions from the set.

$3x + 2y > -6$

$y \leq 3$

**Peter's Solution: Using graph paper**

$x \in R, y \in R$

I assumed both x and y are in the set of real numbers because restrictions on the domain and range were not stated. I knew the graph would have a continuous solution region and could be in all four quadrants.

$3x + 2y > -6$

x-intercept:

$3x + 2(0) = -6$

$\frac{3x}{3} = -\frac{6}{3}$

$x = -2$

$(-2, 0)$

y-intercept:

$3(0) + 2y = -6$

$\frac{2y}{2} = -\frac{6}{2}$

$y = -3$

$(0, -3)$

To graph $3x + 2y > -6$, I identified the x- and y-intercepts of the linear equation of the boundary $3x + 2y = -6$.

Test (0, 0) in $3x + 2y > -6$.

| LS | RS |
|---|---|
| $3x + 2y$ | $-6$ |
| $3(0) + 2(0)$ | |
| 0 | |

Since $0 > -6$,
(0, 0) is in the solution region.

I used the test point (0, 0) to determine which region to shade.

I drew a dashed green line for the boundary since the > sign does not include the possibility of equality and the solution set is continuous.

I shaded the half plane that included (0, 0), since (0, 0) is a solution to the linear inequality. I used green shading to show a continuous solution region.

$y \leq 3$

I knew that I should draw a solid horizontal green boundary because the inequality has one variable, y, the sign is ≤ and the solution set is continuous.

I shaded the half plane below the boundary, since all the points in this region have y-coordinates that are less than 3.

Where the solid and dashed boundaries intersect, I drew an open dot to show that this point is not part of the solution region. It made sense that the intersection point is not included because none of the points on the boundary of $3x + 2y > -6$ are included in its solution region.

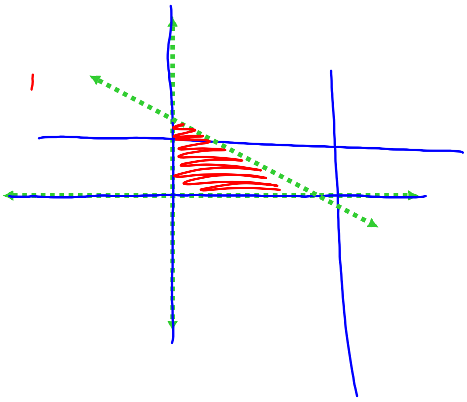
I knew that all the points in the overlapping solution region, which included points along its solid boundary, represented the solution set, because x and y are in the set of real numbers.

The overlapping solution region represents the solution set of the system of linear inequalities. Therefore, (2, −3) and (−1.5, 3) are two possible solutions.

Any point in the solution region is a possible solution.

*Your Turn*

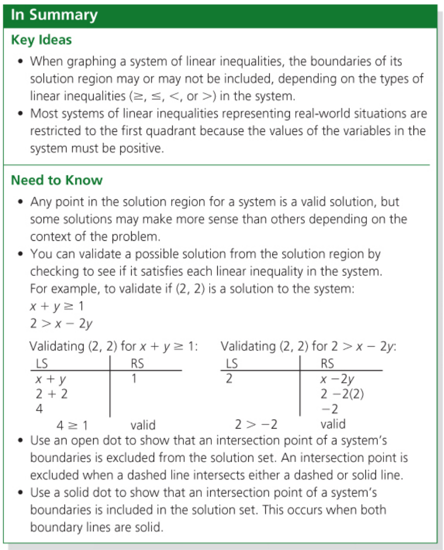
How would the solution region change if $x \in I$ and $y \in I$?
How would it stay the same?



## Let's check out the web...

EXAMPLE #3: Applet from online Math Tutor

" Khan"



**In Summary**

**Key Ideas**

- When graphing a system of linear inequalities, the boundaries of its solution region may or may not be included, depending on the types of linear inequalities ($\geq$, $\leq$, $<$, or $>$) in the system.
- Most systems of linear inequalities representing real-world situations are restricted to the first quadrant because the values of the variables in the system must be positive.

**Need to Know**

- Any point in the solution region for a system is a valid solution, but some solutions may make more sense than others depending on the context of the problem.
- You can validate a possible solution from the solution region by checking to see if it satisfies each linear inequality in the system. For example, to validate if (2, 2) is a solution to the system:
  $x + y \geq 1$
  $2 > x - 2y$

Validating (2, 2) for $x + y \geq 1$:

| LS | RS |
|---|---|
| $x + y$ | 1 |
| $2 + 2$ | |
| 4 | |
| $4 \geq 1$ | valid |

Validating (2, 2) for $2 > x - 2y$:

| LS | RS |
|---|---|
| 2 | $x - 2y$ |
| | $2 - 2(2)$ |
| | $-2$ |
| $2 > -2$ | valid |

- Use an open dot to show that an intersection point of a system's boundaries is excluded from the solution set. An intersection point is excluded when a dashed line intersects either a dashed or solid line.
- Use a solid dot to show that an intersection point of a system's boundaries is included in the solution set. This occurs when both boundary lines are solid.

## HOMEWORK...

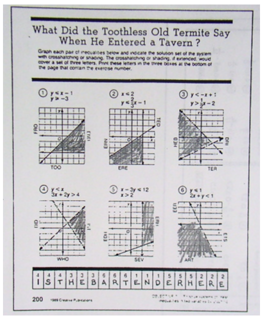
Puzzle Worksheet - Systems of Linear Inequations.docx

Worksheet - Systems of Linear Inequations.docx

page 225 #1,2

## SOLUTIONS...

PUZZLE WORKSHEET:



**What Did the Toothless Old Termite Say When He Entered a Tavern?**

Graph each pair of inequalities below and indicate the solution set of the system with crosshatching or shading. The crosshatching or shading, if extended, would cover a set of three letters. Print these letters in the three boxes at the bottom of the page that contain the exercise number.

