

**In Summary**

**Key Idea**

- A single error in reasoning will break down the logical argument of a deductive proof. This will result in an invalid conclusion, or a conclusion that is not supported by the proof.

**Need to Know**

- Division by zero always creates an error in a proof, leading to an invalid conclusion.
- Circular reasoning must be avoided. Be careful not to assume a result that follows from what you are trying to prove.
- The reason you are writing a proof is so that others can read and understand it. After you write a proof, have someone else who has not seen your proof read it. If this person gets confused, your proof may need to be clarified.

**HOMEWORK...**

p. 42: #1 - 10  
(omit #8)

*Questions!*

*# 2, 4*

2. According to this proof,  $5 = 7$ .  
Identify the error.

**Proof**

$$1 = 1 + 1$$

$$2(1) = 2(1 + 1)$$

$$2(1) + 3 = 2(1 + 1) + 3$$

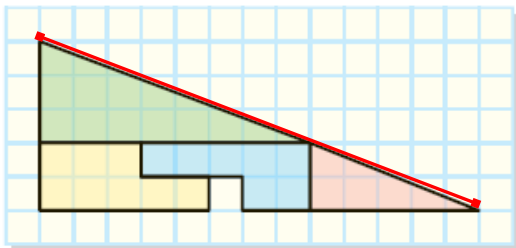
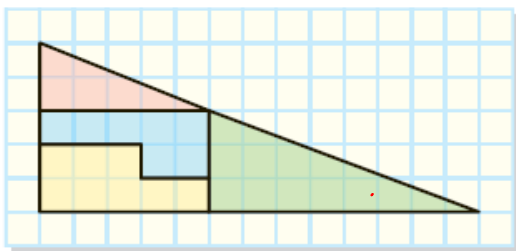
$$2 + 3 = 4 + 3$$

$$5 = 7$$

*false*

**PRACTISING**

4. Noreen claims she has proved that  $32.5 = 31.5$ .



Is Noreen's proof valid? Explain.

6. Connie tried this number trick:

- Write down the number of your street address.
- Multiply by 2.
- Add the number of days in a week.
- Multiply by 50.
- Add your age.
- Subtract the number of days in a year.
- Add 15.

13100

Ans+37	1310350
Ans-365	1310387
Ans+15	1310022
■	1310037



*Handwritten notes:*  
 - A red circle around the '0' in the final result '1310037'.  
 - An arrow pointing from the circled '0' to the word 'Hundreds' written in red.  
 - Another arrow pointing from the circled '0' to the letter 'a' written in red.

Connie's result was a number in which the tens and ones digits were her age and the rest of the digits were the number from her street address. She tried to prove why this works, but her final expression did not make sense.

Let  $n$  represent any house number. ✓  
 $2n$  Multiply by 2. ✓  
 $2n + 7$  Add the number of days in a week. ✓  
 $100n + 350$  Multiply by 50. ✓  
 Let  $a$  represent any age. ✓  
 $100n + 350 + a$  Add your age. ✓  
 $100n + 350 + a - 360$  Subtract the number of days in a year. ✓  
 $100n + a + 5$  Add 15. ✓

*Handwritten corrections and notes:*  
 - A red line with the word 'error' written in red, pointing to the subtraction step in the algebraic proof.  
 - The expression  $100n + 350 + a - 360$  is circled in red.  
 - The expression  $100n + a + 5$  is circled in red.  
 - The number '365' is written in red above the subtraction step.  
 - The number '15 + 15' is written in red above the final addition step.

- Try this number trick to see if you get the same result as Connie.
- Determine the errors in her proof, and then correct them.
- Explain why your final algebraic expression describes the result of this number trick.

