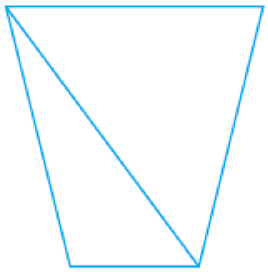
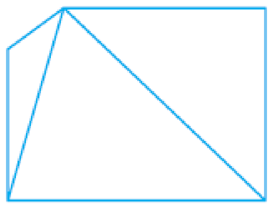
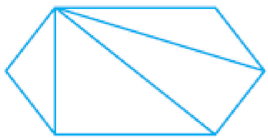
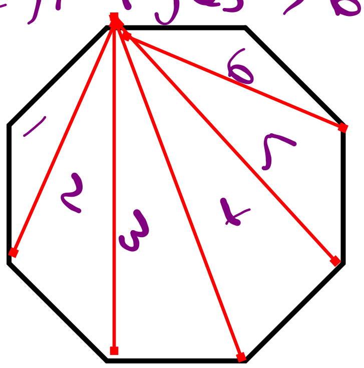
6. Use the evidence given in the chart below to make a conjecture.
Provide more evidence to support your conjecture.

4 5 6

Polygon	quadrilateral	pentagon	hexagon
Fewest Number of Triangles	 2	 3	 4

Conjecture → The # of triangles is always 2 less than the # of sides

8 sided
triangles ⇒ 6



1.3 Using Reasoning to Find a Counterexample to a Conjecture

GOAL

Invalidate a conjecture by finding a contradiction.

To restate what you have read so far, a conjecture is a mathematical statement that has been proposed as a true statement, but not yet proven or disproved.

Once a conjecture is proven, it is a mathematical **fact**.

One method to test a conjecture is to attempt to **disprove** it by using a **counterexample**.

For example:

Conjecture: All prime numbers are odd.
Counterexample: But 2 is a prime number.

The counterexample disproves the conjecture, hence we can conclude that not all prime numbers are odd.

EXAMPLE #2:

$x = 3 \le x^2 = 9$

$x = -5 \le x^2 = 25$

Conjecture:

For all real numbers *x*, the expressions x^2 is greater than or equal to *x*

$x = 0.5 > x^2 = 0.25$

Conjecture: For all real numbers *x*, the expressions x^2 is greater than or equal to *x*

Here is a counterexample:

$(0.5)^2 = 0.25$, and 0.25 is **not** greater than or equal to 0.5

In fact, any number between 0 and 1 is a counterexample. The conjecture is false.

COUNTEREXAMPLE???

PAGE 21

EXAMPLE 3 Using reasoning to find a counterexample to a conjecture

Matt found an interesting numeric pattern:

$1 \cdot 8 + 1 = \mathbf{9}$

$12 \cdot 8 + 2 = \mathbf{98}$

$123 \cdot 8 + 3 = \mathbf{987}$

$1234 \cdot 8 + 4 = \mathbf{9876}$

Matt thinks that this pattern will continue.

Search for a counterexample to Matt's conjecture.

Kublu's Solution

$\mathbf{1} \cdot 8 + \mathbf{1} = 9$

$\mathbf{12} \cdot 8 + \mathbf{2} = 98$

$\mathbf{123} \cdot 8 + \mathbf{3} = 987$

$1234 \cdot 8 + \mathbf{4} = 9876$

The pattern seemed to be related to the first factor (the factor that wasn't 8), the number that was added, and the product.

	A	B
1	1 · 8 + 1	9
2	12 · 8 + 2	98
3	123 · 8 + 3	987
4	1234 · 8 + 4	9876
5	12345 · 8 + 5	98765
6	123456 · 8 + 6	987654
7	1234567 · 8 + 7	9876543
8	12345678 · 8 + 8	98765432
9	123456789 · 8 + 9	987654321

I used a spreadsheet to see if the pattern continued.

The spreadsheet showed that it did.

$12345678\mathbf{910} \cdot 8 + \mathbf{10} = 98\,765\,431\,290$

$123456789\mathbf{0} \cdot 8 + \mathbf{10} = 9\,876\,543\,130$

$12345678\mathbf{910} \cdot 8 + \mathbf{0} = 98\,765\,431\,280$

$123456789\mathbf{0} \cdot 8 + \mathbf{0} = 9\,876\,543\,120$

When I came to the tenth step in the sequence, I had to decide whether to use 10 or 0 in the first factor and as the number to add. I decided to check each way that 10 and 0 could be represented.