① $y < x - 1$
$y > -3$

② $x \leq 2$
$y < \frac{2}{3}x - 1$

③ $y < -x + 1$
$y > \frac{1}{2}x - 2$

④ $y < x$
$3x + 2y > 4$

⑤ $x - 3y \leq 12$
$x > 2$

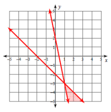
⑥ $y \leq 1$
$2x + y < 1$

| 4 | 4 | 4 | 3 | 3 | 3 | 6 | 6 | 6 | 1 | 1 | 1 | 5 | 5 | 5 | 2 | 2 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I | S | T | H | E | B | A | R | T | E | N | D | E | R | H | E | R | E |

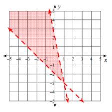
200 ©1989 Creative Publications

WORKSHEET:

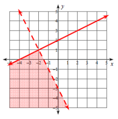
1) $y \leq -x - 2$
$y \geq -5x + 2$



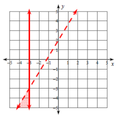
2) $y > -x - 2$
$y < -5x + 2$



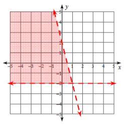
3) $y \leq \frac{1}{2}x + 2$
$y < -2x - 3$



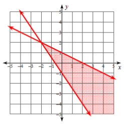
4) $x \leq -3$
$y < \frac{5}{3}x + 2$



5) $4x + y < 2$
$y > -2$



6) $3x + 2y \geq -2$
$x + 2y \leq 2$



**WARM-UP**: Graph the solution and state 2 possible solutions...

$\{(x,y) \mid 2x+y>8, x\varepsilon W, y\varepsilon W\}$

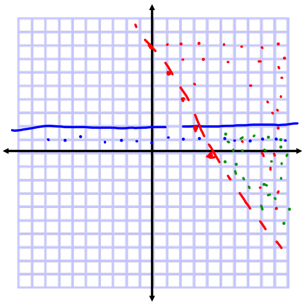
$\{(x,y) \mid y\le 2, x\varepsilon W, y\varepsilon W\}$

$y>-2x+8$

$y\le 2$

$(6,0)$

$(10,-4)$



## **Applications:** Systems Involving Inequalities

**STEP 1** - Declare Variables
State Restrictions

**STEP 2** - Create Linear Inequalities

**STEP 3** - Graph Solution Set

**STEP 4** - Answer question(s)

**EXAMPLE #1:**

To raise funds for $\pi$ - day, the PI Committee has 500 T-shirts to sell.

They have two varieties:

#1. 'I 8 Sum $\pi$' or #2.'$\pi$- DAY 2013'.

They expect to sell at least twice as many of the first as the second.

a) Define the variables and restrictions. Write a system of linear inequalities that models the situation.

x→t-shirt1

y→t-shirt2

$x,y\varepsilon W$

$x\ge 2y$ ← hard bit

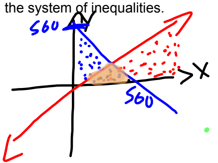
$x+y\le 500$

$2y\le x$

$y\le\frac{1}{2}x$

$y\le -x+500$

b) Graph the system of inequalities.



c) State a combination of T-shirt sales.

| | |
|---|---|
| 200 | 1 |
| 100 | 2 |

### 5.2 Exploring Graphs of Systems of Linear Inequalities

**GOAL**

Explore graphs of situations that can be modelled by systems of two linear inequalities in two variables.

### *EXPLORE the Math*

A nursery school serves morning and afternoon snacks to its students. The morning snacks are fruits, vegetables, and juice, and the afternoon snacks are cheese and milk.

- The school can accommodate 50 students or fewer altogether. Students can attend for just the morning or for a full day.
- The morning snack costs \$1 per student per week, and the afternoon snack costs \$2 per student per week.
- The weekly snack budget is \$120 or less.

? What combinations of morning and full-day students can the school accommodate and stay within the weekly snack budget?

? What combinations of morning and full-day students can the school accommodate and stay within the weekly snack budget?

**Sample Solution**

First, we represented the two unknowns in the problem using $x$ and $y$:

- $x$ is the number of morning students.
- $y$ is the number of full-day students.

$x,y\in W$

Then we wrote a linear inequality to represent each part of the problem:

- The total cost of the snacks, as it relates to the number of students, is the sum of the cost of the morning snack multiplied by the number of morning students and the sum of the cost of the afternoon snack multiplied by the number of full-day students. The total cost is \$120 or less:

  $x + 3y \le 120$ ← cost

- The total number of students can be up to and including 50 students:

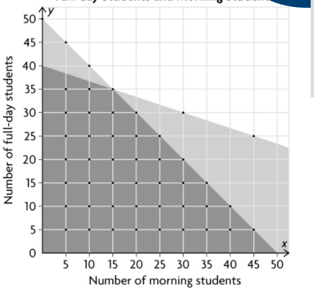
  $x + y \le 50$

Finally, we graphed both linear inequalities on the same coordinate plane.

$x + 3y \le 120$ → $y=\frac{1}{3}x+40$

$x + y \le 50$ $y=-x+50$

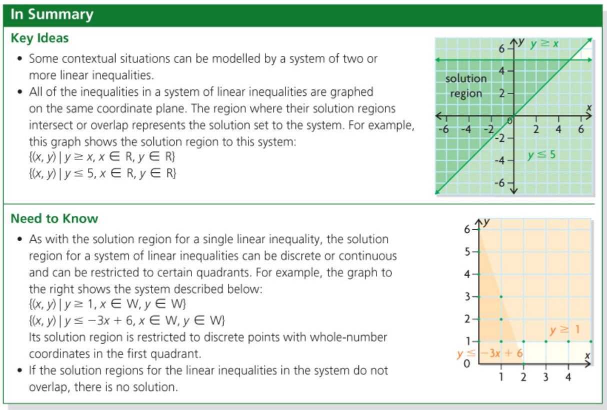
Full-day Students and Morning Studen



We knew that the area where the two solution regions intersect or overlap contains points that represent all the possible combinations of morning and full-day students that will work. We also knew that only whole-number points, such as (24, 24) and (8, 36), make sense.

Pull for Lesson Notes

For the exploration, there is no independent-dependent variable relationship between the two variables. Therefore, students can represent either type of student (morning or full day) along the x-axis. As a result, graphical representations may appear different but yield the same solution. Some students may use guess and check to find several solutions that work for both inequalities. Other students may realize that they can graph each inequality on the same coordinate plane and look for the intersection of the solution region.



**In Summary**

**Key Ideas**

- Some contextual situations can be modelled by a system of two or more linear inequalities.
- All of the inequalities in a system of linear inequalities are graphed on the same coordinate plane. The region where their solution regions intersect or overlap represents the solution set to the system. For example, this graph shows the solution region to this system:
  $\{(x, y) \mid y \geq x, x \in R, y \in R\}$
  $\{(x, y) \mid y \leq 5, x \in R, y \in R\}$

**Need to Know**

- As with the solution region for a single linear inequality, the solution region for a system of linear inequalities can be discrete or continuous and can be restricted to certain quadrants. For example, the graph to the right shows the system described below:
  $\{(x, y) \mid y \geq 1, x \in W, y \in W\}$
  $\{(x, y) \mid y \leq -3x + 6, x \in W, y \in W\}$
  Its solution region is restricted to discrete points with whole-number coordinates in the first quadrant.
- If the solution regions for the linear inequalities in the system do not overlap, there is no solution.