7. According to this proof,  $2 = 1$ . Determine the error in reasoning.

Let  $a = b$ .

$$\begin{aligned}
 a^2 &= ab \\
 a^2 + a^2 &= a^2 + ab \\
 2a^2 &= a^2 + ab \\
 2a^2 - 2ab &= a^2 + ab - 2ab \\
 2a^2 - 2ab &= a^2 - ab \\
 2(a^2 - ab) &= 1(a^2 - ab) \\
 2 &= 1
 \end{aligned}$$

- Multiply by  $a$ . ✓
- Add  $a^2$ . ✓
- Simplify. ✓
- Subtract  $2ab$ . ✓
- Simplify. ✓
- Factor. ✓
- Divide by  $(a^2 - ab)$ . ✗

$$a^2 - ab = 0$$

Divide by 0

9. Brittney said she could prove that a strip of paper has only one side. She took a strip of paper, twisted it once, and taped the ends together. Then she handed her friend Amber a pencil, and asked Amber to start at any point and draw a line along the centre of the paper without lifting the pencil. Does a strip of paper have only one side? Why or why not?



Möbius Strip

10. Brenda was asked to solve this problem:

Three people enjoyed a meal at a Thai restaurant. The waiter brought a bill for \$30. Each person at the table paid \$10.

Later the manager realized that the bill should have been for only \$25, so she sent the waiter back to the table with \$5.

The waiter could not figure out how to divide \$5 three ways, so he gave each person \$1 and kept \$2 for himself.

Each of the three people paid \$9 for the meal.

$$9 \cdot 3 = 27$$

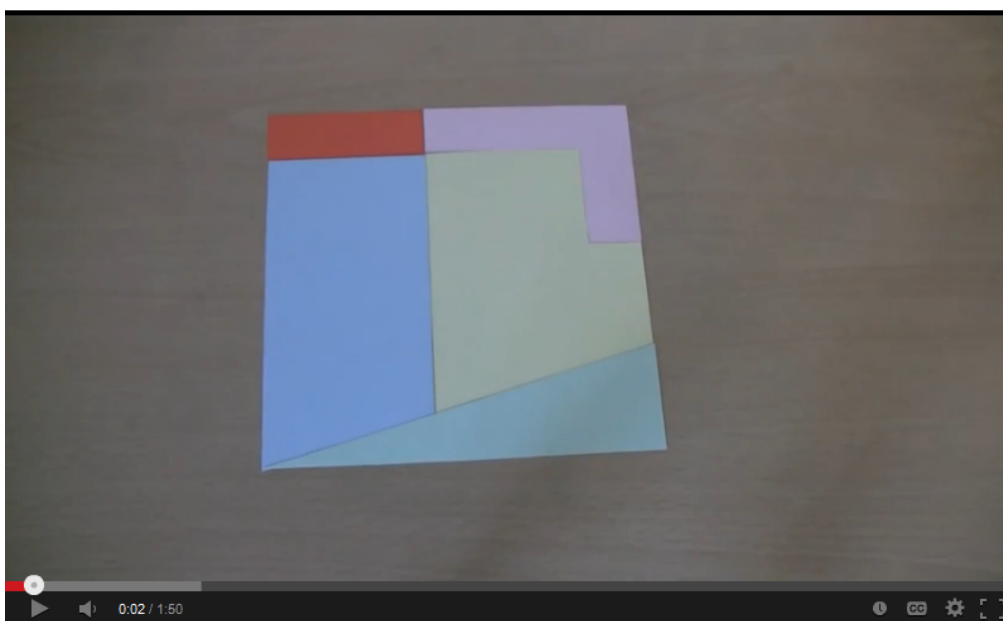
The waiter kept \$2.

$$27 + 2 = 29$$

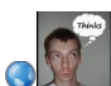


What happened to the other dollar?

Does the question make sense? How should Brenda answer it?



### A Maths Puzzle: The Missing Square Solution

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THE MYSTERY  
OF THE  
VANISHING DOLLAR

Mystery of the Missing Dollar Puzzle

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14 1

## 1.6

## Reasoning to Solve Problems

**GOAL**

Solve problems using inductive or deductive reasoning.

**EXPLORE...**

- Suppose that you are lost in the woods for hours and come upon a cabin. In the cabin, you find a lantern, a candle, a wood stove with wood in it, and a match. What do you light first?

**SAMPLE ANSWER**

I would light the match first. If I didn't, I couldn't light any of the other items. I would light the candle next, since it would stay lit for longer than the match and would allow me to light the other two items. Also, it's less likely that I would make an error or fail when lighting the candle. The lantern and the stove would be more difficult to light.