The pattern holds true until 9 of the 10 digits are included. At the tenth step in the sequence, a counterexample is found.

Since the pattern did not continue, Matt's conjecture is invalid.

Revised conjecture: When the value of the addend is 1 to 9, the pattern will continue.

I decided to revise Matt's conjecture by limiting it.

Your Turn

If Kublu had not found a counterexample at the tenth step, should she have continued looking? When would it be reasonable to stop gathering evidence if all the examples supported the conjecture? Justify your decision.

Answer

If Kublu had not found a counterexample at the 10th step, she could have still stopped there. With the quantity of evidence found to support the conjecture, and a two-digit number further validating the conjecture, the conjecture could be considered strongly supported. If she had wanted to do one more example, then it might have been logical to try a three-digit number to see if the conjecture was valid in that case.

Mathematical HISTORY???

Goldbach's Conjecture

One famous example of an unproven conjecture has remained undecided for nearly 300 years.

In the early 1700's, Christian Goldbach, a Prussian mathematician, noticed that many even numbers greater than 2 can be written as the sum of two primes. Expanding on examples like these, Goldbach wrote the following conjecture:

4 = 2 + 2	10 = 3 + 7	16 = 3 + 13
6 = 3 + 3	12 = 5 + 7	18 = 5 + 13
8 = 3 + 5	14 = 3 + 11	20 = 3 + 17

Conjecture: Every even number greater than 2 can be written as the sum of two primes.

To this day, no one has proven **Goldbach's Conjecture** or found a counterexample to show that it is false. It is still unknown whether this conjecture is true or false. It is known, however, that all even numbers up to 4×10^{18} confirm Goldbach's Conjecture.

Christian Goldbach (1690 – 1764) was a German mathematician famous for his eponymous Conjecture. Goldbach's Conjecture is one of the most infamous problems in mathematics, and states that every even integer number greater than 2 can be expressed as the sum of two prime numbers. For example, 4=2+2, 6=3+3, and 8=3+5. While there have not been any counter-examples found up through 4×10^{18}(as of 2012), the conjecture has not yet been formally proven.

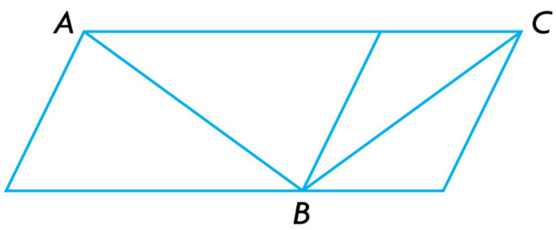
1.2 Exploring the Validity of Conjectures

GOAL

Determine whether a conjecture is valid.

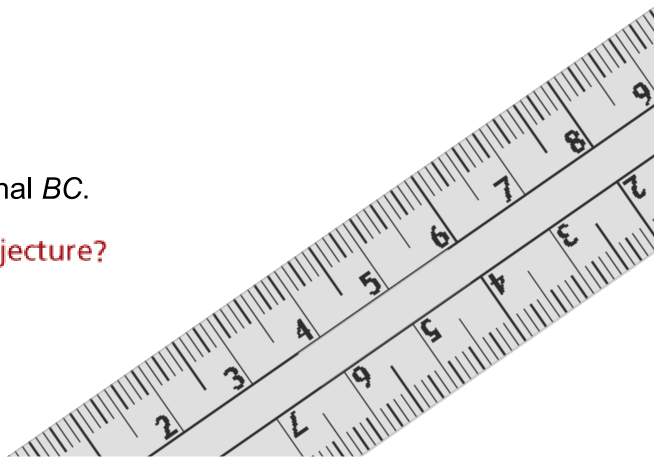
EXPLORE the Math p. 16

Your brain can be deceived.



