1.2 - Validity of Conjectures?

In Summary page 17

Key Idea

- Some conjectures initially seem to be valid, but are shown not to be valid after more evidence is gathered.

Need to Know

- The best we can say about a conjecture reached through inductive reasoning is that there is evidence either to support or deny it.
- A conjecture may be revised, based on new evidence.

1.3 - Counterexamples

In Summary page 22

Key Ideas

- Once you have found a counterexample to a conjecture, you have disproved the conjecture. This means that the conjecture is invalid.
- You may be able to use a counterexample to help you revise a conjecture.

Need to Know

- A single counterexample is enough to disprove a conjecture.
- Even if you cannot find a counterexample, you cannot be certain that there is not one. Any supporting evidence you develop while searching for a counterexample, however, does increase the likelihood that the conjecture is true.

HOMEWORK...

p. 17: #1 & 2

p. 22: #1, 3, 5, 8, 12, 17

1.4 Proving Conjectures: Deductive Reasoning

GOAL

Prove mathematical statements using a logical argument.

Every day, you use **deductive thinking** to **deduce** new information.

In this course, you will use this method to deduce the properties of geometric figures and many geometric relationships. For example:

Step A General Statement	**And**	**Step B Particular Statement**	Thus	**Step C Conclusion**
During a game, five players are used on a basketball team.	And	UNB is playing basketball.	Thus	UNB uses five players on the basketball team.
All isosceles triangles have two equal sides.	And	Triangle ABC is isosceles.	Thus	Two sides of Triangle ABC are equal.

In Step A, based on your earlier knowledge (your experience, what you have learned in life) you accept certain general statements to be true. In Step B you are confronted with a particular case that is related to a general statement. Lastly, in Step C, you deduce a conclusion based upon Step A and B.

KEY TERMS...

deductive reasoning
Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

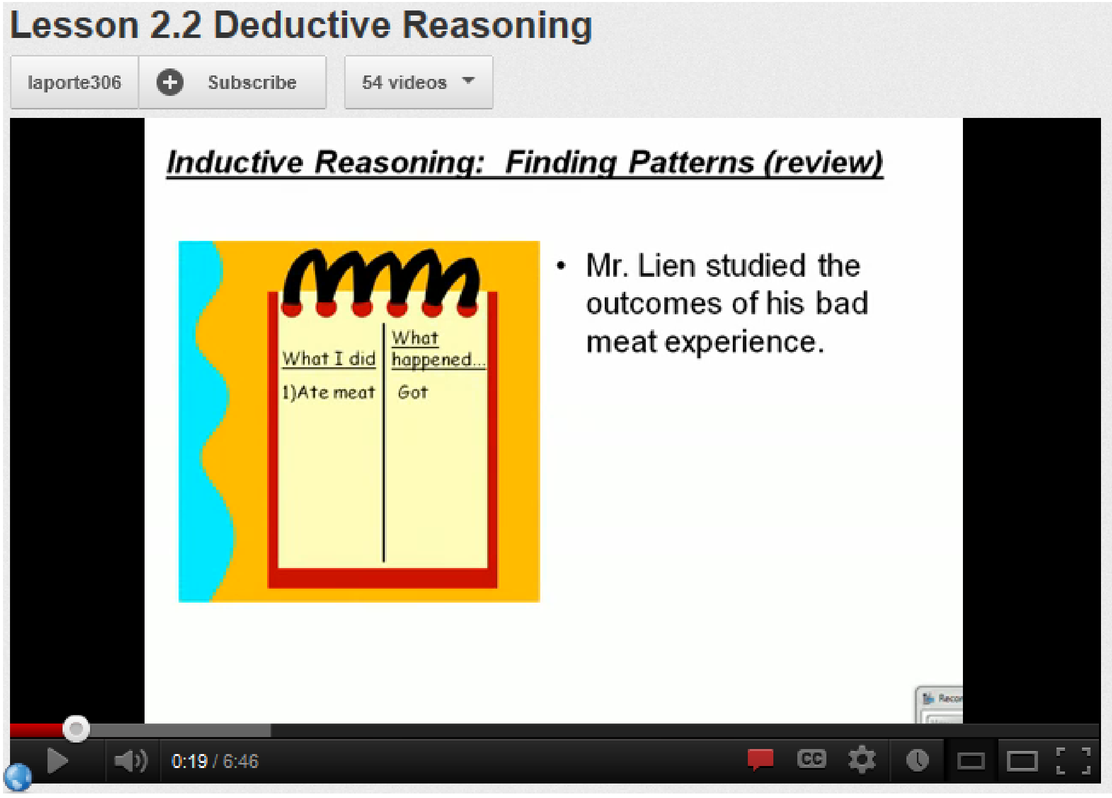
proof
A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

two-column proof
A presentation of a logical argument involving deductive reasoning in which the statements of the argument are written in one column and the justifications for the statements are written in the other column.

generalization
A principle, statement, or idea that has general application.

