APRIL 19, 2016

UNIT 7: SIMILARITY AND TRANSFORMATIONS

7.4: SIMILAR TRIANGLES

M. MALTBY INGERSOLL
MATH 9

WHAT'S THE POINT OF TODAY'S LESSON?

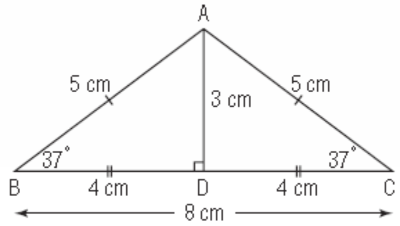
We will continue working on the Math 9 Specific Curriculum Outcome (SCO) "Shape and Space 3" OR "SS3" which states:

"Demonstrate an understanding of similarity of polygons."

BD = 4 cm
So, BC = 2 × 4 cm
= 8 cm

I know that the sum of the angles in a triangle is 180°.
So, I can calculate the measure of ∠BAC.

$\angle BAC + \angle ACD + \angle ABD = 180°$
$\angle BAC + 37° + 37° = 180°$
$\angle BAC + 74° = 180°$
$\angle BAC + 74° - 74° = 180° - 74°$
$\angle BAC = 106°$

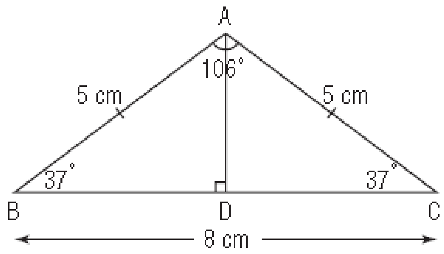


My friend Janelle showed me a different way to calculate.
She recalled that the line AD is a line of symmetry for an isosceles triangle.
So, ΔABD is congruent to ΔACD.

This means that ∠BAD = ∠CAD
Janelle calculated the measure of ∠BAD in ΔABD.

$\angle BAD + 37° + 90° = 180°$
$\angle BAD + 127° = 180°$
$\angle BAD + 127° - 127° = 180° - 127°$
$\angle BAD = 53°$

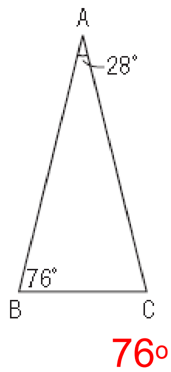
Then, $\angle BAC = 2 \times 53°$
$= 106°$



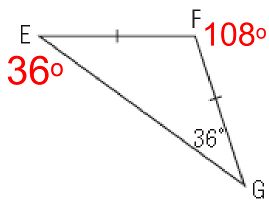
Check

1. Calculate the measure of each angle.

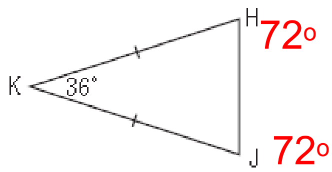
a) ∠ACB



b) ∠GEF and ∠GFE



c) ∠HJK and ∠KHJ



$\angle C = 180 - (76 + 28)$ (SATT)
$= 180 - 104$
$= 76°$

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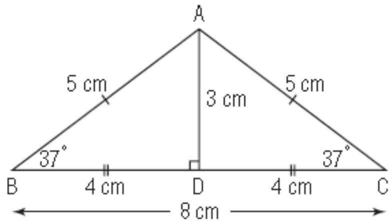

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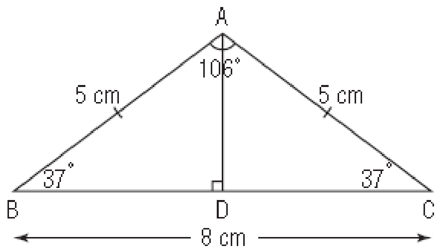