Logic - Hockey Time.doc

**INSTRUCTIONS:**

Complete the grid below by using a '✓' for facts and 'X' for elimination

| | Newcastle | Renous | Tabusintac | Douglastown | Mileron | Cul Tech | Eng | OP | Bio | Math | Van | Tor | Mtl | Edm | Ott |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| March 26 | X | ✓ | X | X | X | | | | | | | | | | |
| May 3 | | X | | | | | | | | | | | | | |
| May 14 | | X | | | | | | | | | | | | | |
| November 13 | | X | | | | | | | | | | | | | |
| November 21 | | X | | | | | | | | | | | | | |
| Van | | | | | | | | | | | | | | | |
| Tor | | | | | | | | | | | | | | | |
| Mtl | | | | | | | | | | | | | | | |
| Edm | | | | | | | | | | | | | | | |
| Ott | | | | | | | | | | | | | | | |
| Cul Tech | | | | | | | | | | | | | | | |
| Eng | | | | | | | | | | | | | | | |
| OP | | | | | | | | | | | | | | | |
| Bio | | | | | | | | | | | | | | | |
| Math | | | | | | | | | | | | | | | |

# HOMEWORK...

Logic - Hockey Time.doc

(Due on FridayFIRST of class)

~~p. 225: #1 & 2~~

p. 235: #2, 5 & 6

QUIZ on Friday!!!

Logic - Hockey Time.doc

**INSTRUCTIONS:**

Complete the grid below by using a '✓' for facts and 'X' for elimination

| | Newcastle | Renous | Tabusintac | Douglastown | Mileron | Cul Tech | Eng | OP | Bio | Math | Van | Tor | Mtl | Edm | Ott |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| March 26 | X | ✓ | X | X | X | X | X | ✓ | X | X | X | X | X | X | ✓ |
| May 3 | X | X | X | ✓ | X | ✓ | X | X | X | X | X | ✓ | X | X | X |
| May 14 | X | X | ✓ | X | X | X | X | X | ✓ | X | X | X | ✓ | X | X |
| November 13 | ✓ | X | X | X | X | X | X | X | X | ✓ | ✓ | X | X | X | X |
| November 21 | X | X | X | X | ✓ | X | ✓ | X | X | X | X | X | X | ✓ | X |
| Van | ✓ | X | X | X | X | X | X | X | X | ✓ | | | | | |
| Tor | X | X | X | ✓ | X | ✓ | X | X | X | X | | | | | |
| Mtl | X | X | ✓ | X | X | X | X | X | ✓ | X | | | | | |
| Edm | X | X | X | X | ✓ | X | ✓ | X | X | X | | | | | |
| Ott | X | ✓ | X | X | X | X | X | ✓ | X | X | | | | | |
| Cul Tech | X | X | X | ✓ | X | | | | | | | | | | |
| Eng | X | X | X | X | ✓ | | | | | | | | | | |
| OP | X | ✓ | X | X | X | | | | | | | | | | |
| Bio | X | X | ✓ | X | X | | | | | | | | | | |
| Math | ✓ | X | X | X | X | | | | | | | | | | |

P236 #6.

2 ham = egg

declare variabks

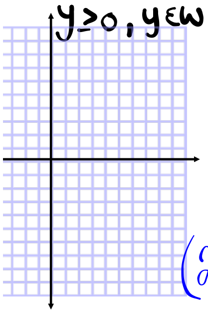
$x \rightarrow$ egg
$y \Rightarrow$ ham-cheese

inequalities

$x + y \leq 450$
$y \leq -x + 450$
$y \geq 2x$

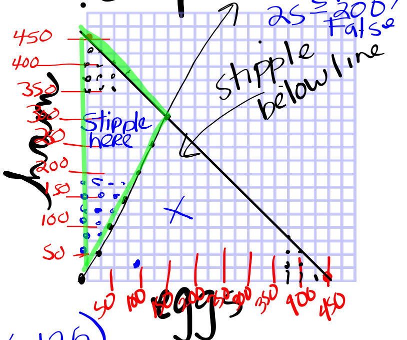
restrictions

$x \geq 0, x \in W$
$y \geq 0, y \in W$



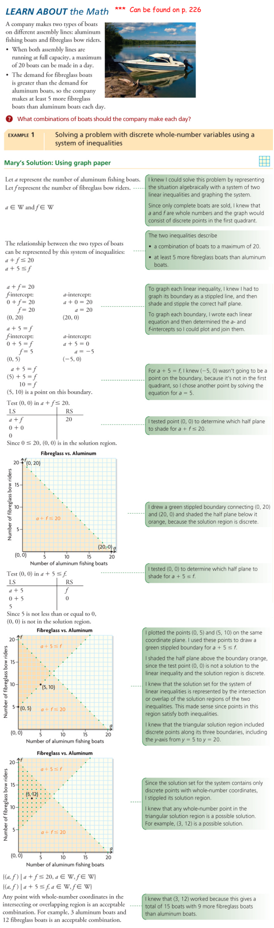
Graph

$25 \geq 2(100)$
$25 \geq 200$
False



(25, 125)

$x + y = 450$
$x \leq 450$



*LEARN ABOUT the Math* *** Can be found on p. 226

A company makes two types of boats on different assembly lines: aluminum fishing boats and fibreglass bow riders.

- When both assembly lines are running at full capacity, a maximum of 20 boats can be made in a day.
- The demand for fibreglass boats is greater than the demand for aluminum boats, so the company makes at least 5 more fibreglass boats than aluminum boats each day.

? What combinations of boats should the company make each day?

**EXAMPLE 1** Solving a problem with discrete whole-number variables using a system of inequalities

**Mary's Solution: Using graph paper**

Let $a$ represent the number of aluminum fishing boats. Let $f$ represent the number of fibreglass bow riders.

$a \in \mathbb{W}$ and $f \in \mathbb{W}$

I knew I could solve this problem by representing the situation algebraically with a system of two linear inequalities and graphing the system.

Since only complete boats are sold, I knew that $a$ and $f$ are whole numbers and the graph would consist of discrete points in the first quadrant.

The relationship between the two types of boats can be represented by this system of inequalities:
$a + f \leq 20$
$a + 5 \leq f$

The two inequalities describe
- a combination of boats to a maximum of 20.
- at least 5 more fibreglass boats than aluminum boats.

$a + f = 20$
$f$-intercept: $0 + f = 20$, $f = 20$, $(0, 20)$
$a$-intercept: $a + 0 = 20$, $a = 20$, $(20, 0)$

$a + 5 = f$
$f$-intercept: $0 + 5 = f$, $f = 5$, $(0, 5)$
$a$-intercept: $a + 5 = 0$, $a = -5$, $(-5, 0)$

To graph each linear inequality, I knew I had to graph its boundary as a stippled line, and then shade and stipple the correct half plane.

To graph each boundary, I wrote each linear equation and then determined the $a$- and $f$-intercepts so I could plot and join them.

$a + 5 = f$
$(5) + 5 = f$
$10 = f$
$(5, 10)$ is a point on this boundary.

For $a + 5 = f$, I knew $(-5, 0)$ wasn't going to be a point on the boundary, because it's not in the first quadrant, so I chose another point by solving the equation for $a = 5$.

Test $(0, 0)$ in $a + f \leq 20$.

| LS | RS |
|---|---|
| $a + f$ | 20 |
| $0 + 0$ | |
| 0 | |

Since $0 \leq 20$, $(0, 0)$ is in the solution region.