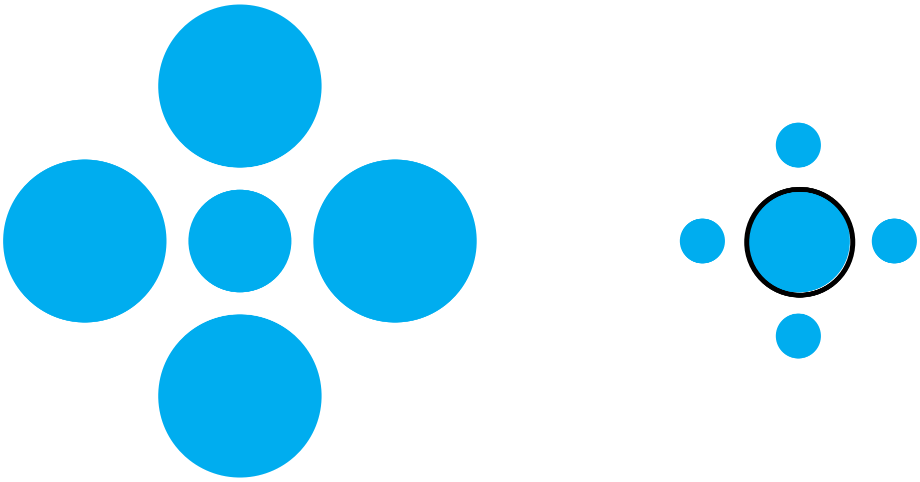
Make a conjecture about diagonal *AB* and diagonal *BC*.

? How can you check the validity of your conjecture?



EXPLORE the Math

Your brain can be deceived.

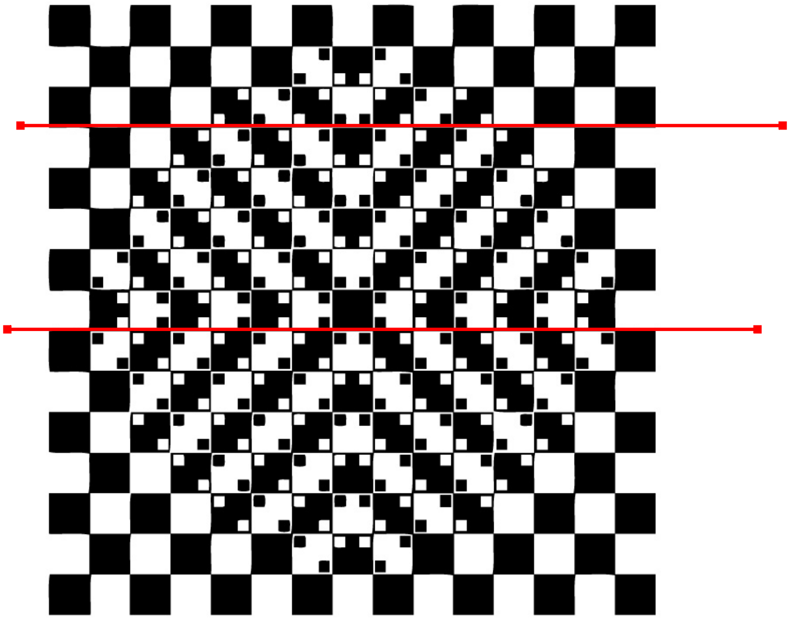


Make a conjecture about the circles in the centre.

? How can you check the validity of your conjecture?

EXPLORE the Math

Your brain can be deceived.

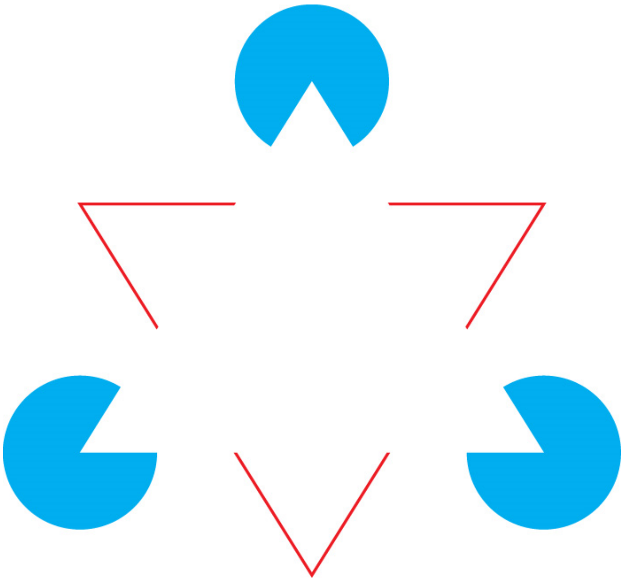


Make a conjecture about the lines.

❓ How can you check the validity of your conjecture?

EXPLORE the Math

Your brain can be deceived.



Make a conjecture about the number of triangles.

How can you check the validity of your conjecture?