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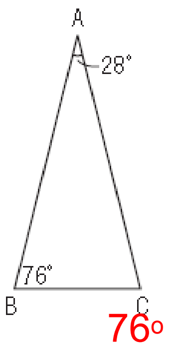

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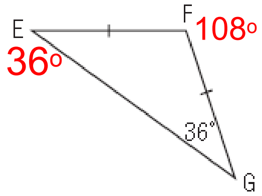

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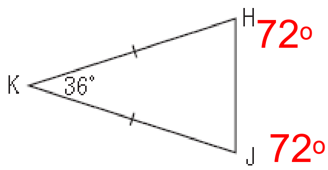

∠E = 36° (ITT)

∠F = 180 − 2(36) (SATT)
= 180 − 72
= 108°

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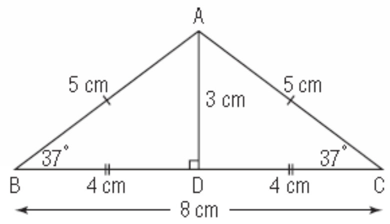

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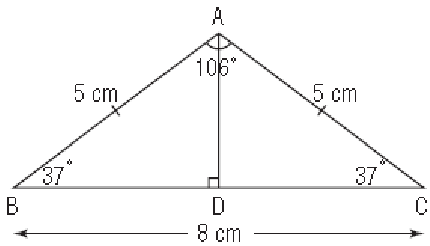

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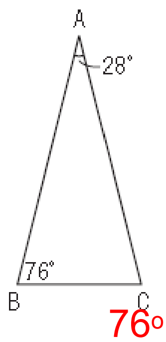

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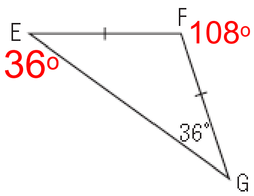

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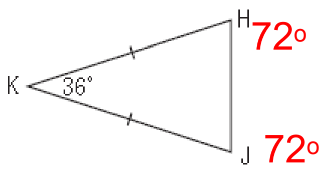


$\angle H = \angle J = \frac{180 - 36}{2}$ (ITT/SATT)
$= \frac{144}{2}$
$= 72°$

SIMILAR TRIANGLES

TO IDENTIFY SIMILAR TRIANGLES:

* **the measures of the 3 pairs of corresponding angles must be EQUAL**

OR

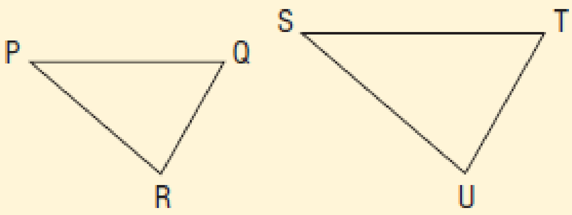
* **the ratios of the lengths of the 3 pairs of corresponding sides must be EQUAL; in other words, corresponding sides are proportional**

***MMS9*, Page 344:**

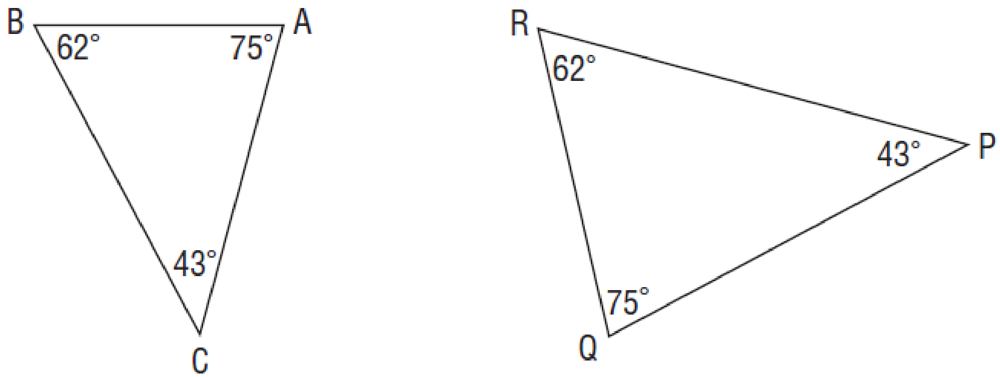
Properties of Similar Triangles

To identify that ΔPQR and ΔSTU are similar, we only need to know that:

- $\angle P = \angle S$ and $\angle Q = \angle T$ and $\angle R = \angle U$; *or*
- $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$

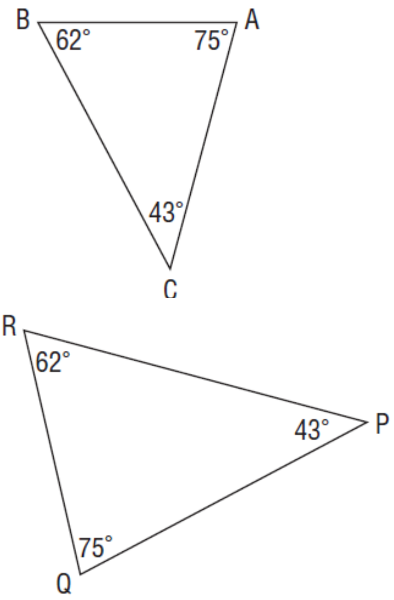


ARE THESE TWO TRIANGLES SIMILAR?



EXAMPLE - How you show PROOF OF SIMILARITY (AAA) in your work:

(NOTE: "AAA" = angle; angle; angle)



$\angle A = \angle Q$ **(GIVEN)**

$\angle B = \angle R$ **(GIVEN)**

$\angle C = \angle P$ **(GIVEN)**

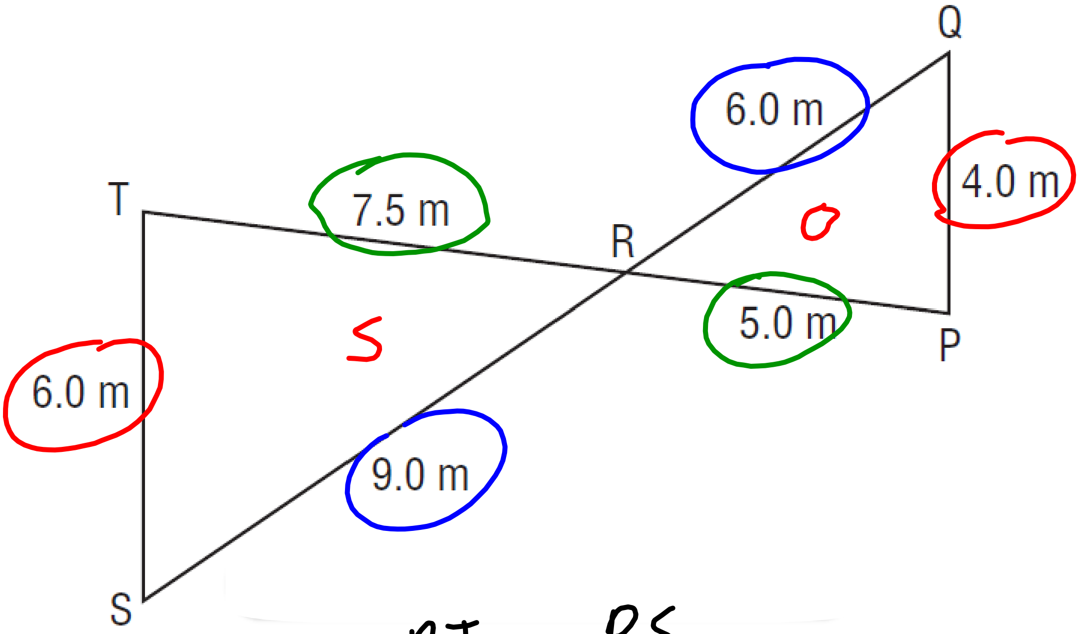
$\therefore \triangle ABC \sim \triangle QRP$ **(AAA)**

$$\Delta ABC \sim \Delta QRP$$

SYMBOL FOR "SIMILAR TO"

THIS IS CALLED A
"SIMILARITY STATEMENT".