I tested point $(0, 0)$ to determine which half plane to shade for $a + f \leq 20$.

Fibreglass vs. Aluminum

I drew a green stippled boundary connecting $(0, 20)$ and $(20, 0)$ and shaded the half plane below it orange, because the solution region is discrete.

Test $(0, 0)$ in $a + 5 \leq f$.

| LS | RS |
|---|---|
| $a + 5$ | $f$ |
| $0 + 5$ | 0 |
| 5 | |

Since 5 is not less than or equal to 0, $(0, 0)$ is not in the solution region.

I tested $(0, 0)$ to determine which half plane to shade for $a + 5 \leq f$.

Fibreglass vs. Aluminum

I plotted the points $(0, 5)$ and $(5, 10)$ on the same coordinate plane. I used these points to draw a green stippled boundary for $a + 5 \leq f$.

I shaded the half plane above the boundary orange, since the test point $(0, 0)$ is not a solution to the linear inequality and the solution region is discrete.

I knew that the solution set for the system of linear inequalities is represented by the intersection or overlap of the solution regions of the two inequalities. This made sense since points in this region satisfy both inequalities.

I knew that the triangular solution region included discrete points along its three boundaries, including the $y$-axis from $y = 5$ to $y = 20$.

Fibreglass vs. Aluminum

Since the solution set for the system contains only discrete points with whole-number coordinates, I stippled its solution region.

I knew that any whole-number point in the triangular solution region is a possible solution. For example, $(3, 12)$ is a possible solution.

$\{(a, f) \mid a + f \leq 20, a \in \mathbb{W}, f \in \mathbb{W}\}$
$\{(a, f) \mid a + 5 \leq f, a \in \mathbb{W}, f \in \mathbb{W}\}$

Any point with whole-number coordinates in the intersecting or overlapping region is an acceptable combination. For example, 3 aluminum boats and 12 fibreglass boats is an acceptable combination.

I knew that $(3, 12)$ worked because this gives a total of 15 boats with 9 more fibreglass boats than aluminum boats.

$2y > x$ $\boxed{y > 2x}$

$20 = 2(10)$
ham egg

$h \geq 2e$
$y \geq 2x$

P237 #8, 10

$30 = 3(10)$
$S = 3r$
$y = 3r$

P241 #6, 7

① declare variables
$x \rightarrow$ school
$y \rightarrow$ rugby

② state restrictions
$x \in W$
$y \in W$

③ Develop inequalities(s) *

$x + y \leq 500$
$y \geq 3x$

④ Graph

## Reflecting

**A.** Is every point on the boundaries of the solution region a possible solution? Explain.

**B.** Are the three points where the boundaries intersect part of the solution region? Explain.

**C.** How would the graph change if fewer than 25 boats were made each day?

**D.** All points with whole-number coordinates in the solution region are valid, but are they all reasonable? Explain.

**Answers**

**A.**

**B.**

**C.**

**D.**

$x \rightarrow$ younger $\quad x \in W$
$y \rightarrow$ older $\quad y \in W$

$x + y \leq 36$

$x \geq 2y$

$\frac{-2y}{-2} \geq \frac{-x}{-2}$

$y \leq \frac{1}{2}x$

$\boxed{p241}$

6. $l < 100 \quad l > 0, l \in R$
$2l + 2w \leq 400 \quad w > 0, w \in R$

7. $P + M \leq 30$
$18P + 10M \leq 470$

*** Can be found on p. 233

**EXAMPLE 3** Solving graphically a problem with continuous positive variables

A sloop is a sailboat with two sails: a mainsail and a jib. When a sail is fully out or up, it is said to be "out 100%." When the winds are high, sailors often reef, or pull in, the sails to be less than their full capability.

- Jim is sailing in winds of 22 knots, so he wants no more than 80% of the mainsail out.
- Jim also wants more mainsail out than jib.

What possible combinations of mainsail and jib can Jim have out?

**Louise's Solution: Using graph paper**

Let $m$ represent the percent of mainsail out.
Let $j$ represent the percent of jib out.

$m \geq 0$ and $j \geq 0$, where $m \in R, j \in R$

I knew that I could solve the problem by representing it algebraically with a system of two linear inequalities and then graphing it.

I knew that the graph would be in the first quadrant since there can't be negative percents of sails out.

I also knew that the solution region would be continuous since decimal percents are possible.

The relationship between the two types of sails can be represented by the following system of two linear inequalities:
$m \leq 80$
$j < m$

The inequalities describe the following information:

- No more than 80% of the mainsail can be out.
- Less jib than mainsail must be out.

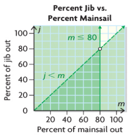
$m \leq 80$
Boundary: $m = 80$
Boundary is a vertical line with an $m$-intercept of 80.

$j < m$
Boundary: $j = m$
Boundary line has a slope of 1 and a $j$-intercept of 0.

I decided to use $m$ as the independent variable.

I examined each inequality to determine its boundary:

- Since $m$ is the independent variable, I knew the boundary for $m \leq 80$ would be a vertical line through $m = 80$.
- I knew the boundary of $j < m$ has a slope of 1 and passes through the point (0, 0).

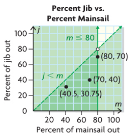


For $m \leq 80$, I drew a solid green vertical line through $m = 80$ and then shaded the half plane to its left green, since the inequality sign is $\leq$.

For $j < m$, I drew a green dashed line through (0, 0) with a slope of 1 and I shaded the half plane below green since the inequality is $<$.

I drew an open dot where the dashed boundary intersects the solid boundary to show that point isn't part of the solution region.

The solution region for the system is a right triangle and consists of all the points in the overlapping region, including the solid boundary and the $m$-axis from 0 to 80.



I looked for several solutions in the solution region. I knew that I could choose points with decimal coordinates since the solution region is continuous.

$\{(m, j) \mid m \leq 80, m \geq 0, j \geq 0, m \in R, j \in R\}$
$\{(m, j) \mid j < m, m \geq 0, j \geq 0, m \in R, j \in R\}$

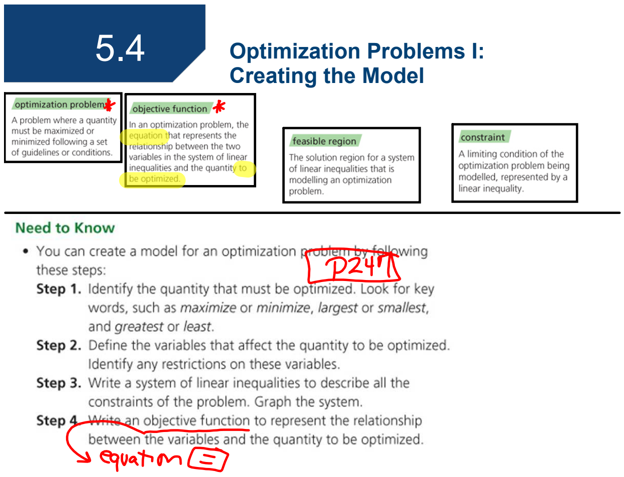
Any point in the solution region represents an acceptable combination. For example,

- 80% of the mainsail and 70% of the jib can be out.
- 70% of the mainsail and 40% of the jib can be out.
- 40.5% of the mainsail and 30.75% of the jib can be out.

