

Geometric Proofs... The 'Two-Column Proof'

Key Terms (in your notes)...

deductive reasoning

Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

proof

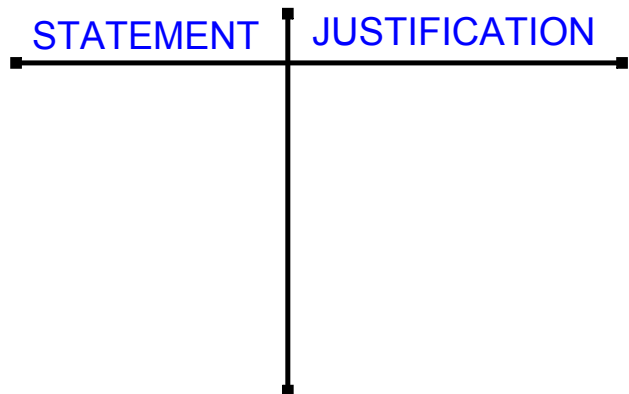
A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

transitive property

If two quantities are equal to the same quantity, then they are equal to each other.
If $a = b$ and $b = c$, then $a = c$.

two-column proof

A presentation of a logical argument involving deductive reasoning in which the statements of the argument are written in one column and the justifications for the statements are written in the other column.



*****ADD this one to your notes...**

converse

A statement that is formed by switching the premise and the conclusion of another statement.

EXAMPLES...

Conjecture: If it is raining outside, then the grass is wet.

CONVERSE: If the grass is wet, then it is raining.

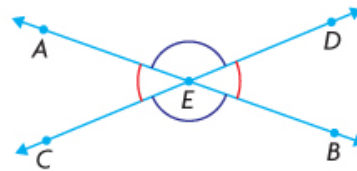
THEOREM: If you have parallel lines, then the corresponding angles are equal.

CONVERSE: If the corresponding angles are equal, then the lines are parallel.

p. 29

EXAMPLE 4 Using deductive reasoning to prove a geometric conjecture

Prove that when two straight lines intersect, the vertically opposite angles are equal.



Jose's Solution: Reasoning in a two-column proof

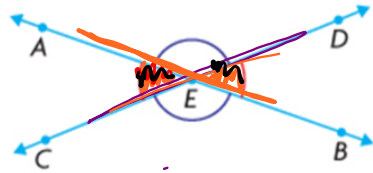
Statement	Justification
$\angle AEC + \angle AED = 180^\circ$	Supplementary angles
$\angle AEC = 180^\circ - \angle AED$	Subtraction property
$\angle BED + \angle AED = 180^\circ$	Supplementary angles
$\angle BED = 180^\circ - \angle AED$	Subtraction property
$\angle AEC = \angle BED$	Transitive property

p. 29

EXAMPLE 4

Using deductive reasoning to prove a geometric conjecture

Prove that when two straight lines intersect, the vertically opposite angles are equal.



STATEMENT	JUSTIFICATION
$\angle AEC + \angle AED = 180^\circ$	SAT
$\angle AEC = 180^\circ - \angle AED$	Rearranged
$\angle DEB + \angle AED = 180^\circ$	SAT
$\angle DEB = 180^\circ - \angle AED$	Rearranged
$\angle AEC = \angle DEB$	Transitive

$$\angle AEC + \angle AED = 180^\circ$$

$$\angle AEC = 180^\circ - \angle AED$$

$$\angle DEB + \angle AED = 180^\circ$$

$$\angle DEB = 180^\circ - \angle AED$$

$$\angle AEC = \angle DEB$$

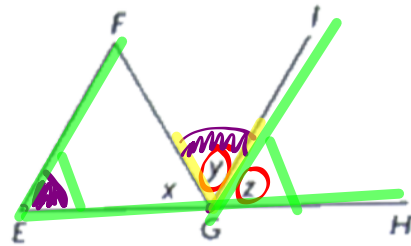
SAT
Rearranged

SAT
Rearranged

Transitive

Example #2:

In $\triangle EFG$, GI bisects $\angle FGH$ *2 equal pieces*
 a) If $\angle E = \angle y$, then prove that $EF \parallel GI$

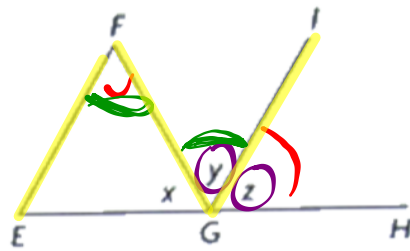


Statement	Justification
$\angle y = \angle z$	Bisected (given)
$\angle E = \angle y$	given
$\angle E = \angle z$	Transitive
$\therefore EF \parallel GI$	CA

"Therefore"

In $\triangle EFG$, GI bisects $\angle FGH$

b) If $\angle F = \angle z$, then prove that $EF \parallel GI$



Statement	Justification
$\angle y = \angle z$	Given (Bisected)
$\angle F = \angle z$	Given
$\angle y = \angle F$	Transitive
$\therefore EF \parallel GI$	$\angle \simeq \angle$

APPLY the Math

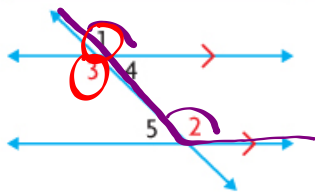
EXAMPLE 1
p. 75

Reasoning about conjectures involving angles formed by transversals

Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.

Tuyet's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the **alternate interior angles** are equal.



I drew two parallel lines and a transversal as shown, and I numbered the angles. I need to show that $\angle 3 = \angle 2$.

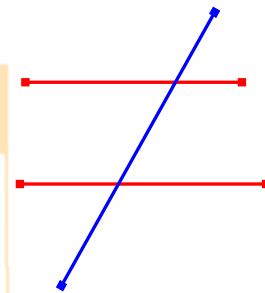
Statement	Justification
$\angle 1 = \angle 2$	Corresponding angles
$\angle 1 = \angle 3$	Vertically opposite angles
$\angle 3 = \angle 2$	Transitive property
My conjecture is proved.	

*Vo.A
O.A.T*

Since I know that the lines are parallel, the corresponding angles are equal.

When two lines intersect, the opposite angles are equal.

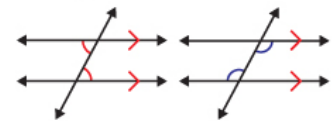
$\angle 2$ and $\angle 3$ are both equal to $\angle 1$, so $\angle 2$ and $\angle 3$ are equal to each other.



Pull for Lesson Notes

alternate interior angles

Two non-adjacent interior angles on opposite sides of a transversal.

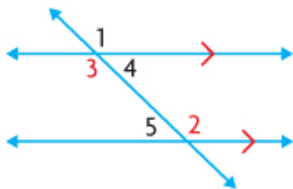


EXAMPLE 1 Reasoning about conjectures involving angles formed by transversals

Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.

Ali's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the interior angles on the same side of the transversal are supplementary.



$$\angle 1 = \angle 2$$

$$\angle 2 + \angle 5 = 180^\circ$$

$$\angle 1 + \angle 5 = 180^\circ$$

$$\angle 1 = \angle 3$$

$$\angle 3 + \angle 5 = 180^\circ$$

My conjecture is proved.

I need to show that $\angle 3$ and $\angle 5$ are supplementary.

Since the lines are parallel, the corresponding angles are equal.

These angles form a straight line, so they are supplementary.

Since $\angle 2 = \angle 1$, I could substitute $\angle 1$ for $\angle 2$ in the equation.

Vertically opposite angles are equal. Since $\angle 1 = \angle 3$, I could substitute $\angle 3$ for $\angle 1$ in the equation.

Homework...

p. 72: #4-6

p. 78: #2, 8, 10, 12, 20

.