Reflecting

A. Describe the steps you took to verify your conjectures.

B. After collecting evidence, did you decide to revise either of your conjectures? Explain.

C. Can you be certain that the evidence you collect leads to a correct conjecture? Explain.

Answers

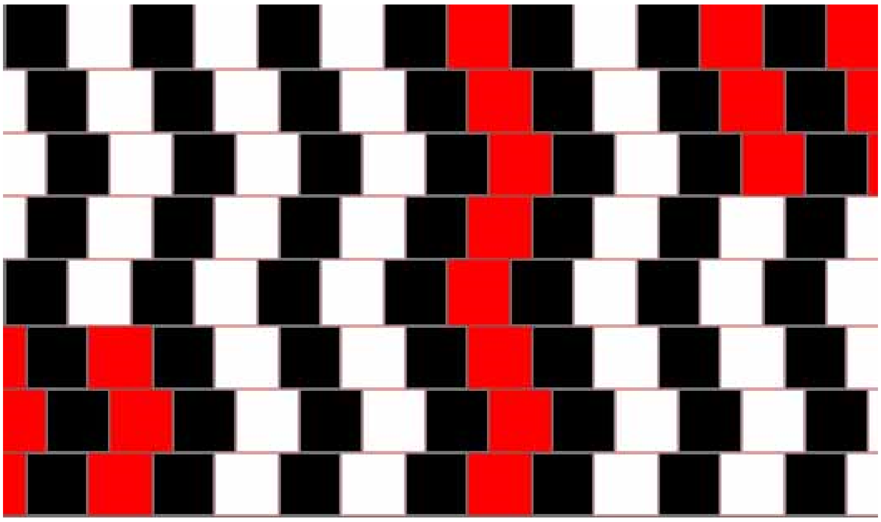
A. Both measurement and visual inspection helped to verify or discredit the conjectures.

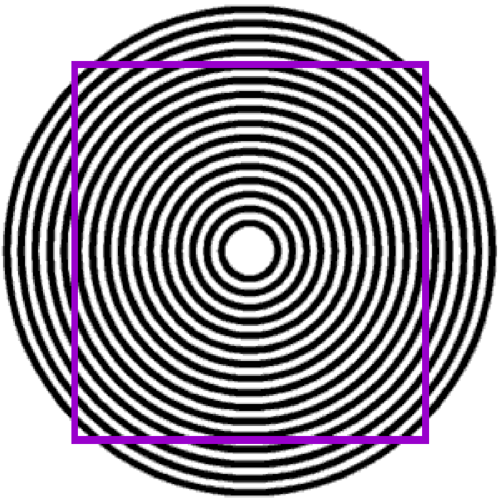
B. My conjectures changed as follows after collecting more evidence:

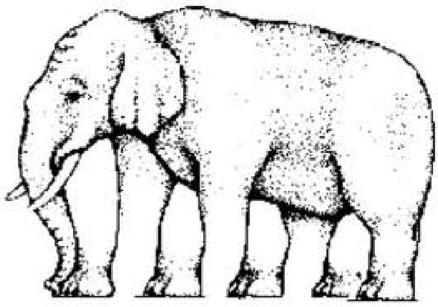
- First image: Both diagonals are the same length.
- Second image: The centre circles of the figures are the same size.
- Third image: The rows and columns of white and black shapes are placed in straight lines.
- Fourth image: There are no triangles in the figure.

C. For these images, the revised conjectures hold true for the accuracy of the tools I used. I cannot be absolutely sure that my new conjectures are valid until the precision of the tools is considered.

Some other optical illusions...







M. C. Escher...

http://www.mcescher.com/

Three Dragons

three_dragons.wmv

1.2 - Validity of Conjectures?

In Summary page 17

Key Idea

- Some conjectures initially seem to be valid, but are shown not to be valid after more evidence is gathered.

Need to Know

- The best we can say about a conjecture reached through inductive reasoning is that there is evidence either to support or deny it.
- A conjecture may be revised, based on new evidence.

1.3 - Counterexamples

In Summary page 22

Key Ideas

- Once you have found a counterexample to a conjecture, you have disproved the conjecture. This means that the conjecture is invalid.
- You may be able to use a counterexample to help you revise a conjecture.

Need to Know

- A single counterexample is enough to disprove a conjecture.
- Even if you cannot find a counterexample, you cannot be certain that there is not one. Any supporting evidence you develop while searching for a counterexample, however, does increase the likelihood that the conjecture is true.

HOMEWORK...

p. 17: #1 & 2
p. 22: #1, 3, 5, 8, 12, 17

Attachments

1s3e3 final.mp4

three_dragons.wmv