**HOMEWORK...**

p. 236: #7 - 10
*NOTE*: Each question requires a graph to
get possible solutions!

Quiz on graphing inequalities on<u>TOMORROW</u>!!!



## 5.4 Optimization Problems I: Creating the Model

**optimization problem** *
A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

**objective function** *
In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized.

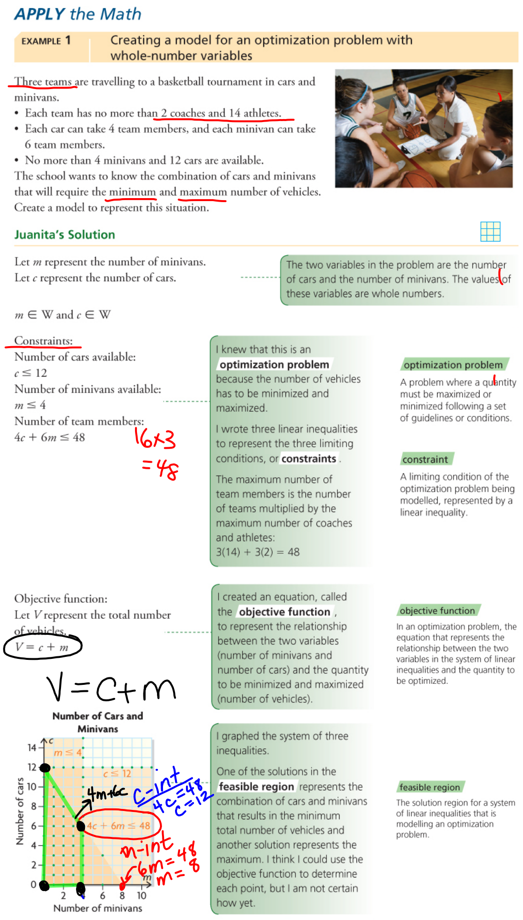
**feasible region**
The solution region for a system of linear inequalities that is modelling an optimization problem.

**constraint**
A limiting condition of the optimization problem being modelled, represented by a linear inequality.

### Need to Know

- You can create a model for an optimization problem by following these steps: $\boxed{P247}$
  **Step 1.** Identify the quantity that must be optimized. Look for key words, such as *maximize* or *minimize*, *largest* or *smallest*, and *greatest* or *least*.
  **Step 2.** Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.
  **Step 3.** Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.
  **Step 4.** Write an objective function to represent the relationship between the variables and the quantity to be optimized.

equation $\boxed{=}$



### *APPLY* the Math

**EXAMPLE 1** Creating a model for an optimization problem with whole-number variables

Three teams are travelling to a basketball tournament in cars and minivans.

- Each team has no more than 2 coaches and 14 athletes.
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available.

The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.

**Juanita's Solution**

Let $m$ represent the number of minivans.
Let $c$ represent the number of cars.

The two variables in the problem are the number of cars and the number of minivans. The values of these variables are whole numbers.

$m \in \mathbb{W}$ and $c \in \mathbb{W}$

Constraints:
Number of cars available:
$c \leq 12$
Number of minivans available:
$m \leq 4$
Number of team members:
$4c + 6m \leq 48$

$16 \times 3$
$= 48$

I knew that this is an **optimization problem** because the number of vehicles has to be minimized and maximized.

I wrote three linear inequalities to represent the three limiting conditions, or **constraints**.

The maximum number of team members is the number of teams multiplied by the maximum number of coaches and athletes:
$3(14) + 3(2) = 48$

**optimization problem**
A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

**constraint**
A limiting condition of the optimization problem being modelled, represented by a linear inequality.

Objective function:
Let $V$ represent the total number of vehicles.
$V = c + m$

$V = C + m$

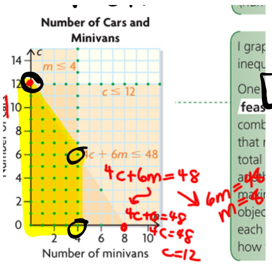
I created an equation, called the **objective function**, to represent the relationship between the two variables (number of minivans and number of cars) and the quantity to be minimized and maximized (number of vehicles).

**objective function**
In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized.

I graphed the system of three inequalities.

One of the solutions in the **feasible region** represents the combination of cars and minivans that results in the minimum total number of vehicles and another solution represents the maximum. I think I could use the objective function to determine each point, but I am not certain how yet.

**feasible region**
The solution region for a system of linear inequalities that is modelling an optimization problem.



$m = 4$
$c = 6$
$\boxed{V = m + c}$
$V = 4 + 6$

$V = 10$

### *Your Turn*

Suppose that the greatest number of athletes changed from 14 to 12 per team. How would Juanita's model change?

### *Answer*

[redacted] n the
42 (12
f the model
follows:

EXAMPLE 2 **Creating a model for a maximization problem with positive real-number variables**

A refinery produces oil and gas.

- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for \$1.10 per litre. Heating oil is projected to sell for \$1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

#### Umberto's Solution

Let $h$ represent the number of litres of heating oil.
Let $g$ represent the number of litres of gasoline.

I knew that this is an optimization problem because the total revenue has to be maximized.

The two variables in the problem are the volume of heating oil and the volume of gasoline, both in litres. Litres are measured using positive real numbers.

Restrictions:
$h \geq 0$ and $g \geq 0$, where $h \in \mathrm{R}$ and $g \in \mathrm{R}$

Constraints:
Ratio of gasoline produced to oil produced:
$g \geq 2h$
Amount of gasoline that can be produced:
$g \leq 6\ 000\ 000$
Amount of oil that can be produced:
$h \leq 9\ 000\ 000$

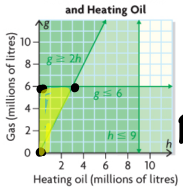
I created inequalities to represent the five constraints of the problem.

I treated the restrictions on each variable as a constraint.

don't graph

Let $R$ represent total revenue from sales of gasoline and heating oil.
Objective function to maximize:
$R = 1.10g + 1.75h$

I wrote an objective function to represent the relationship between the two variables (volume of heating oil and volume of gasoline) and the quantity to be maximized (total revenue).



~~(0,0)~~
~~(0,6)~~
(3,6)
$R = 1.10(6) + 1.75(3)$

I graphed the system of inequalities in the 1st quadrant because of the restrictions on the variables. The feasible region is a right triangle and includes all points on its boundaries.

I think I can use the objective function to determine which point in the feasible region represents the combination of oil and gas that will result in the maximum revenue, but I am not sure how yet.