We can often use the **transitive property** in deductive reasoning. According to this property, **if two things are equal to the same thing, then they are equal to one another**. We can express this property mathematically:

If $a = b$ and $b = c$, then $a = c$.


Lesson 2.2 Deductive Reasoning
laporte306
Subscribe
54 videos
Inductive Reasoning: Finding Patterns (review)
What I did
What happened...
1)Ate meat
Got
• Mr. Lien studied the outcomes of his bad meat experience.
0:19 / 6:46

LEARN ABOUT the Math

Jon discovered a pattern when adding integers:

$$1 + 2 + 3 + 4 + 5 = 15$$
$$(-15) + (-14) + (-13) + (-12) + (-11) = -65$$
$$(-3) + (-2) + (-1) + 0 + 1 = -5$$

×5

Deductive

middle # → x

$(x-2)+(x-1)+(x)+(x+1)+(x+2)$

$5x$

sum will be 5 times the middle

He claims that whenever you add five consecutive integers, the sum is always 5 times the median of the numbers.

? How can you prove that Jon's conjecture is true for all integers?

p. 27

EXAMPLE 1 Connecting conjectures with reasoning

Prove that Jon's conjecture is true for all integers.

Pat's Solution

$$5(3) = 15$$
$$5(-13) = -65$$
$$5(-1) = -5$$

The median is the middle number in a set of integers when the integers are arranged in consecutive order. I observed that Jon's conjecture was true in each of his examples.

$$210 + 211 + 212 + 213 + 214 = 1060$$
$$5(212) = 1060$$

I tried a sample with greater integers, and the conjecture still worked.

Let x represent any integer.
Let S represent the sum of five consecutive integers.
$S = (x - 2) + (x - 1) + x + (x + 1) + (x + 2)$

I decided to start my **proof** by representing the sum of five consecutive integers. I chose x as the median and then wrote a **generalization** for the sum.

proof
A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

generalization
A principle, statement, or idea that has general application.

$$S = (x + x + x + x + x) + (-2 + (-1) + 0 + 1 + 2)$$
$$S = 5x + 0$$

I simplified by gathering like terms.

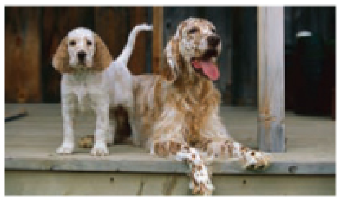
$S = 5x$
Jon's conjecture is true for all integers.

Since x represents the median of five consecutive integers, $5x$ will always represent the sum.

p. 29

EXAMPLE 3 Using deductive reasoning to make a valid conclusion

All dogs are mammals. All mammals are vertebrates. Shaggy is a dog. What can be deduced about Shaggy?

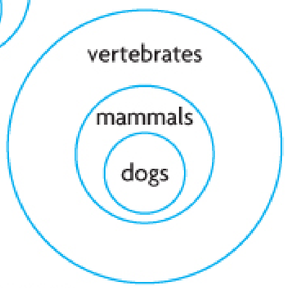


Oscar's Solution

Shaggy is a dog.

All dogs are mammals.

These statements are given. I represented them using a Venn diagram.



All mammals are vertebrates.

This statement is given. I modified my diagram.

Therefore, through deductive reasoning, Shaggy is a mammal and a vertebrate.

Transitive Property...

a = b AND b = c therefore, a = c

In Summary

Key Idea

- Inductive and deductive reasoning are useful in problem solving.

Need to Know

- Inductive reasoning involves solving a simpler problem, observing patterns, and drawing a logical conclusion from your observations to solve the original problem.
- Deductive reasoning involves using known facts or assumptions to develop an argument, which is then used to draw a logical conclusion and solve the problem.

HOMEWORK...

p. 48: #1 - ~~13~~ 9
(OMIT #5, 8, ~~10, 11~~)