ARE THESE TWO TRIANGLES SIMILAR?



$$\frac{ST}{PQ} = \frac{RT}{RP} = \frac{RS}{RQ}$$

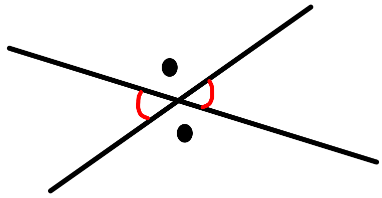
$$\frac{6}{4} = \frac{7.5}{5} = \frac{9}{6}$$

$$1.5 = 1.5 = 1.5$$

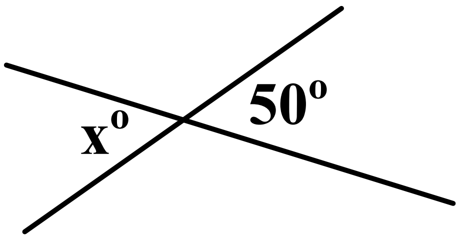
$\therefore \triangle RST \sim \triangle RQP$ (Ratios)

There are two angle theorems that you will need for your similar triangles proofs:

1. **OPPOSITE ANGLES THEOREM (OAT):**
   opposite angles are EQUAL



Ex.:



$\angle x = 50°$ (OAT)

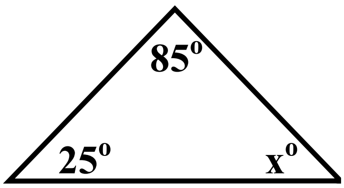
2. SUM OF THE ANGLES IN A TRIANGLE THEOREM (SATT): the sum of the angles in a triangle is 180°.

Ex.: Calculate the unknown angle measure.

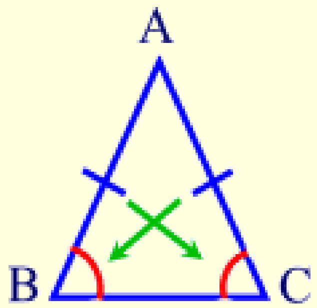
$\angle x = 180 - (25 + 85)$ (SATT)

$= 180 - 110$

$= 70^\circ$



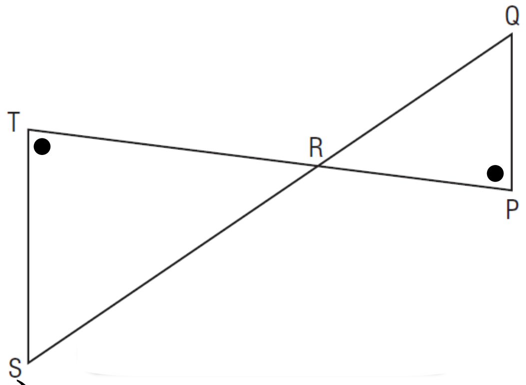
3. ISOSCELES TRIANGLE THEOREM (ITT) : The two angles that are opposite to the two congruent sides in an isosceles triangle are also congruent.



If : $\overline{AB} \cong \overline{AC}$

then : $\measuredangle B \cong \measuredangle C$

EXAMPLE: **PROVE** **that the triangles in the diagram below are SIMILAR.**



<T = <P (given)
<R = <R (OAT)
<S = <Q (SATT)

$\therefore \triangle RST \sim \triangle RQP$ **(AAA)**

CONCEPT REINFORCEMENT:

MMS9:

PAGE 349: #4 (should say, "Are the triangles in each pair..."), #5 & #6 (no proofs required - told the triangles similar; #6b,c on pg 350)