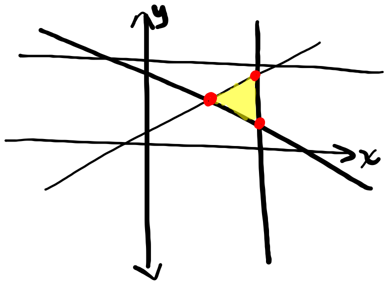
#### In Summary

**Key Ideas**

- To solve an optimization problem, you need to determine which combination of values of two variables results in a maximum or minimum value of a related quantity.
- When creating a model, the first step is to represent the situation algebraically. An algebraic model includes these parts:
  - a defining statement of the variables used in your model
  - a statement describing the restrictions on the variables
  - a system of linear inequalities that describes the constraints
  - an objective function that shows how the variables are related to the quantity to be optimized
- The second step is to represent the system of linear inequalities graphically.
- In optimization problems, any restrictions on the variables are considered constraints. For example, if you are working with positive real numbers, $x \geq 0$ and $y \geq 0$ are constraints and should be included in the system of linear inequalities.

**Need to Know**

- You can create a model for an optimization problem by following these steps:
  - **Step 1.** Identify the quantity that must be optimized. Look for key words, such as *maximize* or *minimize*, *largest* or *smallest*, and *greatest* or *least*.
  - **Step 2.** Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.
  - **Step 3.** Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.
  - **Step 4.** Write an objective function to represent the relationship between the variables and the quantity to be optimized.



## HOMEWORK...

Stapk "hockey"

**Page 248: #1abc, #2, #3**

**NOTE:**

Create a model means graph the solution region

Quiz → Tuesday

## QUIZ TIME...

en finished pass your quiz in and work on the following:

**GOAL**

Solve optimization problems.

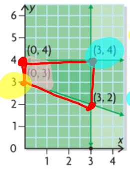
$\boxed{T=5x+y}$

**EXPLORE...**

- The following system of linear inequalities has been graphed below:

System of linear inequalities:
$y \geq 0$
$x \geq 0$
$y \leq 4$
$x \leq 3$
$3y \geq -x + 9$

$y \geq -\frac{1}{3}x+3$



$T=5(0)+4=4$
$T=5(0)+3=3$
$T=5(3)+2=17$
$T=5(3)+4=19$

**a)** For each objective function, what points in the feasible region represent the minimum and maximum values?
**i)** $T = 5x + y$
**ii)** $T = x + 5y$
**b)** What do you notice about the optimal points for the two objective functions? Why do you think this happened?

**SAMPLE ANSWER**

**a) i)** For $T = 5x + y$,

| If (x, y) is... | Then... | |
|---|---|---|
| (3, 2) | $T = 5(3) + 2$<br>$T = 17$ | |
| (3, 4) | $T = 5(3) + 4$<br>$T = 19$ | maximum |
| (0, 3) | $T = 5(0) + 3$<br>$T = 3$ | minimum |
| (0, 4) | $T = 5(0) + 4$<br>$T = 4$ | |

**ii)** For $T = x + 5y$,

| If (x, y) is... | Then... | |
|---|---|---|
| (3, 2) | $T = 3 + 5(2)$<br>$T = 13$ | minimum |
| (3, 4) | $T = 3 + 5(4)$<br>$T = 23$ | maximum |
| (0, 3) | $T = 0 + 5(3)$<br>$T = 15$ | |
| (0, 4) | $T = (0) + 5(4)$<br>$T = 20$ | |

**b)** I noticed that the values of the coefficients of the variables and the values of the variables themselves all contribute to the value of the objective function. For $T = 5x + y$, the $x$-value is multiplied by 5 and the $y$-value is multiplied by 1. For $T = x + 5y$, the $x$-value is multiplied by 1 and the $y$-value is multiplied by 5. In each case, the greater the coordinate that is multiplied by 5, the greater the value of the objective function is. The converse is true for the least values.

### EXAMPLE of an OPTIMIZATION Problem...

Mick and Keith make ~~MP3~~ iphone covers to sell, using beads and stickers.

- At most, 45 covers with stickers and 55 bead covers can be made per day.
- Mick and Keith can make 45 or more covers, in total, each day.
- It costs $0.75 to make a cover with stickers, $1.00 to make one with beads.

Let x represent the number of covers with stickers and let y represent the number of bead covers.

Let C represent the cost of making the covers.

RESTRICTIONS:

$x \in W$
$y \in W$

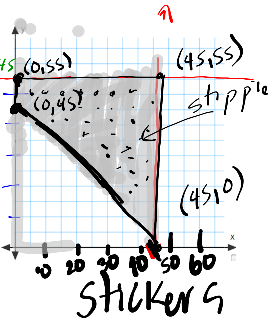
CONSTRAINTS:

$x \leq 45$
$y \leq 55$
$x+y \geq 45$
$y \geq -x+45$

OBJECTIVE FUNCTION: $C=0.75x+1y$

a) Graph the solution set.

Beads



$C=0.75x+1y$
$C=0.75(0)+1(55)=55$
$C=0.75(0)+1(45)=45$
$C=0.75(45)+1(0)=33.75$
$C=0.75(45)+1(55)=88.75$

b) What are the vertices of the feasible region?

c) Which point would result in the maximum value of the objective function?

d) Which point would result in the minimum value of the objective function?

## HOMEWORK...

p. 252: #1 - 3

p. 248: #4 - 6

## 5.5 Optimization Problems II: Exploring Solutions

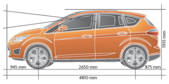
### Need to Know

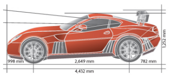
- The solution to an optimization problem is usually found at one of the vertices of the feasible region.
- To determine the optimal solution to an optimization problem using linear programming, follow these steps:
  **Step 1.** Create an algebraic model that includes:
    - a defining statement of the variables used in your model
    - the restrictions on the variables
    - a system of linear inequalities that describes the constraints
    - an objective function that shows how the variables are related to the quantity to be optimized

  **Step 2.** Graph the system of inequalities to determine the coordinates of the vertices of its feasible region.
  **Step 3.** Evaluate the objective function by substituting the values of the coordinates of each vertex.
  **Step 4.** Compare the results and choose the desired solution.
  **Step 5.** Verify that the solution(s) satisfies the constraints of the problem situation.

**optimal solution**
A point in the solution set that represents the maximum or minimum value of the objective function.

**GOAL**
Explore the feasible region of a system of linear inequalities.





### EXPLORE the Math

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs \$8 to make a racing car and \$12 to make a sport-utility vehicle.

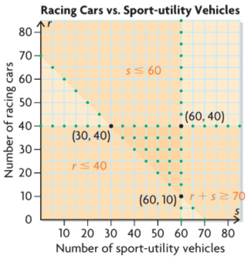
There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

The following model represents this situation. The feasible region of the graph represents all the possible combinations of racing cars ($r$) and sport-utility vehicles ($s$).