### INVESTIGATE the Math

Emma was given this math trick:

- Choose a number.
- Multiply by 6.
- Add 4.
- Divide by 2.
- Subtract 2.

Emma was asked to use inductive reasoning to make a conjecture about the relationship between the starting and ending numbers, and then use deductive reasoning to prove that her conjecture is always true. Here is her response to the problem:

**Inductive reasoning:**

#	$\times 6$	$+4$	$\div 2$	$-2$
5	30	34	17	15
-3	-18	-14	-7	-9
0	0	4	2	0
24	144	148	74	72

I followed the steps to work through four examples.

Conjecture: It is 3 times.

$$\frac{6d + 4}{2}$$

$$3d + 2 - 2$$

$3d$   
 ↑  
 3 times the original

**Deductive reasoning:**

I chose  $d$ .

Then I multiplied, added, divided, and subtracted to get an expression.

$$\left(\frac{6d + 4}{2}\right) - 2$$

It worked.

It simplified to  $3d$ .

**?** How can Emma's communication about her reasoning be improved?

- With a partner, explain why Emma might have chosen the values she did.
- What details are missing from the deductive reasoning Emma used to arrive at the expression  $3d$ ?
- Improve Emma's conjecture, justifications, and explanations.

**Answers**

- Emma might have chosen the four values because each value represents a different attribute. One value is positive, another is negative, another is zero, and the last is a larger number. With this variety, Emma might have thought that she had sampled sufficiently from the range of possible values.
- The explanation does not include reasons for each step, nor does it show what each step looks like. It provides only a summary.
- Conjecture: The resulting value will always be three times the starting value. Justification and explanation:

Let $d$ represent any number.	$d$
Multiply by 6.	$6d$
Add 4.	$6d + 4$
Divide by 2.	$\frac{6d + 4}{2} = 3d + 2$
Subtract 2.	$3d + 2 - 2 = 3d$
The resulting value is three times the starting value.	$3d$

## In Summary

### Key Idea

- Inductive and deductive reasoning are useful in problem solving.

### Need to Know

- Inductive reasoning involves solving a simpler problem, observing patterns, and drawing a logical conclusion from your observations to solve the original problem.
- Deductive reasoning involves using known facts or assumptions to develop an argument, which is then used to draw a logical conclusion and solve the problem.

## **HOMEWORK...**

**p. 48: #10 - 16 (omit 14)**

**p. 55: #5, 7, 10**

WARM UP PROBLEM: Need 4 gallons using only a 3 and 5 gallon jugs???



- ① fill 5 gal
- ② fill 3 gal
- ③ empty 3 gal
- ④ pour 2 gal into 3 gal
- ⑤ fill 5 gal  
fill 3 gal

SOLUTIONS...

there is an alternate way to solve this:

1. fill the 3 gallon jug
2. pour that 3 gallons into the 5 gallon jug
3. refill the 3 gallon jug
4. fill the 5 gallon jug to the top, leaving 1 gallon in the 3 gallon jug
5. empty the 5 gallon jug
6. pour the 1 gallon from the 3 gallon jug into the 5 gallon jug
7. refill the 3 gallon jug
8. pour that 3 gallons into the 5 gallon jug which already has 1 gallon in it for a total of 4 gallons.

Step 1. Fill 5 gallon jug

Step 2. Pour 5 gallon jug into 3 gallon jug, leaving 2 remaining gallons in 5 gallon jug.

Step 3. Empty 3 gallon jug.

Step 4. Pour 2 gallons from 5 gallon jug into 3 gallon jug, leaving 1 gallon of empty space.

Step 5. Refill 5 gallon jug.

Step 6. Pour water from 5 gallon jug into 3 gallon jug, which already has 2 gallons in it, and only 1 gallon of empty space, leaving exactly 4 gallons in the 5 gallon jug.



**Die Hard III**

Maths problem-solving with Bruce Willis and Samuel L. Jackson

0:05 / 2:06

Die Hard III - the water jug problem.

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