Variables:
Let $s$ represent the number of sport-utility vehicles.
Let $r$ represent the number of racing cars.
Let $C$ represent the cost of production.

Restrictions:
$s \in W, r \in W$

Constraints:
$s \geq 0$
$r \geq 0$
$r \leq 40$
$s \leq 60$
$r + s \geq 70$



Objective function to optimize:
$C = 12s + 8r$

❓ How can you use patterns in the feasible region to predict the combinations of sport-utility vehicles and racing cars that will result in the minimum and maximum values of the objective function?

Value of $C = 8r + 12s$

- as $s$ increases: 720 at (40, 30), 840 at (50, 30), 960 at (60, 30)
- as $r$ increases: 860 at (45,40), 780 at (45, 30) 740 at (45, 25)
- in the middle of the solution region: 820 at (45, 35), 880 at (50, 35), 840 at (50, 30)
- at the corners of the solution region: 800 at (60, 10), 1040 at (60, 40), 680 at (30, 40)

### Reflecting

**A.** With a partner, discuss the pattern in the value of $C$ throughout the feasible region. Is the pattern what you expected? Explain.

**B.** As you move from left to right across the feasible region, what happens to the value of $C$?

**C.** As you move from the bottom to the top of the feasible region, what happens to the value of $C$?

**D.** What points in the feasible region result in each **optimal solution**?
**i)** the maximum possible value of $C$
**ii)** the minimum possible value of $C$

**E.** Explain how you could verify that your solutions from part D satisfy each constraint in the model.

### Answers

**A.**

**B.**

**C.**

**D. i)**
**ii)**

**E. i)**

**ii)**

### EXAMPLE #1...

The vertices of the feasible region of a graph of a system of linear inequalities are (-4, -8); (5, 0) and (1, -6). Which point would result in the minimum value of the objective function C = 0.50x + 0.60y?

---

$C = 0.50(-4)+0.60(-8) = -6.8$
$C = 0.50(5)+0.60(0) = 2.5$
$C = 0.50(1)+0.60(-6) = -3.1$

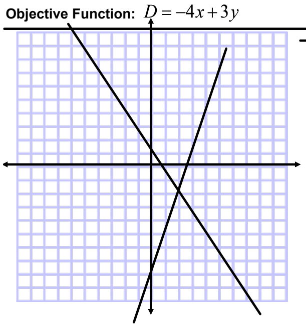
## EXAMPLE #2...

The following model represents an optimization problem. Determine the maximum solution.

**Restrictions:** $x \varepsilon R$ and $y \varepsilon R$

**Constraints:** $y \le 1$; $2y \ge -3x + 2$; $y \ge 3x - 8$

**Objective Function:** $D = -4x + 3y$



---

## EXAMPLE #2...

The following model represents an optimization problem. Determine the maximum solution.

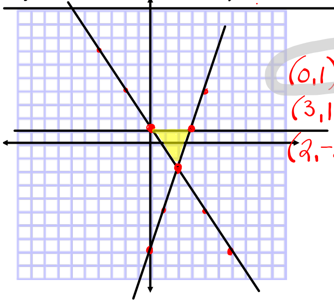
**Restrictions:** $x \varepsilon R$ and $y \varepsilon R$

**Constraints:** $y \le 1$; $2y \ge -3x + 2$; $y \ge 3x - 8$

$y \ge -\frac{3}{2}x + 1$

① ② ③

**Objective Function:** $D = -4x + 3y$



$(0,1) \rightarrow D = -4(0) + 3(1) = 3$

$(3,1) \rightarrow D = -4(3) + 3(1) = -9$

$(2,-2) \rightarrow D = -4(2) + 3(-2) = -14$

---

Four MVHS teams are travelling to a basketball tournament in cars and minivans.

- ~~Each team~~ has no more than 2 coaches and 10 athletes
- Each car can take 4 team members. Each minivan can take 6 team members.
- No more than 6 cars are available, but more than 3 minivans are available.

Mr. Watters wants to know the combination of cars and minivans that will require the maximum number of vehicles...

~~a) Create an algebraic model to represent this situation.~~

b) Graph the model.

$4x + 6y \le 48$

$x \le 6$

$y > 3$

$\boxed{V = x + y}$

c) What combination of cars/minivans will result in the maximum number of vehicles?

d) How many team members can travel in the maximum number of vehicles?

$4x + 6y = 48$

x-int

$4x + 6(0) = 48$

$4x = 48$

$x = 12$

$(12,0)$

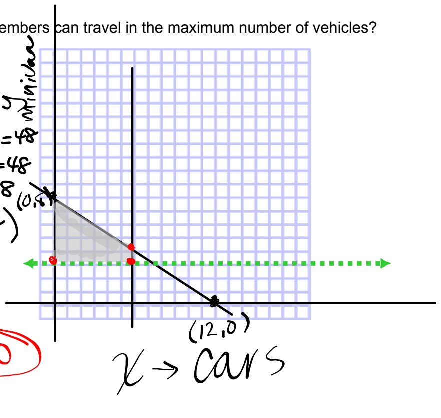
y-int

$4(0) + 6y = 48$

$6y = 48$

$y = 8$

$(0,8)$



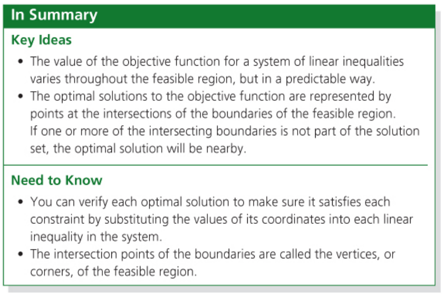
$V = x + y$

~~(0,8) V=8~~

(6,4) V = 10

(0,3)

(6,3?)



**In Summary**

**Key Ideas**

- The value of the objective function for a system of linear inequalities varies throughout the feasible region, but in a predictable way.
- The optimal solutions to the objective function are represented by points at the intersections of the boundaries of the feasible region. If one or more of the intersecting boundaries is not part of the solution set, the optimal solution will be nearby.

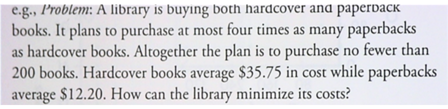
**Need to Know**

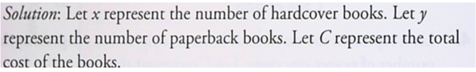
- You can verify each optimal solution to make sure it satisfies each constraint by substituting the values of its coordinates into each linear inequality in the system.
- The intersection points of the boundaries are called the vertices, or corners, of the feasible region.

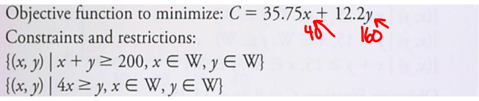
## HOMEWORK...

- Page 259: #1-4

## WARM UP - use graph paper


e.g., *Problem*: A library is buying both hardcover and paperback books. It plans to purchase at most four times as many paperbacks as hardcover books. Altogether the plan is to purchase no fewer than 200 books. Hardcover books average \$35.75 in cost while paperbacks average \$12.20. How can the library minimize its costs?


*Solution*: Let $x$ represent the number of hardcover books. Let $y$ represent the number of paperback books. Let $C$ represent the total cost of the books.

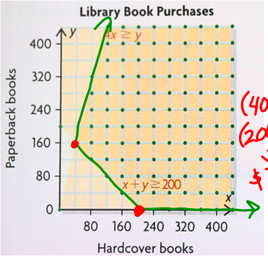

Objective function to minimize: $C = 35.75x + 12.2y$

40 160

Constraints and restrictions:

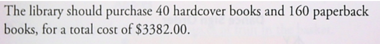
$\{(x, y) \mid x + y \geq 200, x \in W, y \in W\}$

$\{(x, y) \mid 4x \geq y, x \in W, y \in W\}$



(40,160) = \$3382

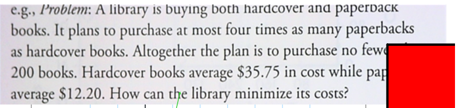
(200,0) = \$7150


The library should purchase 40 hardcover books and 160 paperback books, for a total cost of \$3382.00.

Restrictions

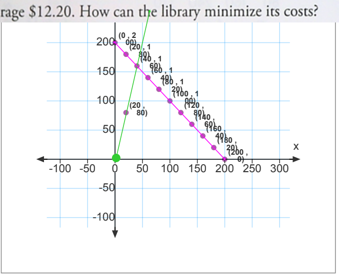
Constraints

ObjectiveFunction


e.g., *Problem*: A library is buying both hardcover and paperback books. It plans to purchase at most four times as many paperbacks as hardcover books. Altogether the plan is to purchase no fewe

200 books. Hardcover books average \$35.75 in cost while pap

average \$12.20. How can the library minimize its costs?



## ONE MORE...

Malia and Lainey are baking cupcakes and banana mini-loaves to sell at a school fundraiser...

- No more than 60 cupcakes and 35 mini-loaves can be made each day.
- Malia and Lainey can mke no more than 80 baked goods, in total, each day.
- It costs \$0.50 to make a cupcake and \$0.75 to make a mini-loaf.

Determine the maximum cost to produce the baked goods.

---

**Variables & restrictions**

c → cupcakes $c \in W$

m → miniloaf $m \in W$

**Constraints**

$c \geq 0$ $c \leq 60$

$m \geq 0$ $m \leq 35$

$m + c \leq 80$

} Graph shade

**Objective Function**

$C = 0.5c + 0.75m$

**ONE MORE...**

Malia and Lainey are baking cupcakes and banana mini-loaves to sell at a school fundraiser...

- No more than 60 cupcakes and 35 mini-loaves can be made each day.
- Malia and Lainey can mke no more than 80 baked goods, in total, each day.
- It costs $0.50 to make a cupcake and $0.75 to make a mini-loaf.

Determine the minimum cost to produce the baked goods.

---

Restrictions

C → cupcakes $C \in W$

m → mini Loaf $m \in W$

Objective

$0.50C + 0.75m = C$

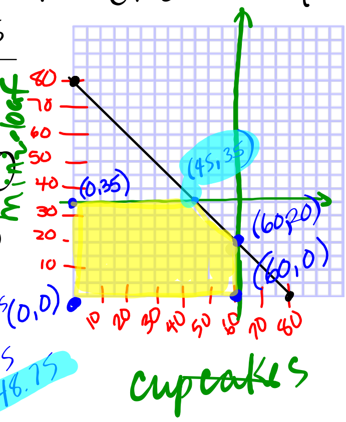
Constraints

$C \leq 60$

$m \leq 35$

$C + m \leq 80$

$m \leq -C + 80$



$0.5(0) + 0.75(0) = 0$

$0.5(0) + 0.75(35) = 26.25$

$0.5(60) + 0.75(0) = 30$

$0.5(60) + 0.75(20) = 45$

$0.5(45) + 0.75(35) = 48.75$

**HOMEWORK...**

p. 261: #5, 7, 8, 11, 13

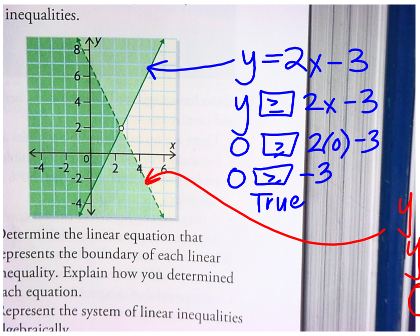
## HOMEWORK: Test is on THURSDAY!!

Review/Practice Questions...

- p. 239: Mid-Chapter Review (Frequently Asked Questions) *
- p. 241: Mid-Chapter Practice Questions
- p. 266: Chapter Review (Frequently Asked Questions) *
- p. 267: Chapter Practice Questions
- p. 265: Chapter Self-Test (Do this AFTER you practice)

LOGIC PROBLEM is due Tuesday!

Logic - Inequality Sudoku #2.doc



inequalities.

Determine the linear equation that
epresents the boundary of each linear
nequality. Explain how you determined
ach equation.
Represent the system of linear inequalities
lgebraically

$y = 2x - 3$

$y \boxed{\geq} 2x - 3$

$0 \boxed{\geq} 2(0) - 3$

$0 \boxed{\geq} -3$

True

$y = -2x + 7$

$y \boxed{<} -2x + 7$

$0 < 7$

True

p268.

#7. m→male $m \in W$

f→female $f \in W$

$$\boxed{\begin{array}{c} m \geq 3f \\ m+f \leq 28 \end{array}}$$

graph!

find vertices

Objective function

$R = 115m + 90f$

$m \geq 3f$

$12 = 3(4)$

Max $( \quad , \quad )$

## Attachments

NOTES - Graphing Linear Relationships.docx

Puzzle Worksheet - Graphing Lines.docx

Puzzle Worksheet - Graphing Inequalities with One Variable.docx

fm6s1-p5.tns

6Ws1e1.mp4

6Ws1e2.mp4

Worksheet - Graphing Linear Inequalities.pdf

fm6s1-p9.tns

6Ws1e3.mp4

6Ws3e2.mp4

6Ws4e1.mp4

Puzzle Worksheet - Graphing Linear Inequalities with Two Variables.pdf

Worksheet - Solving Systems of Linear Inequalities.pdf

Warm Up - Prior Knowledge for Coordinate Geometry.docx

NOTES - Graphing a Linear Inequation.docx

Logic - Three Little Pigs.doc

Puzzle Worksheet - Systems of Linear Inequations.docx

Worksheet - Systems of Linear Inequations.docx

Example - Application of a Linear Inequality.docx

Logic - Hockey Time.doc

Logic - Inequality Sudoku #2.doc